LABOR MATCHING MODELS: EFFICIENCY PROPERTIES

APRIL 19, 2012

LABOR-MATCHING EFFICIENCY

- **Social Planning problem**
  - **Social Planner also subject to matching "technology"**

$$\max_{c_t, v_t, N_{t+1}} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b$$

Fix *h* = 1

$$N_{t+1} = (1 - \rho^x) N_t + m(u_t, v_t)$$

And *N* = 1 – *u*

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Multipliers

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b$$ **Fix $h = 1$** — λ_t

$$N_{t+1} = (1 - \rho^x) N_t + m(1 - N_t, v_t)$$ **And $N = 1 - u$** — μ_t

- **FOCs**

$$u'(c_t) - \lambda_t = 0$$

$$-\lambda_t \gamma + \mu_t m_2(1 - N_t, v_t) = 0$$

$$-\mu_t + \beta E_t \left\{ \lambda_{t+1} \left[z_{t+1} - b \right] \right\} + \beta E_t \left\{ \mu_{t+1} \left[(1 - \rho^x) - m_1(1 - N_t, v_t) \right] \right\} = 0$$

↓ **Eliminate multipliers**

LABOR-MATCHING EFFICIENCY

- **Social Planning problem**

$$\frac{\gamma}{m_2(1-N_t,v_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - b - \frac{\gamma m_1(1-N_t,v_t)}{m_2(1-N_t,v_t)} + \frac{(1-\rho^x)\gamma}{m_2(1-N_t,v_t)}\right]\right\}$$

Cobb-Douglas matching

$$m(u,v) = u^{\alpha}v^{1-\alpha}$$

$$m_1(u,v) = \alpha u^{\alpha-1}v^{1-\alpha} = \alpha\theta^{1-\alpha}$$

$$m_2(u,v) = (1-\alpha)u^{\alpha}v^{-\alpha} = (1-\alpha)\theta^{-\alpha}$$

AND

$$k^h(\theta) = \frac{m(u,v)}{u} = m(1,\theta) = \theta^{1-\alpha}$$

$$k^f(\theta) = \frac{m(u,v)}{v} = m(\theta^{-1},1) = \theta^{-\alpha}$$

AND $m_1(u,v) = \alpha k^h(\theta)$ $\quad m_2(u,v) = (1-\alpha)k^f(\theta)$

Labor-Matching Efficiency

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AND $m_1(u,v) = \alpha k^h(\theta)$ $\quad m_2(u,v) = (1-\alpha)k^f(\theta)$

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - \left(\alpha\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\alpha)b\right) + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right]\right\}$$

KEY IDEAS

Taking the pricing kernel as given, the only unknown process here is θ_t!

Efficiency in vacancy-postings is governed by "getting market tightness right!"

LABOR-MATCHING EFFICIENCY

- Socially-efficient vacancy posting described by

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- Recall decentralized vacancy posting described by

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right)\right\}$$

AND

$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

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- **Efficiency in vacancy posting requires $\eta = \alpha$!**

HOSIOS CONDITION

- **Cobb-Douglas matching technology + Nash bargaining**
  - **Efficient level of job-creation requires $\eta = \alpha$**
  - **Hosios (1990 *ReStud*)**

- **Intuition: search activity generates externalities**
  - **One extra individual (firm) searching for a job (worker) lowers the probability that *all other* individuals (firms) will find a match…**
  - **…but raises the probability that *all other* firms (individuals) will find a match**
  - **Congestion externality – search imposes both positive and negative externalities (on opposite sides of the market)**

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- **Nash bargaining: η governs the private returns to search**
  - **Share of total match surplus kept by individual**
- **Cobb-Douglas matching: α governs the social returns to search**
  - **Elasticity of aggregate number of matches with respect to *u***

- **Efficiency requires equating private and social returns: $\eta = \alpha$**

HOSIOS CONDITION

- **Also holds under some more general conditions**
  - **Endogenous search intensity**
  - **Endogenous "vacancy posting intensity" (Pissarides Chapter 5)**

- **Pissarides (2000, p. 198): "..we are not likely to find intuition for it…"**

- **RSW (2005 *JEL* p. 982): "…genuinely surprising result…"**

- **Is the Hosios condition empirically relevant?**
  - **Who knows?...it's a nongeneric parameterization…**
  - **Nonetheless has become a focal point for calibrated models**

