Calibrating the (RBC + Solow) Model

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STEADY STATE

- Deterministic steady state the natural point of approximation
- Shut down all shocks and set exogenous variables at their means
- The idea: let economy run for many (infinite) periods
  - Time eventually "doesn't matter" any more
  - Drop all time indices

$$-\frac{u_n(\bar{c},\bar{n})}{u_c(\bar{c},\bar{n})} = \bar{z}F_n(\bar{k},\bar{n})$$

$$u_c(\bar{c},\bar{n}) = \beta u_c(\bar{c},\bar{n})\left[\bar{z}F_k(\bar{k},\bar{n}) + 1 - \delta\right]$$

$$\bar{c} + \delta\bar{k} = \bar{z}F(\bar{k},\bar{n})$$

$$\ln\bar{z} = (1-\rho_z)\ln\bar{z} + \rho_z\ln\bar{z} \Rightarrow \bar{z} = \bar{z} \quad \text{(a parameter of the model)}$$

- Given functional forms and parameter values, solve for $(\bar{c},\bar{n},\bar{k})$
  - The steady state of the model
  - Taylor expansion around this point

CALIBRATION – PHILOSOPHY

- An economic model is a measuring device
- If model makes "believable" predictions along some important dimensions (i.e., "matches some key data")…
- …then maybe its predictions are "believable" along the novel dimensions of the model
- Getting some "partial derivatives" of the model in known directions correct…
- …may build credibility that its "partial derivatives" in novel directions are at least not grossly incorrect
- Make model match some data of interest – often long-run (i.e., time-averaged data) growth facts
  - Preferably well-accepted "stylized facts"
  - Solow growth model in the background
  - Natural candidate: Kaldor growth facts
- Calibration vs. Estimation

CALIBRATION OF BASELINE RBC MODEL

- **Must take a stand on three (related) points**
  - **Which data do we want model to match? (even constructing data is challenging…)**
  - **Functional forms (utility, production)**
  - **Parameter values**

- **Choose functional forms consistent with "Kaldor-plus facts"**
  - **(K1) Capital income share and labor income share of GDP are stationary**
  - **(K2) All real quantity variables grow at same rate in the long run**
  - **(K3) Real interest rate is stationary**
  - **(K4) Hours per worker are stationary**
  - **(K5) (K2) requires trend productivity to be labor-augmenting (Phelps 1966)**

- **Often start with RBC model that abstracts from long-run growth**

- **But "true" calibration begins with model featuring *only* long-run growth**
  - **Puts restrictions on instantaneous utility and production forms**
  - **Use (K1)-(K5) to obtain these restrictions**

- **Richer models: more calibration targets and/or treating data differently**
  - **Monopoly markups (e.g., Dixit-Stiglitz and sticky price models)**
  - **Probability of finding a job (e.g., labor matching models)**
  - **Durable consumption vs. non-durable consumption**

RBC Model with Growth

- **Absent shocks, TFP grows at deterministic rate γ**
- **Planner problem / perfect competition**

$$\max E_0 \sum_{t=0}^{\infty} b^t u(C_t, n_t) \quad \text{subject to}$$

Trend productivity is labor-augmenting (Harrod-neutral) (Makes use of fact (K5))

$$C_t + K_{t+1} - (1-\delta)K_t = z_t F(K_t, n_t X_t)$$

Flow resource constraint

$$X_t = \gamma X_{t-1}, \qquad \gamma \geq 1$$

Evolution of deterministic component of productivity

Red indicates variables or parameters that will be modified when detrending the model

given stochastic process for evolution of z_t and (K_{-1}, z_0, X_0)

- **Suppose z_t = 1 always, so only deterministic growth**
- **Deterministic dynamics of (C_t, K_{t+1}, n_t, X_{t+1}) governed by**

(1) $$-\frac{u_n(C_t, n_t)}{u_c(C_t, n_t)} = X_t F_2(K_t, n_t X_t)$$

Labor supply function (aka consumption-labor optimality)

