Economics 8861.01

Project 1 – Suggested Solutions

Professor Sanjay Chugh

Fall 2015

Objective

As a building block to working with some set(s) of general equilibrium and/or partial equilibrium models (both in this class and, should your research interests eventually take you in that direction, your own continued research), you will compute a **first-order approximation** to the decisions rules of a DSGE/RBC economy. You will use the Schmitt-Grohe and Uribe (2004 *Journal of Economic Dynamics and Control*) algorithm.

Because the primary methodological objective here is **to learn how to implement** such solutions yourself, you are **not** permitted to use off-the-shelf programs provided by Schmitt-Grohe and Uribe, Uhlig, or others or packaged programs such as Dynare.

A good, complete submission (excluding code) should be relatively brief. A few guidelines:

1. Length: 5 – 10 pages excluding Appendices.
2. Lengthy, step-by-step derivations in the main text may distract readers. If any detailed derivations are needed, place them in Appendices.

Regardless of submission length, your first FIVE sentences, AT MOST, should contain a BRIEF summary of the findings (and I am the one who decides whether or not it is "brief"). This should be the ABSTRACT of your text.

This abstract should include (very importantly!) CLEAR, GOOD economic INTUITION, and (if it helps you explain something important) either one or at most two important numerical results. (Note: is "intuition" simply a verbal description of the numerical results?). Further details about "what to submit" appear at the end of this document. Note that, despite a very long project description, "what to submit" is actually a fairly SHORT amount of results from everybody; and then a section, to be determined by each person individually, that might further expand on the results.

The Problem

Use exactly the framework described in Project 0 and use only Parameter Set B. The exogenous TFP process evolves as

$$\ln z_{t+1} = \rho_z \ln z_t + \varepsilon_{t+1}^z ,$$

in which ε_{t+1}^z is distributed as i.i.d. $N(0,\sigma_z^2)$. The persistence and standard deviation parameters are, respectively $\rho_z = 0.95$ and $\sigma_z = 0.007$.

Simulations

Having computed the matrices g_x and h_x, the next step is to conduct simulations of your model(s). In order to generate simulations, recall that the first-order approximations are given by

$$\begin{aligned} y_t &= g(x_t,\sigma) \approx g(\bar{x},0) + g_x \cdot (x_t - \bar{x}) \\ x_{t+1} &= h(x_t,\sigma) \approx h(\bar{x},0) + h_x \cdot (x_t - \bar{x}) + \eta\sigma\varepsilon_{t+1} \end{aligned}$$

in which it is easiest to set the perturbation parameter σ = 1, in which case the matrix η must contain the standard deviations of the model's exogenous state variables. You will be provided with sequences of shocks for the process z_t which is the forcing process for your time-series simulations. Specifically, you will be provided with 200 sequences each of length 200 periods (quarters). These shocks are drawn from an *iid* $N(0, 1)$ distribution, which, when pre-multiplied with the appropriate row of the matrix η yields an *iid* $N(0, \sigma_z^2)$.[1]

Using **both the non-filtered AND the HP-filtered cyclical components of your simulated time series** (specifically, the net percentage deviation (aka log deviation) of each simulated series from its respective trend), calculate, for each time series of interest in a given simulation, standard deviations, first-order serial correlations, and contemporaneous correlation of each variable with GDP.[2,3] Then, compute and report the means and standard deviations of these means, the means and standard deviations of these standard deviations, and the means and standard deviations of these correlations across all simulations. These sets of second-moment statistics (along with the steady state values of the endogenous variables you decide are interesting/relevant to analyze) are what you should report as your simulation-based results (in some appropriate and informative combination of tables and/or graphs and/or text).

[1] The shocks were generated using Matlab's built-in `randn` function. For this project, use the provided sequence of shocks for your simulations (for the sake of some comparability). In subsequent projects of your own, you can use the `randn` function to generate your own random numbers.
[2] Note that some series (such as GDP) may have to be constructed residually if you do not include them as part of your state or costate vectors.
[3] You will be provided with Matlab files that implement the HP filter.

Compare and contrast the business-cycle moments you find with two tables – Table 1and Table 3 on p. 957 – provided in King and Rebelo (1999 *Handbook of Macroeconomics*).

Impulse Responses

Plot impulse response functions for variables of economic interest. (Hint: it would likely be more informative for the reader of your short paper if several IRFs could be contained in one diagram – see `help subplot` in Matlab for more.)

(Some) Computational/Programming Guidance

Using Matlab's `fsolve` function to solve for the matrices g_x and h_x is once again the key computational step, as in Project 0.

