LABOR MATCHING MODELS: BASIC DSGE IMPLEMENTATION

APRIL 12, 2012

FIRM VACANCY-POSTING PROBLEM

- Dynamic firm profit-maximization problem

$$\max_{v_t, n_{t+1}^f} \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(y_t - w_t n_t^f h_t - g(v_t) \right) \right]$$

Discount factor between time 0 and *t* because *dynamic* firm problem; in equilibrium, = household stochastic discount factor

Number of vacancies to post (how many "job advertisements")

Desired target *future* firm employment

Total output – sold in perfectly-competitive goods market

Total wage bill depends on both extensive and intensive employment

Total cost of posting *v* vacancies

  - Subject to (perceived) law of motion for firm's employment stock

- Baseline model
  - Shut down intensive margin: $h_t = 1$
  - Linear posting costs: $g(v) = \gamma v$
  - Firm production function: $y_t = z_t * n_t$
  - Wage-setting (process) taken as given when posting vacancies

Firm Vacancy-Posting Problem

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_{t+1}^f} \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f - w_t n_t^f - \gamma v_t \right) \right]$$

$$\text{s.t. } n_{t+1}^f = (1-\rho^x) n_t^f + v_t k^f(\theta_t)$$

Perceived law of motion for evolution of employment stock

Number of existing jobs that do not end: ρ^x exogenous separation rate, but can also endogenize

Each vacancy has probability $k^f(\theta)$ of attracting a prospective employee: depends on a *market* variable, θ, so taken as given

↓ **FOCs with respect to v_t, n_{t+1}**

$$-\gamma + \mu_t k^f(\theta_t) = 0$$

$$-\mu_t + E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + (1-\rho^x)\mu_{t+1} \right) \right\} = 0$$

↓ **Combine**

FIRM VACANCY-POSTING PROBLEM

- **Vacancy posting condition (aka job creation condition)**

$$\gamma = k^f(\theta_t) E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

γ / k^f is capital value of an existing employee – because one *less* worker firm has to find in the future

EMPLOYEES ARE ASSETS

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of attracting a worker) x (expected future benefit of an additional worker)

= marginal output – wage payment + expected asset value of an additional worker

- **Vacancy-posting is a type of investment decision**
  - **Intertemporal dimension makes discount factor potentially important**
    - **i.e., makes general equilibrium effects potentially important**

- **Two prices affect posting decision (aside from intertemporal price)**
  - **(Future) wage**
  - **Matching probability (loosely interpret probabilities as prices) which depends on the market variable θ**

HOUSEHOLD PROBLEM

- **Dynamic household utility-maximization problem**
  - **A continuum [0, 1] of households (a standard assumption)**
  - **A continuum [0, 1] of atomistic individuals live in each household**
  - **Thus representative household has a continuum of "family members"**

$$\max_{c_t, a_t} \left[E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$\text{s.t.} \quad c_t + a_t = \underbrace{n_t w_t h_t}_{} + \underbrace{(1-n_t)b}_{} + R_t a_{t-1}$$

An (arbitrary) asset to make pricing interest rates explicit

Wage (-setting process) taken as given by household

Measure n_t of family members earn labor income (because they work) (and recall we've normalized h = 1)

Measure 1-n_t of family members receive unemployment benefits and/or engaged in home production

Household Problem

- Dynamic household utility-maximization problem
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KEY: Assuming infinite *family* structure delivers *full consumption insurance* – i.e., all employed and unemployed individuals have equal consumption!