(2) $$\frac{u_C(C_t, n_t)}{b u_C(C_{t+1}, n_{t+1})} = F_1(K_{t+1}, n_{t+1} X_{t+1}) + 1 - \delta$$

Capital supply function (aka consumption-savings optimality)

(3) $$C_t + K_{t+1} - (1-\delta)K_t = F(K_t, n_t X_t)$$

(4) $$X_t = \gamma X_{t-1}$$

Normalize X_0 = 1

RESTRICTIONS ON FUNCTIONAL FORMS

- **Deterministic dynamics of (C_t, K_{t+1}, n_t, X_t) governed by**

(1) $$-\frac{u_n(C_t, n_t)}{u_c(C_t, n_t)} = X_t F_2(K_t, n_t X_t)$$

(3) $$C_t + K_{t+1} - (1-\delta)K_t = F(K_t, n_t X_t)$$

(2) $$\frac{u_C(C_t, n_t)}{b u_C(C_{t+1}, n_{t+1})} = F_1(K_{t+1}, n_{t+1} X_{t+1}) + 1 - \delta$$

(4) $$X_t = \gamma X_{t-1}$$

- **(K1) Capital income share and labor income share of GDP are stationary**
  And viewing economic profits as zero

$$\Rightarrow F(K, nX) = K^{\alpha}(nX)^{1-\alpha} \qquad (\alpha \approx 0.4)$$

Restrictions on Functional Forms

- **Deterministic dynamics of (C_t, K_{t+1}, n_t, X_t) governed by**

(1) $$-\frac{u_n(C_t,n_t)}{u_c(C_t,n_t)}=(1-\alpha)X_t\left(\frac{K_t/X_t}{n_t}\right)^{\alpha}$$

(2) $$\frac{u_C(C_t,n_t)}{bu_C(C_{t+1},n_{t+1})}=\alpha\left(\frac{K_{t+1}/X_{t+1}}{n_{t+1}}\right)^{\alpha-1}+1-\delta$$

(3) $$C_t+K_{t+1}-(1-\delta)K_t=K_t^{\alpha}(n_tX_t)^{1-\alpha}$$

(4) $$X_t=\gamma X_{t-1}$$

- **(K2) All real quantity variables grow at same rate in the long run**

$$\Rightarrow \frac{Y_{t+1}}{Y_t}=\frac{C_{t+1}}{C_t}=\frac{K_{t+1}}{K_t}=\frac{X_{t+1}}{X_t}=\gamma, \quad \forall t$$

$$\Rightarrow y_t\equiv\frac{Y_t}{X_t}=\overline{y}, \quad k_t\equiv\frac{K_t}{X_t}=\overline{k}, \quad c_t\equiv\frac{C_t}{X_t}=\overline{c}, \quad \forall t$$

- **And scale (3) by X_t to make stationary**

$$-\frac{u_n(C_t,n_t)}{u_c(C_t,n_t)}=(1-\alpha)X_t\left(\frac{k_t}{n_t}\right)^{\alpha}$$

$$\frac{u_C(C_t,n_t)}{bu_C(C_{t+1},n_{t+1})}=\alpha\left(\frac{k_{t+1}}{n_{t+1}}\right)^{\alpha-1}+1-\delta$$

Note long run growth rate affects capital accumulation even in stationary representation!

$$c_t+\gamma k_{t+1}-(1-\delta)k_t=k_t^{\alpha}n_t^{1-\alpha}$$

$$X_t=\gamma X_{t-1}$$

Restrictions on Functional Forms

- **Deterministic dynamics of (C_t, n_t, X_t) governed by**

(1) $$-\frac{u_n(C_t,n_t)}{u_c(C_t,n_t)} = (1-\alpha)X_t\left(\frac{\bar{k}}{n_t}\right)^{\alpha}$$

(3) $$\bar{c} + \gamma\bar{k} - (1-\delta)\bar{k} = \bar{k}^{\alpha} n_t^{1-\alpha}$$

(2) $$\frac{u_C(C_t,n_t)}{bu_C(C_{t+1},n_{t+1})} = \alpha\left(\frac{\bar{k}}{n_{t+1}}\right)^{\alpha-1} + 1 - \delta$$

(4) $$X_t = \gamma X_{t-1}$$

- **(K4) Hours per worker are stationary**

$$\Rightarrow n_t = \bar{n}$$

along deterministic path.

