SIMPLE DSGE MODELS OF "MONEY" PART I

FEBRUARY 15, 2012

BASIC ISSUES

- Money/monetary policy issues an enduring fascination in macroeconomics
- How can/should central bank "control" the economy? Should it/ can it at all?
- Roles of "money"
  - Medium of exchange (transactions role)
  - Unit of account (numeraire role)
  - Store of value (asset role)

Basic Issues

- Money/monetary policy issues an enduring fascination in macroeconomics
- How can/should central bank "control" the economy? Should it/ can it at all?
- Roles of "money"
  - Medium of exchange (transactions role) ← Highlighted in CIA, MIU, and money-search approaches
  - Unit of account (numeraire role) ← Highlighted in New Keynesian approach
  - Store of value (asset role)
- How to "model money" in DSGE environment?
  - Which role to model?
  - Which role is tractable to model?
  - Which role is most relevant for conduct of monetary policy?

Households

- Household optimization $\max_{c_t, n_t, M_t, B_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$

s.t.

$$P_t c_t + M_t + B_t = P_t w_t n_t + M_{t-1} + (1+i_{t-1})B_{t-1} + T_t$$

Flow budget constraint

- P_t the nominal price of c_t – equivalently, the nominal price level
- $P_t c_t$: Nominal consumption spending
- M_t: Nominal money holdings
- B_t: Nominal bond holdings
- $P_t w_t n_t$: Nominal labor income
- i_{t-1}: Nominal interest rate (on previously-accumulated nominal bonds)
- T_t: Lump-sum monetary injection/contraction

Households

- **Household optimization** $\max_{c_t,n_t,M_t,B_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t,n_t)$

s.t.

$$P_t c_t + M_t + B_t = P_t w_t n_t + M_{t-1} + (1+i_{t-1})B_{t-1} + T_t$$

Flow budget constraint

P_t the nominal price of c_t – equivalently, the nominal price level

Nominal consumption spending

Nominal money holdings

Nominal bond holdings

Nominal labor income

Nominal interest rate (on previously-accumulated nominal bonds)

Lump-sum monetary injection/contraction

$$P_t c_t \le M_t$$

"Cash-in-advance" (CIA) constraint

- "forces" consumers to "hold money"
- articulates a transactions motive

Households

- **Household optimization** $\max_{c_t,n_t,M_t,B_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t,n_t)$

s.t.

$$P_t c_t + M_t + B_t = P_t w_t n_t + M_{t-1} + (1+i_{t-1})B_{t-1} + T_t$$

Flow budget constraint

P_t the nominal price of c_t – equivalently, the nominal price level

Nominal consumption spending

Nominal money holdings

Nominal bond holdings

Nominal labor income

Nominal interest rate (on previously-accumulated nominal bonds)

Lump-sum monetary injection/contraction

$$P_t c_t \le M_t$$

"Cash-in-advance" (CIA) constraint

- "forces" consumers to "hold money"
- articulates a transactions motive

- **CIA constraint a friction on economy**
  - **Pareto-optimal allocations do not require it**
  - **Money not "essential" as in models of Kiyotaki and Wright (1993), Lagos and Wright (2005)**

Does not ENDOGENOUSLY EXPAND consumers' set of feasible trades. Because underlying DSGE model features full set (including over *all* state-date pairs) of Arrow-Debreu securities – complete markets! Trade does not require "money"…

Simple Policy Analysis

- **Removing monetary friction…**
  - **….requires an allocation that features a zero multiplier on CIA constraint…**
  - **…implies zero nominal interest rate**
- **Friedman Rule**
  - **Benchmark result in monetary theory**
  - **Completely relaxing "monetary friction" requires eliminating any (opportunity) cost of holding money**

Simple Policy Analysis

- **Removing monetary friction…**
  - **….requires an allocation that features a zero multiplier on CIA constraint…**
  - **…implies zero nominal interest rate**
- **Friedman Rule**
  - **Benchmark result in monetary theory**
  - **Completely relaxing "monetary friction" requires eliminating any (opportunity) cost of holding money**
  - **Other Interpretations**
    - **Eliminate the wedge between alternative nominal assets: i = 0 makes money and nominal bonds equivalent assets (in terms of their cost and benefit properties)**
    - **Eliminate the wedge in the consumption-leisure optimality condition**

    Really the same thing…
  - **Are monetary frictions empirically important?…and thus, is the Friedman Rule of practical use for advising monetary policy?**

SIMPLE POLICY ANALYSIS

- Household optimality conditions

hh multiplier on CIA constraint → ϕ_t; hh multiplier on budget constraint → λ_t

$$\phi_t = \lambda_t \left[\frac{i_t}{1+i_t} \right]$$

No-arbitrage between money and nominal bonds

(Assumption: i_t in the information set of time t)

$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = w_t \left[1 + \frac{i_t}{1+i_t} \right]^{-1}$$

Consumption-leisure optimality condition
- relative price depends on w_t AND i_t

Efficiency requires C-L optimality depends on real wage....