Thus individual family members are risk-neutral with respect to their labor-market realization

Analogy with Hansen-Rogerson structure (see Andolfatto 1996 *AER*)

$$\max_{c_t, a_t} \left[E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

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Measure n_t of family members earn labor income (because they work) (and recall we've normalized h = 1)

Measure 1-n_t of family members receive unemployment benefits and/or engaged in home production

- Consumption-savings optimality condition: $1 = R_t E_t \underbrace{\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}}_{}$

Stochastic discount factor

- No labor-supply/part. margin in basic model
  - Each family member either works or is looking for work

WAGE BARGAINING

- **(Generalized) Nash Bargaining**

$$\max_{w_t} \left(W(w_t) - U(w_t)\right)^{\eta} \left(J(w_t) - V(w_t)\right)^{1-\eta}$$

Bargaining over how to divide the surplus

Net payoff to an individual/household of agreeing to wage *w* and beginning production

Net payoff to a firm of agreeing to wage *w* and beginning production

- **Asset values**
  - ***W*: value to (representative) household of having one additional member employed**
  - ***U*: value to (representative) household of having one additional member unemployed and searching for work**
  - ***J*: value to (representative) firm of having one additional employee**
  - ***V*: value to (representative) firm of having a job that goes unfilled**
    - **Free entry in vacancy-posting → *V* = 0**

- **Define *W* and *U* in terms of household problem**
  - **i.e., based on envelope conditions of household value function**

WAGE BARGAINING

- **(Generalized) Nash Bargaining**

$$\max_{w_t} \underbrace{\left(W(w_t) - U(w_t)\right)^{\eta}}\ \underbrace{J(w_t)}^{1-\eta}$$

Bargaining over how to divide the surplus

Net payoff to an individual/household of agreeing to wage *w* and beginning production

Net payoff to a firm of agreeing to wage *w* and beginning production

- **The Nash surplus-sharing rule**

$$\eta\left(W'(w_t) - U'(w_t)\right)J(w_t) = (1-\eta)(-J'(w_t))\left(W(w_t) - U(w_t)\right)$$

(FOC with respect to w_t)

  - **Present in any model with Nash bargaining**
    - **(Most) labor matching models**
    - **(Most) monetary search models**
    - **Political bargaining games (Albanesi 2007 *JME*)**

- **Must specify value equations *W*(.), *U*(.), *J*(.)**

VALUE EQUATIONS

- **Individual/household value equations (constructed from household problem)**

$$W(w_t) = \underbrace{w_t}_{\text{Contemporaneous return is wage}} + E_t\left\{ \underbrace{\Xi_{t+1|t}\left[(1-\rho^x)W(w_{t+1}) + \rho^x U(w_{t+1}) \right]}_{\text{Expected future return takes into account transition probabilities}} \right\}$$

Value to household of having the marginal individual employed

$$U(w_t) = \underbrace{b}_{\text{Contemporaneous return is unemployment benefit/home production}} + E_t\left\{ \underbrace{\Xi_{t+1|t}\left[k^h(\theta_t)W(w_{t+1}) + (1-k^h(\theta_t))U(w_{t+1}) \right]}_{\text{Expected future return takes into account transition probabilities}} \right\}$$

Value to household of having the marginal individual unemployed and searching

VALUE EQUATIONS

- **Individual/household value equations (constructed from household problem)**

Each searching individual has probability $k^h(\theta)$ of finding a job opening: depends on a *market* variable, θ, so taken as given

$$W(w_t) = w_t + E_t\left\{\Xi_{t+1|t}\left[(1-\rho^x)W(w_{t+1}) + \rho^x U(w_{t+1})\right]\right\}$$

Value to household of having the marginal individual employed

Contemporaneous return is wage

Expected future return takes into account transition probabilities

$$U(w_t) = b + E_t\left\{\Xi_{t+1|t}\left[k^h(\theta_t)W(w_{t+1}) + (1-k^h(\theta_t))U(w_{t+1})\right]\right\}$$

Value to household of having the marginal individual unemployed and searching

Contemporaneous return is unemployment benefit/ home production

Expected future return takes into account transition probabilities

- **Firm value equation**

$$J(w_t) = z_t - w_t + E_t\left\{\Xi_{t+1|t}(1-\rho^x)J(w_{t+1})\right\}$$

Value to firm of the marginal employee

Contemporaneous return is marginal output net of wage payment

Expected future return takes into account transition probabilities

WAGE BARGAINING

- **The Nash surplus-sharing rule**

$$\eta\left(W'(w_t)-U'(w_t)\right)J(w_t)=(1-\eta)(-J'(w_t))\left(W(w_t)-U(w_t)\right)$$