BUT $\bar{n}$ is endogenous…

RESTRICTIONS ON FUNCTIONAL FORMS

- Deterministic dynamics of C_t governed by

(1) $$-\frac{u_n(C_t,\bar{n})}{u_c(C_t,\bar{n})}=(1-\alpha)X_t\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha}$$

(3) $$\bar{c}+\gamma\bar{k}-(1-\delta)\bar{k}=\bar{k}^{\alpha}\bar{n}^{1-\alpha}$$

(2) $$\frac{u_C(C_t,\bar{n})}{bu_C(C_{t+1},\bar{n})}=\alpha\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha-1}+1-\delta$$

- Implied already (RHS of (2)) is
  - (K3) Real interest rate is stationary
- Final step – functional form for utility?
- Observations
  - Optimal choice of labor ($\bar{n}$) must be independent of X_t (from (1))
    - Requires offsetting income and substitution effects of wages (long-run productivity) on labor supply
  - IMRS can only depend on C_{t+1}/C_t (from (2)), which in turn = γ
- Two requirements together imply

$$u(C_t,n_t)=\begin{cases}\dfrac{[C_t v(n_t)]^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma>0,\ \sigma\neq 1\\ \ln C_t+v(n_t) & \text{if } \sigma=1\end{cases}$$

King, Plosser, Rebelo (1988 *JME*)

STEADY STATE

- Steady state $\left(\bar{c},\bar{n},\bar{k}\right)$ solves (1), (2), (3)
- A dynamic phenomenon!
  - Not static!
  - Economy is moving exactly along its long-run (i.e., deterministic) growth path
  - Balanced growth path
- Scale of absolute quantity outcomes within model is meaningless
  - What does, e.g., $\bar{c}=1.56$ mean?
- Relative quantity outcomes are interpretable
  - Provide calibration targets
  - e.g., $\bar{c}/\bar{y}=0.70, \quad \bar{k}/\bar{y}=2.5$ (if annual measurement)
- Time use and intertemporal price outcomes within model are interpretable
  - Provide calibration targets
  - $\bar{n}$ is fraction of time spent in paid market work
    - Empirical: $n \approx 0.30$
  - Return on capital $\alpha\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha-1}+1-\delta$
- Use ss calibration targets to set parameter values, given functional forms

RBC MODEL WITHOUT GROWTH

- Often start instead with

$$u(c_t, n_t) = \begin{cases} \frac{[c_t v(n_t)]^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma > 0,\ \sigma \neq 1 \\ \ln c_t + v(n_t) & \text{if } \sigma = 1 \end{cases}$$ and $$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

$c_t = C_t / X_t$

- For this transformed model to deliver same steady state (relative quantities, time use, and *r*), require
  - Resource constraint $c_t + \gamma k_{t+1} - (1-\delta)k_t = z_t k_t^{\alpha} n_t^{1-\alpha}$
  - Subjective discount factor $\beta \equiv b\gamma^{1-\sigma}$ (King and Rebelo, p. 945)
- Typical assumption $\gamma = 1$ omits growth altogether
- What if trend growth rate fluctuates, γ_t?
  - Typical representation cannot accommodate trend shocks because $\gamma = 1$
  - Trend shocks fairly common in small-open-economy models
  - Affects discount factor and capital accumulation equation

BASELINE RBC MODEL

- **Assuming γ = 1…**
- **…complete calibration?**

- **Data: long-run labor income share of GDP ≈ 0.60**
  - **Cobb-Douglas *F*(.) implies**

$$\frac{wn}{F(k,n)} = \frac{(1-\alpha)k^{\alpha}n^{1-\alpha}}{k^{\alpha}n^{1-\alpha}} = 1-\alpha \quad \Rightarrow \quad \alpha = 0.40$$

