Nominal Rigidities in a DSGE Model: Basic Calvo-Yun Model

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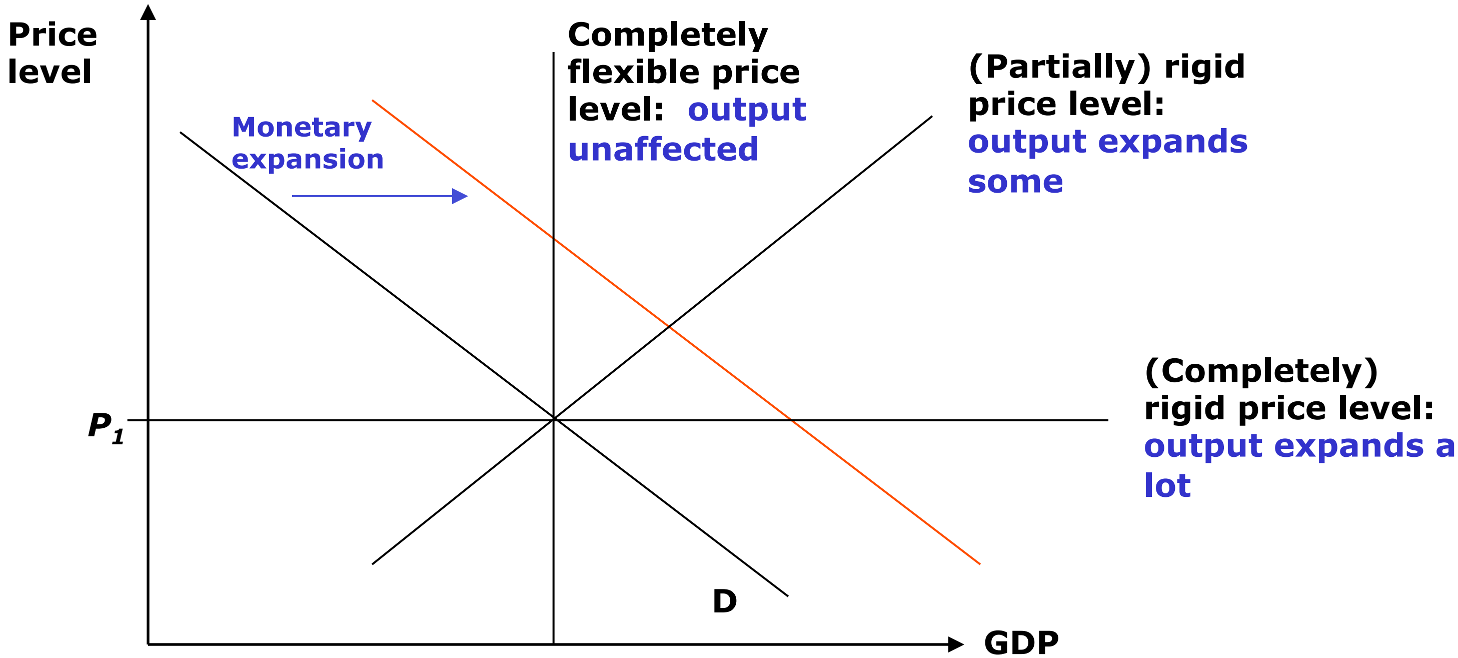
BUSINESS CYCLE IMPLICATIONS OF MONEY

- Conventional Keynesian view: nominal rigidities (in price and/or wage level) cause monetary shifts to have real effects



- Do money demand distortions have a role in policy transmission?
  - Cooley and Hansen (1989, 1991): Not very much…

GENERAL ISSUES

- **How often do prices change empirically?...**
  - **Wide heterogeneity across goods/categories of goods**
    - **Bils and Klenow (2004 *JPE*), Nakamura and Steinsson (2007), Kehoe and Midrigan (2007), Klenow and Krystov (2007), many others...**
  - **Median ~ 2-3 quarters...**

- **...and which price changes are most relevant for macro phenomena?**
  - **Aggregate price level?**
  - **Individual goods/sectors? Thus require a model of heterogeneity in pricing outcomes?**

