Simple DSGE Models of "Money"
Part I

October 3, 2013

Basic Issues

- Money / monetary policy issues an enduring fascination in macroeconomics
- How can / should central bank "control" the economy? Should it / can it at all?
- Roles of "money"
  - Medium of exchange (transactions role)
  - Unit of account (numeraire role)
  - Store of value (asset role)

Highlighted in CIA, MIU, and money-search approaches

Highlighted in New Keynesian approach

- How to "model money" in DSGE environment?
  - Which role to model?
  - Which role is tractable to model?
  - Which role is most relevant for conduct of monetary policy?

Households

- **Household optimization** $\max_{c_t, n_t, M_t, B_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$

Lump-sum monetary injection / contraction

s.t.

P_t the nominal price of c_t – equivalently, the nominal price level

$$P_t c_t + M_t + B_t = P_t w_t n_t + M_{t-1} + (1 + i_{t-1}) B_{t-1} + T_t$$

Flow budget constraint

Nominal consumption spending

Nominal money holdings

Nominal bond holdings

Nominal labor income

Nominal interest rate (on previously-accumulated nominal bonds)

$$P_t c_t \leq M_t$$

"Cash-in-advance" (CIA) constraint

- "forces" consumers to "hold money"

- **CIA constraint a friction on economy**
  - Pareto-optimal allocations do not require it
  - Money not "essential" as in models of Kiyotaki and Wright (1993), Lagos and Wright (2005)

Does not ENDOGENOUSLY EXPAND consumers' set of feasible trades. Because underlying DSGE model features full set (including over *all* state-date pairs) of Arrow-Debreu securities – complete markets! Trade does not require "money"…

SIMPLE POLICY ANALYSIS

- Removing monetary friction…
  - ….requires an allocation that features a zero multiplier on CIA constraint…
  - …implies zero nominal interest rate

- Friedman Rule
  - Benchmark result in monetary theory
  - Completely relaxing "monetary friction" requires eliminating any (opportunity) cost of holding money
  - Other Interpretations
    - Eliminate the wedge between alternative nominal assets: i = 0 makes money and nominal bonds equivalent assets (in terms of their cost and benefit properties)
    - Eliminate the wedge in the consumption-leisure optimality condition
  - Are monetary frictions empirically important?…and thus, is the Friedman Rule of practical use for advising monetary policy?

Really the same thing…

SIMPLE POLICY ANALYSIS

- **Household optimality conditions**

hh multiplier on CIA constraint

hh multiplier on budget constraint

$$\phi_t = \lambda_t \left[\frac{i_t}{1+i_t} \right]$$

No-arbitrage between money and nominal bonds

(Assumption: i_t in the information set of time t)

SIMPLE POLICY ANALYSIS

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Note disutility of labor appears in intertemporal MRS…

$$-\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)} = w_t \left[1 + \frac{i_t}{1+i_t} \right]^{-1}$$

Consumption-leisure optimality condition

- relative price depends on w_t AND i_t

If monetary friction were "shut down," would have u_c here "as usual."

Either through Friedman Rule or through "cashless" New Keynesian environment (later…)

Efficiency requires C-L optimality depends on real wage….

…but not on monetary aspects of economy (non-technology)

Friedman Rule achieves Pareto efficiency along this margin

$$\frac{u_n(c_t,n_t)}{w_t} = (1+i_t)\beta E_t \left[\frac{u_n(c_{t+1},n_{t+1})}{w_{t+1}} \cdot \frac{P_t}{P_{t+1}} \right]$$

Consumption-savings optimality condition (aka bond Euler equation) (aka Fisher equation!)

Simple Policy Analysis

- **Household optimality conditions (continued)**

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Consumption-savings optimality condition
(aka bond Euler equation)
(aka Fisher equation)

$$c_t = \frac{M_t}{P_t}$$

Binding CIA constraint

Obvious if $i_t > 0$ (why hold excess money?)

Also assume it even in states where $i_t = 0$: pins down a monetary equilibrium level of M_t, hence is an equilibrium selection device

Simple Policy Analysis

- **Household optimality conditions (continued)**

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- **Rest of the environment**
  - w_t = marginal product of labor (linear production + competitive factor market)
  - Govt budget: $T_t = M_t - M_{t-1} = (1+\mu_t)M_{t-1}$ Resource constraint: $c_t = z_t n_t$

SIMPLE POLICY ANALYSIS

- **Household optimality conditions (continued)**

$$\phi_t = \lambda_t \left[\frac{i_t}{1+i_t} \right]$$

No-arbitrage between money and nominal bonds

Define

$\pi_{t+1} = P_{t+1} / P_t - 1$

$\mu_{t+1} = M_{t+1} / M_t - 1$

$$-\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)} = w_t \left[1 + \frac{i_t}{1+i_t} \right]^{-1}$$