(FOC with respect to w_t)

↓ **Insert marginal values**

$$\eta J(w_t)=(1-\eta)\left(W(w_t)-U(w_t)\right)$$

Firm's surplus *J* a constant fraction of household's surplus *W – U*

NOTE: NOT a general property of Nash bargaining; here due to the linearity of *W, U,* and *J* with respect to wage

↓ **Using definitions of *W, U,* and *J,* the job-creation condition, and some algebra**

$$w_t=\eta\left[z_t+\gamma\theta_t\right]+(1-\eta)b$$

Bargained wage a convex combination of gains from consummating the match and the gains from walking away from the match

Contemporaneous marginal output…

…and a term that captures the social savings on future posting costs if match continues

NOTE: With CRS matching function,

$\theta = k^h(\theta)/k^f(\theta)$

LABOR MARKET MATCHING

- **Aggregate matching function displays CRS**

$$m(u_t, v_t)$$

u_t = 1 – n_t is measure of individuals searching for work

- **For any given individual vacancy or individual (partial equilibrium), matching probabilities depend only on *v/u***

$$\theta_t \equiv \frac{v_t}{u_t}$$

Market tightness: measures relative number of traders on opposite sides of market

LABOR MARKET MATCHING

- Aggregate matching function displays CRS

$$m(u_t, v_t)$$

$u_t = 1 - n_t$ is measure of individuals searching for work

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NOTE: With CRS matching function,

$\theta = k^h(\theta)/k^f(\theta)$

In matching models, θ is the key driving force of efficiency and therefore optimal policy prescriptions (Hosios 1990 *ReStud* the key reference)

$$\frac{m(u_t, v_t)}{v_t} = m\left(\frac{u_t}{v_t}, 1\right) = m\left(\theta_t^{-1}, 1\right) \equiv k^f(\theta_t)$$

Probability a given vacancy/job posting attracts a worker

$$\frac{m(u_t, v_t)}{u_t} = m\left(1, \frac{v_t}{u_t}\right) = m\left(1, \theta_t\right) \equiv k^h(\theta_t)$$

Probability a given individual finds a job opening

$$\theta_t \equiv \frac{v_t}{u_t}$$

Market tightness: measures relative number of traders on opposite sides of market

- Market tightness an allocational signal
  - Because matching probabilities depend on it
  - e.g., the higher (lower) is ***v/u,*** the easier (harder) it is for a given individual to find a job opening

LABOR-MARKET EQUILIBRIUM

- **Aggregate law of motion of employment**

$$N_{t+1} = (1-\rho^x)N_t + m(u_t, v_t)$$

- **Flow equilibrium conditions (an accounting identity…)**

$$m(u_t, v_t) = u_t k^h(\theta_t) = v_t k^f(\theta_t)$$

- **Vacancy-posting (aka job-creation) condition**

$$\gamma = k^f(\theta_t) E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

- **Wage determination**

$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

- **Basic labor-theory literature: impose ss on these and analyze, do comparative statics, etc. (exogenous real interest rate)**
  - **Pissarides Chapter 1, RSW 2005 *JEL***

GENERAL EQUILIBRIUM

- ❒ **Aggregate law of motion for employment**
- ❒ **Vacancy-posting (aka job-creation) condition**
- ❒ **Wage determination**

The labor market equilibrium (*partial* equilibrium from the perspective of the entire environment)

- ❒ **Consumption-savings optimality condition (endogenizes real interest rate)**

$$1 = R_t E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}$$

- ❒ **Aggregate resource constraint**

Often interpreted as the output of a home production sector – only the unemployed produce in the home sector