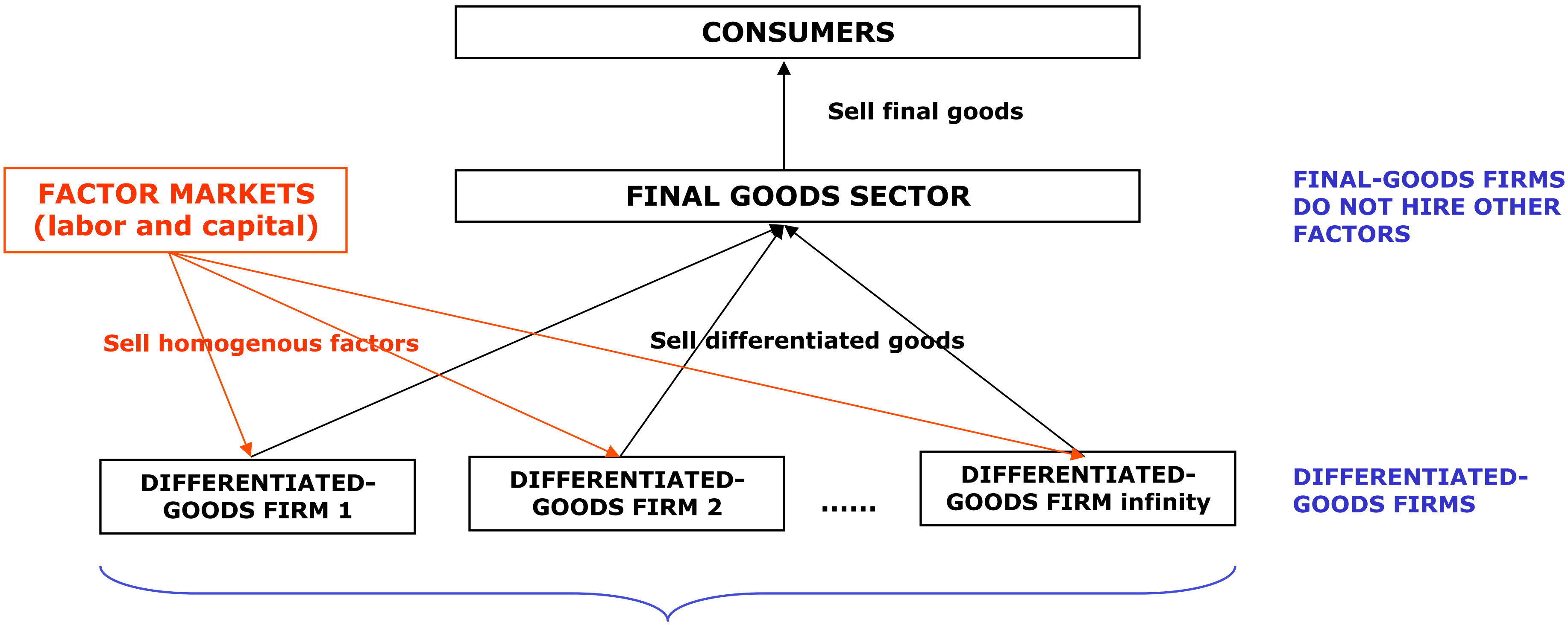
- **How to introduce nominal rigidities in basic DSGE model?**
  - **Time-dependent: firms (re-)set prices according to some exogenous time interval** ← **Much more tractable to model**
  - **State-dependent: firms (re-)set prices according to endogenous (potentially firm-specific) state**

CANONICAL DSGE STICKY-PRICE MODEL

Though events of past few years suggest *liquidity* issues not irrelevant

- **Cashless environment**
  - **Ignore money demand altogether (i.e., no CIA, no MIU, no transactions costs)**
  - **Woodford (2000): "…effectiveness of monetary policy does not depend on the ability of the central bank to manipulate significant market distortions…"**
- **Nominal prices (hence nominal price level) move sluggishly**
  - **Due to some "costs" or other "timing impediments"**
- **Built on Dixit-Stiglitz foundation**
  - **Infrequent (re-)setting of price requires imperfect competition**
  - **D-S framework readily tractable in DSGE environment**
- **Common sticky-price mechanisms**
  - **Calvo-Yun: firm receives exogenous "signal" to re-optimize price**
  - **Taylor: firm can re-optimize price every *T* periods**
  - **Rotemberg: firm can re-optimize price every period, but subject to a quadratic "menu cost"**

CANONICAL DSGE STICKY-PRICE MODEL



Measure-one continuum of firms, each of which can (re-)set its *nominal* price only infrequently (Calvo and Taylor models) or at some cost (Rotemberg model)