- **Hosios efficiency emerges endogenously in competitive search equilibrium (CSE) concept**
  - **Moen (1997 *JPE*): basic static partial labor search model**
  - **A well-understood concept in labor theory, but little incorporation into DSGE models**

Competitive Search Equilibrium (CSE)

- Question: can a "competitive" notion of wage-setting be entertained in a search and matching model?
  - Would get away from the non-genericity of the Hosios bargaining parameterization
  - May be apriori an appealing way of describing labor markets
    - Locating a firm or a worker is costly and time-consuming…
    - …but once matched, wages are more or less determined by "market forces," perhaps with little/no room for "bargaining"

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- **Moen (1997 *JPE*) and Shimer (1996) the original implementations of CSE**
  - **Static partial-equilibrium labor search models**

- **Will implement in the context of our full DSGE labor-search model**
  - **Only recently have started to become incorporated into DSGE search models….**
  - **…but goods-search models, not labor-search (Arseneau and Chugh (2007), Gourio and Rudanko (2009) (Menzio and Shi (2010 *JET*) a labor-search application)**

CSE – Basics of Environment

- **Need "many markets" and "many firms"**
  - **To rationalize "competition," so can operationalize decentralized wage-formation process**

- **Index continuum of labor "submarkets" by *j* – e.g., local labor markets**

- **Within a submarket *j*, many firms looking to hire workers**
  - **Even within a "local" labor market, coordination frictions in finding workers may exist**
  - **Index by *i***

- **Unemployed individuals *direct* their job search ("send an application") to a particular submarket**
  - **Based on wages announced by firms in that submarket, and on likelihood of getting a job in that submarket**
  - **Not random search – directed search is key for concept of CSE**
  - **Once search is directed, random matching process governs whether an individual gets a job – match formation is still subject to frictions**

- **Wages determined before search, not after search**
  - **All parties direct search according to "posted" wages**

CSE – Basics of Environment

- Wages determined before search, not after search
  - All parties direct search according to "posted" wages
- Several equivalent ways to implement
  - Perfectly-competitive "market-maker" sector
  - Individuals announce wages before firms search for workers
  - Firms announce wages before individuals search for jobs
    - The implementation we will pursue
  - See RSW 2005 *JEL* survey for alternative implementations

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  - See RSW 2005 *JEL* survey for alternative implementations
- Idea of firm wage-posting/wage-announcement implementation
  - Define (expected) payoff function to firm *ij* of finding an additional worker
  - Define (expected) payoff function to individual searching for/applying to a job at firm *ij*
  - Firm *ij* maximizes its payoff subject to the reaction function defined by the individual's payoff function
    - i.e., firm *internalizes* the effect of wages on the other side of the market…
    - …can already see how congestion *externality* issues will be taken care of…
- Internalizing congestion externalities would also be achieved by…
  - Individuals announcing wages taking into account reactions by firms
  - "Market maker" calling out wages taking account reactions by both sides of market

CSE – IMPLEMENTATION

- **Firm *ij* payoff function described by vacancy-posting decision!**

$$\gamma = k^f(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]$$

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of matching with a worker) x (contemporaneous payoff + continuation payoff)

Note *ij* subscripts:

Matching probability depends on tightness of "applications" at firm *ij*…

…but future asset value of employee depends on market *j* conditions (i.e., replacement value depends on (sub-)market conditions)

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- **Value equations for an individual searching for a match at firm *ij***

$$W(w_{ijt}) = w_{ijt} + E_t\left\{\Xi_{t+1|t}\left[(1-\rho^x)W(w_{ijt+1}) + \rho^x U_{t+1}\right]\right\}$$

With probability $k^h(\theta_{ijt})$, individual gets this payoff

$$U_t = b + E_t\left\{\Xi_{t+1|t}\left[k^h(\theta_t)W(w_{t+1}) + (1-k^h(\theta_t))U_{t+1}\right]\right\}$$

With probability $1-k^h(\theta_{ijt})$, individual gets this payoff

- **With individuals (households) optimally directing their search, the expected payoff of searching for/applying to a job at firm *ij* is**

$$k^h(\theta_{ijt})W(w_{ijt}) + (1-k^h(\theta_{ijt}))U_t = X$$

Payoff of searching at another firm or another submarket independent of *ij*

CSE – IMPLEMENTATION

- **Firm *ij* maximizes**

$$\gamma = k^f(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{ \Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]$$

taking as constraint

$$k^h(\theta_{ijt})W(w_{ijt}) + (1-k^h(\theta_{ijt}))U_t = X$$

- **Choice variables: w_{ijt} and θ_{ijt} (isomorphic to choosing v_{ijt} for a given number of searchers u_{ijt})**