- **Data: long-run ratio of (annual) gross investment to capital stock ≈ 0.07**

$$\frac{k-(1-\delta)k}{k} = \frac{\delta k}{k} = 0.07 \quad \Rightarrow \quad \delta = 0.07 \ \text{(annual) or } 0.018 \ \text{(quarterly)}$$

- **Data: long-run ratio of (annual) output to capital stock ≈ 0.4**
  - **Steady-state Euler equation**

$$1 = \beta\left[\frac{\alpha k^{\alpha}n^{1-\alpha}}{k} + 1 - \delta\right] = \beta\left[\frac{\alpha F(k,n)}{k} + 1 - \delta\right] \quad \Rightarrow \quad \beta = 0.95 \ \text{(annual) or } 0.99 \ \text{(quarterly)}$$

OR **Data: avg. net real return on capital ≈ 5% per year (e.g, return on S&P500)**

- **Steady-state Euler equation**

$$f_k(k,n) = \frac{1}{\beta} - 1 + \delta \quad \Rightarrow \quad \beta = 0.96 \ \text{(annual) or } 0.99 \ \text{(quarterly)}$$

BASELINE RBC MODEL

- **Utility parameters**

$$u(c_t,n_t)=\begin{cases}\dfrac{[c_t v(n_t)]^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma>0,\ \sigma\neq 1\\ \ln c_t+v(n_t) & \text{if } \sigma=1\end{cases}$$

- **Data: IES is around unity(?) or lower**
  - **Implies σ > 1**
  - **(Recall: IES = 1/σ for time-separable CRRA utility)**
  - **σ = 1 a conventional value**

- **Labor subutility**
  - **Common form**

$$v(n)=-\frac{\psi}{1+1/\eta}n^{1+1/\eta}$$

  - **η measures Frisch elasticity of labor supply (use C-L optimality condition)**
  - **Calibrate Ψ to hit** $\bar{n}\approx 0.3$
  - **Empirical evidence on Frisch elasticity?**

BASELINE RBC MODEL

- **Labor supply elasticity "controversial"**

- **Micro evidence: very low – η (substantially) smaller than one**
- **Macro evidence: very high – η (substantially) larger than one**

- **"Tension" between macro and micro evidence not useful way to frame the "controversy"**

- **Micro studies pick up intensive margin of labor supply**
- **Macro studies pick up (mostly) extensive margin of labor supply**
  - **And other frictions in allocation of workers to jobs…**

- **Chetty (2011 *Econometrica*): uses both macro and micro evidence to put bounds on labor supply elasticity**

- **Common in DSGE models: $\eta > 1$**

BASELINE RBC MODEL

- **Exogenous process for TFP (deviations from long-run trend productivity)**

$$\ln z_{t+1} = (1-\rho_z)\ln \bar{z} + \rho_z \ln z_t + \varepsilon_{t+1}$$

$$\varepsilon_{t+1} \sim \text{iid } N(0,\sigma_z^2)$$

- **Normalize** $\bar{z} = 1$
  - **Only governs absolute scale of model, which is arbitrary**
  - **What does, e.g.,** $\bar{c} = 1.56$ **mean?**

- **Construct time-series for** z_t **using**
  - **Data on labor, (detrended) capital, and (detrended) output**

- **AR(1) estimation**
  - **Quarterly frequency**

$$\Rightarrow \rho_z = 0.95 \quad \text{and} \quad \sigma_z = 0.006$$

USING THE RBC (OR ANY DSGE) MODEL

1. Dream up / construct / write fully-articulated model
   - Ideally to answer questions motivated by data and with hypotheses

2. Choose parameter values
   - Perhaps extremely rigorously, if goal is to match certain empirical facts very precisely
   - Perhaps adopting generally-accepted values, if goal is to illustrate some insight

3. Solve for deterministic steady state (balanced growth path)

4. Solve for dynamic decision rules (e.g., linear approximation, second-order approximation, global approximation)

5. Conduct informative battery of experiments (impulse responses, simulations, etc.) to try to falsify hypotheses

6. Tabulate results, write a (good!) paper, get it published