Final-Goods Firms

- **Aggregator** $y_t = \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$

- **Nominal profit-maximization problem**

P_t the nominal price of final good; in equilibrium, P_t is nominal *price level*

$$\max_{\{y_{it}\}_{i=0}^{1}} P_t y_t - \int_0^1 P_{it} y_{it} di$$

P_{it} the nominal price of differentiated good *i*

↓ **Substitute in CES final-goods aggregator**

$$\max_{\{y_{it}\}_{i=0}^{1}} P_t \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{it} y_{it} di$$

↓ **Profit-maximization**

$$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \cdot y_t$$

TAKEN AS GIVEN BY DIFFERENTIATED FIRM *i*

Relative price of firm *i*'s output

Aggregate output a *shifter* of firm *i*'s demand function

Each differentiated firm *i* chooses its nominal P_i to maximize profit…but choice of P_i subject to frictions…

DIFFERENTIATED-GOODS FIRMS

- **Two-stage optimization problem**
  - **Stage 1: Choose optimal nominal P_{it} subject to the pricing friction**
  - **(Intermediate "stage"): "choose" to produce the y_{it} corresponding to the implied value of P_{it}/P_t**
  - **Stage 2: Choose factor inputs to produce y_{it} at minimum cost**

i.e., total production y_i is simply "read off the demand curve"

- **Differentiated producer *i* production technology**

Usual CRS

$$y_{it} = z_t f\left(k_{it}, n_{it}\right) - \Phi$$

Assume = 0 as before → mc = ac CONSTANT (with respect to quantity)

- **Dynamic profit maximization problem**

Note distinction between *t* and *s* subscripts!!!

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} \left\{ P_{it} y_{is} - P_s mc_s y_{is} \right\} \right\}$$

***Exogenous probability* of *not* being able to (re-)set price – the "Calvo fairy"**

***Discount factor* between *t* and *s* because *dynamic* firm problem; in equilibrium, = household stochastic discount factor**

Substitute in demand function

Differentiated-Goods Firms

- **Dynamic profit maximization problem**

Price change opportunities arrive according to Poisson process

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} \left\{ P_{it} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s - P_s mc_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \right\} \right\}$$

Rewrite

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} \left\{ P_{it}^{1-\varepsilon} P_s^{\varepsilon} - P_{it}^{-\varepsilon} P_s^{1+\varepsilon} mc_s \right\} y_s \right\}$$

- **First-order condition**

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} \left[(1-\varepsilon) \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s + \varepsilon \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0$$

Continue manipulating

DIFFERENTIATED-GOODS FIRMS

- **Optimal-pricing condition**

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[(1-\varepsilon)+\varepsilon\left(\frac{P_{it}}{P_s}\right)^{-1}mc_s\right]\right\}=0$$

Rewrite: multiply by -1/ε

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}-\left(\frac{P_{it}}{P_s}\right)^{-1}mc_s\right]\right\}=0$$

Rewrite: multiply each term by P_s/P_s and multiply entire expression by P_{it}

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}-mc_s\right]\right\}=0$$

If prices are completely flexible (i.e., if $\alpha = 0$) →

Standard *static* Dixit-Stiglitz pricing condition

$$\frac{P_{it}}{P_t}=\frac{\varepsilon}{\varepsilon-1}\cdot mc_t$$

DIFFERENTIATED-GOODS FIRMS

- Optimal-pricing condition

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\underbrace{\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}}_{\text{Real marginal revenue}}-mc_s\right]\right\}=0$$

- **With sticky prices, optimal P_i balances current and future marginal revenues against current and future marginal costs until the next (expected) price re-optimization**

- **Differentiated firm *i*'s (and hence the aggregate) markup will be time-varying**
  - **As "initial marginal revenues" > "initial marginal costs" to balance against "later marginal revenues" < "later marginal costs"**
  - **See King and Wolman (1999)**

  As inflation erodes the *relative* price of firm *i*

- **Conduct full non-linear analysis (around distorted steady state)**
  - **"Textbook" New Keynesian analysis is around efficient steady state**

DIFFERENTIATED-GOODS FIRMS

- **Optimal-pricing condition**

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}-mc_s\right]\right\}=0$$

- **Define**

$$P_t x_t^1 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}\right\}$$

PDV of nominal marginal revenues until next price change

$$P_t x_t^2 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s mc_s\right\}$$

PDV of nominal marginal costs until next price change

- **Optimal-pricing condition:** $x_t^1 = x_t^2$
  - **Emphasizes that optimal P_i balances current and future *mr* against current and future *mc***

- **Write x_t^1, x_t^2 recursively (following SGU (2005 *NBER Macro Annual*))**