- **First-order conditions**

1)

$$-k^f(\theta_{ijt}) - \varphi_{ijt}k^h(\theta_{ijt})W'(w_{ijt}) = 0$$

2)

$$\frac{\partial k^f(\theta_{ijt})}{\theta_{ijt}}\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{ \Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right] - \varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\theta_{ijt}}\left[W(w_{ijt}) - U_t \right] = 0$$

Taking into account how matching probabilities are affected by tightness is the central idea

CSE – IMPLEMENTATION

- **First-order conditions**

1) $$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) W'(w_{ijt}) = 0 \xrightarrow{W'(.)=1} \varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}$$

2) $$\frac{\partial k^f(\theta_{ijt})}{\theta_{ijt}} \underbrace{\left[z_t - w_{ijt} + (1-\rho^x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]} - \varphi_{ijt} \frac{\partial k^h(\theta_{ijt})}{\theta_{ijt}} \left[W(w_{ijt}) - U_t \right] = 0$$

= J_{ijt} if firms are optimizing

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Combine and rearrange

(Competition within submarket *j* and symmetry across submarkets: drop *ij* indices)

$$\boxed{\alpha\left(W(w_t) - U_t\right) = (1-\alpha)J(w_t)}$$

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$$\alpha\left(W(w_t) - U_t\right) = (1-\alpha)J(w_t)$$

Exactly the Nash-bargaining sharing rule with endogenous emergence of Hosios condition ($\eta = \alpha$)!!!

Inserting value equations and solving explicitly for wage obviously gives same outcome as Nash-bargained wage with $\eta = \alpha$...

CSE – Interepretations

- Mortensen and Pissarides (1999 *Handbook Chapter* p. 2589-2592)
  - "Price of time" priced efficiently by markets in CSE
  - "Price of time" generically mispriced in bargaining equilibrium
  - ("Price of time" = matching probabilities, which reflect congestion externalities)
  - Bargaining equilibrium features a particular type of market incompleteness: workers and firms cannot contract on efficient surplus sharing before meeting
  - CSE effectively fills in this missing market…
  - …provided we're willing to assume/believe the strong degree of commitment built into CSE model
    - (i.e., each side of a job-match would have an incentive to try to "renegotiate" the "posted" wage once they actually meet)
    - An open question in search theory
- CSE in principle an alternative equilibrium concept in search models
  - But turns out to be equivalent to bargaining equilibrium with Hosios condition
  - (At least in simple environments….will equivalence hold in richer environments?…)
- Little explored in DSGE contexts
  - Question: Would some types of market frictions, tax issues, etc break the equivalence between CSE and Nash-Hosios bargaining?…

RELEVANCE OF HOSIOS CONDITION IN DSGE

- **Optimal policy (monetary and/or fiscal) will depend on whether or not $\eta = \alpha$**
  - **Yet another distortion (if $\eta = \alpha$ not satisfied) for policy to respond to**
  - **Deviation from Friedman Rule can be used to correct search externalities (Cooley and Quadrini (2004 *JET*), Arseneau and Chugh (2008 *JME*; 2010), Faia (2008 *JEDC*))**

- **Model dynamics can depend (noticeably) on whether or not $\eta = \alpha$**
  - **Walsh (2005 *RED*)**

- **Hosios issues arise in any DGE model with any type of search market**
  - **Money search models**
    - **Rocheteau and Wright (2005 *Econometrica*)**
    - **Aruoba and Chugh (2010 *JET*)**
  - **Product search models**
    - **Hall (2007)**
    - **Arseneau and Chugh (2007)**

DSGE (Labor) Search Models

- Search models articulate trading frictions – cannot instantaneously/costlessly find trading partners
  - An appealing description of labor markets
  - Maybe of other markets also
- Tractable to incorporate in DSGE models because of assumption of aggregate matching function
- Too ad-hoc or "reduced-form" because of assumption of (black box) aggregate matching friction?
- The Shimer Puzzle and attempted answers continue…(do they?…)
- …as do New Keynesian modelers' incorporation of labor matching structure
  - Enables talking meaningfully about the tradeoffs between inflation and unemployment…
  - …i.e., seemingly resuscitates the original Phillips Curve, not the NK Phillips Curve (which links inflation to marginal costs…)