...but not on monetary aspects of economy (non-technology)

Friedman Rule achieves Pareto efficiency along this margin

SIMPLE POLICY ANALYSIS

- Household optimality conditions

hh multiplier on CIA constraint → ϕ_t; hh multiplier on budget constraint → λ_t

$$\phi_t = \lambda_t \left[\frac{i_t}{1+i_t} \right]$$

No-arbitrage between money and nominal bonds

(Assumption: i_t in the information set of time t)

Note disutility of labor appears in intertemporal MRS…

$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = w_t \left[1 + \frac{i_t}{1+i_t} \right]^{-1}$$

Consumption-leisure optimality condition
- relative price depends on w_t AND i_t

If monetary friction were "shut down," would have u_c here "as usual."

Either through Friedman Rule or through "cashless" New Keynesian environment (later…)

Efficiency requires C-L optimality depends on real wage....

...but not on monetary aspects of economy (non-technology)

Friedman Rule achieves Pareto efficiency along this margin

$$\frac{u_n(c_t, n_t)}{w_t} = (1+i_t)\beta E_t \left[\frac{u_n(c_{t+1}, n_{t+1})}{w_{t+1}} \cdot \frac{P_t}{P_{t+1}} \right]$$

Consumption-savings optimality condition
(aka bond Euler equation)
(aka Fisher equation!)

Simple Policy Analysis

- **Household optimality conditions (continued)**

$$\phi_t = \lambda_t \left[\frac{i_t}{1+i_t} \right]$$

No-arbitrage between money and nominal bonds

$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = w_t \left[1 + \frac{i_t}{1+i_t} \right]^{-1}$$

Consumption-leisure optimality condition

$$\frac{u_n(c_t, n_t)}{w_t} = (1+i_t)\beta E_t \left[\frac{u_n(c_{t+1}, n_{t+1})}{w_{t+1}} \cdot \frac{P_t}{P_{t+1}} \right]$$

Consumption-savings optimality condition
(aka bond Euler equation)
(aka Fisher equation)

$$c_t = \frac{M_t}{P_t}$$

Binding CIA constraint

Obvious if $i_t > 0$ (why hold excess money?)

Also assume it even in states where $i_t = 0$: pins down a monetary equilibrium level of M_t, hence is an equilibrium selection device

Basic CIA Model

Simple Policy Analysis

- **Household optimality conditions (continued)**

$$\phi_t = \lambda_t \left[\frac{i_t}{1+i_t} \right]$$

No-arbitrage between money and nominal bonds

$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = w_t \left[1 + \frac{i_t}{1+i_t} \right]^{-1}$$

Consumption-leisure optimality condition

$$\frac{u_n(c_t, n_t)}{w_t} = (1+i_t)\beta E_t \left[\frac{u_n(c_{t+1}, n_{t+1})}{w_{t+1}} \cdot \frac{P_t}{P_{t+1}} \right]$$

Consumption-savings optimality condition
(aka bond Euler equation)
(aka Fisher equation)

$$c_t = \frac{M_t}{P_t}$$

Binding CIA constraint

Obvious if $i_t > 0$ (why hold excess money?)

Also assume it even in states where $i_t = 0$: pins down a monetary equilibrium level of M_t, hence is an equilibrium selection device

- **Rest of the environment**
  - w_t = marginal product of labor (linear production + competitive factor market)
  - Govt budget: $T_t = M_t - M_{t-1} = (1+\mu_t)M_{t-1}$ Resource constraint: $c_t = z_t n_t$

SIMPLE POLICY ANALYSIS

- **Household optimality conditions (continued)**

$$\phi_t = \lambda_t \left[\frac{i_t}{1+i_t} \right]$$ No-arbitrage between money and nominal bonds

Define
$\pi_{t+1} = P_{t+1} / P_t - 1$
$\mu_{t+1} = M_{t+1} / M_t - 1$

$$-\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)} = w_t \left[1 + \frac{i_t}{1+i_t} \right]^{-1}$$ Consumption-leisure optimality condition

$$\frac{u_n(c_t,n_t)}{w_t} = (1+i_t)\beta E_t \left[\frac{u_n(c_{t+1},n_{t+1})}{w_{t+1}} \cdot \frac{1}{1+\pi_{t+1}} \right]$$ Consumption-savings optimality condition (aka bond Euler equation) (aka Fisher equation)

Combine t and t-1 (binding) CIA constraints →

$$\frac{c_t}{c_{t-1}} = \frac{1+\mu_t}{1+\pi_t}$$ Equilibrium link between money growth and inflation