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b$$

Vacancy posting costs and "outside option" are real uses of resources

- ❒ **Exogenous LOMs for any driving processes (TFP, etc)**

Steady State of Labor Market

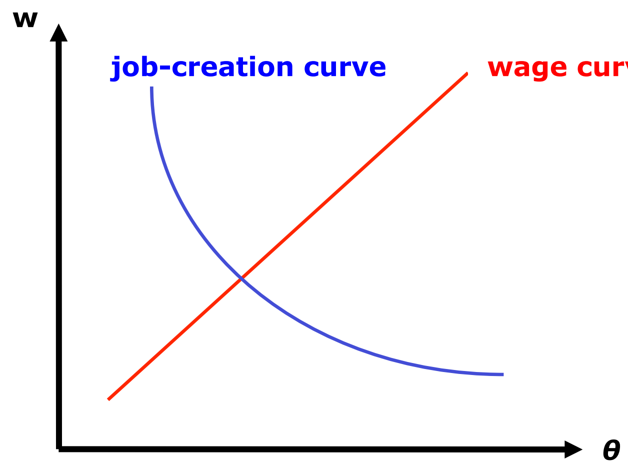
- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1) $$1-u=(1-\rho^x)(1-u)+m(u,v)$$ (using $N = 1 - u$)

(2) $$\gamma=\beta k^f(\theta)\left(z-w+\frac{(1-\rho^x)\gamma}{k^f(\theta)}\right)$$ w negatively and nonlinearly related to θ (given CRS matching function)

(3) $$w=\eta[z+\gamma\theta]+(1-\eta)b$$ w positively and linearly related to θ

Pissarides Figure 1.1



"Labor supply curve" and "labor demand curve" replaced by "wage curve" and "job-creation curve"

The relevant "quantity" variable θ – but can also loosely think of θ as a "price" because it governs matching probabilities…

STEADY STATE OF LABOR MARKET

- **Imposing deterministic steady state on labor-market equilibrium conditions**

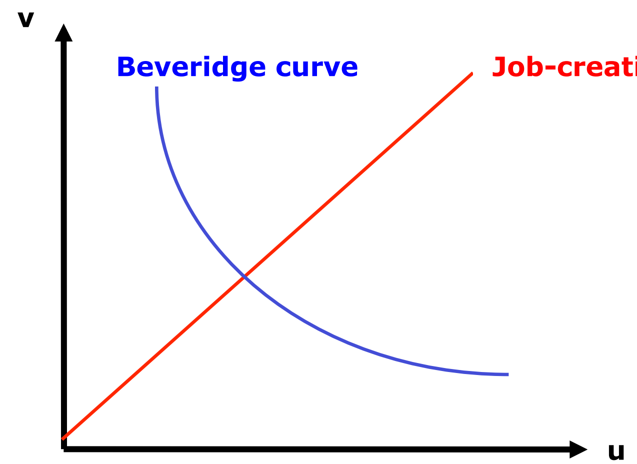
(1)
$$u = \frac{m(u,v)+\rho^x}{\rho^x}$$

For a given (w,θ), v and u negatively related (given CRS matching function)

(2)
$$\gamma = \beta k^f\left(\frac{v}{u}\right)\left(z - w + \frac{(1-\rho^x)\gamma}{k^f\left(\frac{v}{u}\right)}\right)$$

For a given (w,θ), v and u positively related (given CRS matching function)

Pissarides Figure 1.2



BEVERIDGE CURVE: Empirical relationship in both long run and short run (i.e., cyclical)

STEADY STATE OF LABOR MARKET

- **Labor-market equilibrium is (w, u, θ) satisfying (1), (2), (3)**

- **Comparative statics**
  - **A rise in b…**
    - **…raises w**
    - **…lowers θ**
    - **…lowers v and raises u**

Higher value (ue benefit) of unemployment requires a higher wage to induce individuals to work, which reduces firm incentives to create jobs

STEADY STATE OF LABOR MARKET

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  - **A fall in β (or a rise in ρ^x)…**
    - **…lowers w**
    - **…lowers θ**
    - **…raises u**
    - **…ambiguous effect on v**

  Higher real rate and/or faster job separations (i.e., "faster depreciation of employment stock") makes posting jobs (FOR FIXED u) less attractive for firms (both erode firm profits)

- **See Pissarides Chapter 1 and RSW (2005 *JEL*) for more**

- **Next: dynamic stochastic partial equilibrium (Shimer 2005, Hall 2005, and Hagedorn and Manovskii 2008)**