Consumption-leisure optimality condition

$$\frac{u_n(c_t,n_t)}{w_t} = (1+i_t)\beta E_t \left[\frac{u_n(c_{t+1},n_{t+1})}{w_{t+1}} \cdot \frac{1}{1+\pi_{t+1}} \right]$$

Consumption-savings optimality condition
(aka bond Euler equation)
(aka Fisher equation)

Combine t and t-1 (binding) CIA constraints $\longrightarrow$

$$\frac{c_t}{c_{t-1}} = \frac{1+\mu_t}{1+\pi_t}$$

Equilibrium link between money growth and inflation

Articulates a quantity-theoretic channel

- **Rest of the environment**
  - w_t = marginal product of labor (linear production + competitive factor market)
  - Govt budget: $T_t = M_t - M_{t-1} = (1+\mu_t)M_{t-1}$ Resource constraint: $c_t = z_t n_t$

Examine steady-state equilibrium

Simple Policy Analysis

- **Household optimality conditions in deterministic steady state**

$$\phi = \lambda\left[\frac{i}{1+i}\right]$$

No-arbitrage between money and nominal bonds

$$-\frac{u_n(c,n)}{u_c(c,n)} = w\left[1+\frac{i}{1+i}\right]^{-1}$$

Consumption-leisure optimality condition

Friedman Rule: $i = 0 \rightarrow \pi = \beta - 1$

BUT ONLY IN STEADY STATE!

NOT (necessarily) dynamically….

$$1+\pi = \beta(1+i)$$

Consumption-savings optimality condition
(aka bond Euler equation)
(aka Fisher equation)

…and optimal policy calls for $\mu = \beta - 1$

(i.e., SHRINK nominal money supply!)

$$1 = \frac{1+\mu}{1+\pi}$$

Equilibrium link between money growth and inflation

Articulates a quantity-theoretic channel

- **Rest of the environment**
  - ***w* = marginal product of labor (linear production + competitive factor market)**
  - **Govt budget:** $T/P = (1+\mu)(M/P)(1/(1+\pi))$ **Resource constraint:** $c = n$

OTHER ANALYSIS

- Other aspects of equilibrium

Imply $\varphi < 0$, i.e., money NOT valued for exchange

  - $\mu < \beta - 1$ (in steady-state!) inconsistent with monetary equilibrium
  - Dynamic analog: $i_t < 0$ inconsistent with monetary equilibrium
    - Zero-lower-bound constraint

- Model's "policy rate" typically identified with a (short-run Euler equation) market interest rate
  - Whether CIA models, MIU models, New Keynesian models, money search models
  - Model mechanism: change in policy rate (potentially) affects intertemporal incentives (i.e., the real interest rate)
  - A valid empirical identification? Term-structure issues? Other issues? See Canzoneri, Cumby, and Diba (2007 *JME*)…

OTHER VARIANTS OF CIA

- **Cash good / credit good model**
  - **Lucas and Stokey (1983)**
  - **Foundation for Ramsey models of optimal fiscal and monetary policy – see Chari and Kehoe (1999 *Macro Handbook* chapter)**
  - **Subset of goods (c_1) require "cash in advance"**
  - **Subset of goods (c_2) do not require cash in advance**

$$\frac{u_{c_1}}{u_{c_2}} = 1 + i$$

$\text{MRS}_{\text{cash / credit}}$ = gross nominal interest rate

Monetary policy creates a STATIC wedge!....

- **Investment in CIA constraint**
  - **Stockman (1981): long-run inflation lowers long-run capital stock**

$$c_t + k_{t+1} - (1-\delta)k_t \leq \frac{M_t}{P_t}$$

- **Basic Idea: Positive nominal interest rate taxes whatever is in the CIA constraint**

ALTERNATIVE MONETARY MODELS

- Alternatives to CIA
  - Money in the utility function (MIU) models

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, \frac{M_t}{P_t}\right)$$

  - Shopping-time & transactions costs models
    - Nominal money holdings reduce "cost" of acquiring goods

Feenstra (1986 *JME*) shows conditions under which CIA, MIU, shopping-time are equivalent

  - Go "cashless"
    - New Keynesian models don't model "money demand" at all (or, at best, as an appendage separate from the "main" equilibrium)

Can think of as "Friedman Rule running in the background"

  - Go for deep micro-foundations
    - Kiyotaki and Wright (1989, 1993)
    - Lagos and Wright (2005), Aruoba, Waller, and Wright (2011 *JME*)