Articulates a quantity-theoretic channel

- **Rest of the environment**
  - w_t = marginal product of labor (linear production + competitive factor market)
  - Govt budget: $T_t = M_t - M_{t-1} = (1+\mu_t)M_{t-1}$ Resource constraint: $c_t = z_t n_t$

Examine steady-state equilibrium

SIMPLE POLICY ANALYSIS

- **Household optimality conditions in deterministic steady state**

$$\phi = \lambda \left[\frac{i}{1+i} \right]$$ No-arbitrage between money and nominal bonds

$$-\frac{u_n(c,n)}{u_c(c,n)} = w \left[1 + \frac{i}{1+i} \right]^{-1}$$ Consumption-leisure optimality condition

Friedman Rule: $i = 0 \rightarrow \pi = \beta - 1$
BUT ONLY IN STEADY STATE!
NOT (necessarily) dynamically….

$$1+\pi = \beta(1+i)$$ Consumption-savings optimality condition (aka bond Euler equation) (aka Fisher equation)

$$1 = \frac{1+\mu}{1+\pi}$$ Equilibrium link between money growth and inflation

Articulates a quantity-theoretic channel

- **Rest of the environment**
  - w = marginal product of labor (linear production + competitive factor market)
  - Govt budget: $T/P = (1+\mu)(M/P)(1/(1+\pi))$ Resource constraint: $c = n$

SIMPLE POLICY ANALYSIS

- **Household optimality conditions in deterministic steady state**

$$\phi = \lambda \left[\frac{i}{1+i} \right]$$

No-arbitrage between money and nominal bonds

$$-\frac{u_n(c,n)}{u_c(c,n)} = w\left[1 + \frac{i}{1+i}\right]^{-1}$$

Consumption-leisure optimality condition

$$1+\pi = \beta(1+i)$$

Consumption-savings optimality condition (aka bond Euler equation) (aka Fisher equation)

Friedman Rule: $i = 0 \rightarrow \pi = \beta - 1$
BUT ONLY IN STEADY STATE!
NOT (necessarily) dynamically….

$$1 = \frac{1+\mu}{1+\pi}$$

Equilibrium link between money growth and inflation

Articulates a quantity-theoretic channel

…and optimal policy calls for $\mu = \beta - 1$
(i.e., SHRINK nominal money supply!)

- **Rest of the environment**
  - **w = marginal product of labor (linear production + competitive factor market)**
  - **Govt budget:** $T/P = (1+\mu)(M/P)(1/(1+\pi))$ **Resource constraint:** $c = n$

OTHER ANALYSIS

- **Other aspects of equilibrium**
  - **$\mu < \beta - 1$ (in steady-state!) inconsistent with monetary equilibrium**
  - **Dynamic analog: $i_t < 0$ inconsistent with monetary equilibrium**
    - **Zero-lower-bound constraint**

Imply $\varphi < 0$, i.e., money NOT valued for exchange

- **Model's "policy rate" typically identified with a (short-run Euler equation) market interest rate**
  - **Whether CIA models, MIU models, New Keynesian models, money search models**
  - **Model mechanism: change in policy rate (potentially) affects intertemporal incentives (i.e., the real interest rate)**
  - **A valid empirical identification? Term-structure issues? Other issues? See Canzoneri, Cumby, and Diba (2007 *JME*)…**

OTHER VARIANTS OF CIA

- Cash good/credit good model
  - Lucas and Stokey (1983)
  - Foundation for Ramsey models of optimal fiscal and monetary policy – see Chari and Kehoe (1999 *Macro Handbook* chapter)
  - Subset of goods (c_1) require "cash in advance"
  - Subset of goods (c_2) do not require cash in advance

$$\frac{u_{c_1}}{u_{c_2}} = 1 + i$$

$MRS_{cash/credit}$ = gross nominal interest rate

Monetary policy creates a STATIC wedge!....

- Investment in CIA constraint
  - Stockman (1981): long-run inflation lowers long-run capital stock

$$c_t + k_{t+1} - (1-\delta)k_t \leq \frac{M_t}{P_t}$$

- Basic Idea: Positive nominal interest rate taxes whatever is in the CIA constraint

ALTERNATIVE MONETARY MODELS

- Alternatives to CIA
  - Money in the utility function (MIU) models

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, \frac{M_t}{P_t}\right)$$

  - Shopping-time & transactions costs models
    - Nominal money holdings reduce "cost" of acquiring goods

Feenstra (1986 *JME*) shows conditions under which CIA, MIU, shopping-time are equivalent

  - Go "cashless"
    - New Keynesian models don't model "money demand" at all (or, at best, as an appendage separate from the "main" equilibrium)

Can think of as "Friedman Rule running in the background"

  - Go for deep micro-foundations
    - Kiyotaki and Wright (1989, 1993)
    - Lagos and Wright (2005), Aruoba, Waller, and Wright (2011 *JME*)