In order to conduct simulations using the sequences of shocks with which you will be provided, you must essentially proceed "iteratively" through each simulation. To do so, begin with k_0 (which is simply the deterministic steady state value $\bar{k}$) and the "first realization" of the shocks to z and γ (that is, the first (period-zero) shocks to log z and log γ) and compute the period-zero equilibrium outcome using

$$y_0 = g(\bar{x},0) + g_x \cdot (x_0 - \bar{x})$$
$$x_1 = h(\bar{x},0) + h_x \cdot (x_0 - \bar{x}) + \eta\sigma\varepsilon_1$$

Once you have the period-zero equilibrium outcome of the model in hand, compute the period-one equilibrium outcome of the model using

$$y_1 = g(\bar{x},0) + g_x \cdot (x_1 - \bar{x})$$
$$x_2 = h(\bar{x},0) + h_x \cdot (x_1 - \bar{x}) + \eta\sigma\varepsilon_2$$

Continue this way through all periods of the simulation, and then repeat this for each of the simulations. In conducting these simulations, you can and should try to cleverly arrange matrices and vectors in a way that takes advantage of Matlab's comparative advantage (compared to other software programs) in performing matrix manipulations. Be careful about issues such as matrix conformability, in particular with your g_x and h_x matrices.

A "sensibility check" you may want to try on your programs is to check the convergence (to the deterministic steady state) implied by your computed g_x and h_x matrices. To check this, begin with some arbitrary k_0 (say, perhaps 1% above or below the steady state $\bar{k}$) and construct a vector of zeros for the sequence of TFP shocks and deterministic productivity shocks. Iteratively apply your approximated decision rules (as described above) to construct a time-series simulation of the model – the difference, of course, is that this will be a *deterministic simulation* because each period the TFP shock and the trend shock is by assumption zero. If you have computed the correct g_x and h_x, your model variables should clearly converge to their deterministic steady state counterparts.

If you do not find convergence to the deterministic steady state (and you are convinced you are conducting the simulations correctly), there likely is an error in your computed g_x and/or h_x matrix. One "simple" error is that you have found the explosive root of the system (i.e., an eigenvalue outside the unit circle). You can check this using the command `eig(hx).`

What To Submit

Your submission should be a stand-alone, complete paper – i.e., one should be able to read it independent of knowing what the "description" of "Project 1" was. As before, your submission **must be typed, not hand-written.**

Abstract

Section 1. Introduction (see next page)

Section 2. Model

- Provide the "story" of the model, with only the minimum of equations necessary to convey the primitives of the model and the basic intuition.
- Provide in an Appendix as many of the background steps of the algebraic derivations as you judge necessary.
- A formal definition of general equilibrium (this is certainly part of the "minimum set of equations")

Section 3. Parameterization

- Your calibration strategy for the parameter psi
- Your general calibration strategy (even though you are taking them as given…) for other parameters

Section 4. Numerical Results

- BRIEF note(s) about HOW you approximate the model (note: no need to provide the details of the SGU algorithm)
- Steady state results and intuition you can offer
- DO PRESENT **the numerical results for the g_x and h_x matrices**
- Impulse responses to z(t) shocks
- Simulation-based HP-filtered business cycle statistics
- Simulation-based non-HP-filtered business cycle statistics
- Highlights of economic intuition behind impulse responses and business cycle statistics including comparison of your results with those presented in King and Rebelo's (1999) Table 1 and Table 3.

Section 5. Conclusion

- Brief summary of paper and possible future research based on your findings

As for Project 0, attach a print-out of your code to your submission.

<u>Tips on Writing</u>

Because what you will be submitting is a complete research paper, some advice regarding "how to" write good research papers:

1. **Get to the main point(s) quickly.** Different researchers have different writing styles, and you will develop your own as you gain experience, but here are concrete guidelines for you to follow and then later develop from: **by the end of the first two paragraphs of the introduction of your paper,** the following points should be made very clear:
    a. Big-picture motivation(s).
    b. The precise question(s) you ask in the project
    c. A very brief description of the method(s) you use to address your precise question(s)
    d. The main result(s) your analysis yields
    e. Big-picture conclusion(s) you can draw from your analysis and results
    f. Where your work lies in the context of a larger literature (or even what literature it lies in), along with the **marginal contribution** of your paper.

2. **There is no need for a history lesson in the introduction.** Closely related to the "get to the main point(s) quickly" theme is the fact that, although many things may be new to you as you are learning various techniques and literatures, there is little place in a research paper (and certainly not in the introduction) for a long-winded discussion of how and why you or someone might come to ask a particular research question(s).

3. **This isn't a creative writing exercise.** Which returns back to point #1 above – get to the main points without excessive or flowery language and without repeating the same thing(s) over and over.

Solutions:

Given state and costate vectors $x_t = [k_t, z_t]'$ and $y_t = [c_t, l_t]'$, the first-order approximated (in level-linear terms) matrices are $g_x = \begin{pmatrix} 0.0300 & 0.4800 \\ -0.0030 & 0.1823 \end{pmatrix}$ and $h_x = \begin{pmatrix} 0.9724 & 1.2028 \\ 0 & 0.95 \end{pmatrix}$.

The impulse response profile in Figure 1displays the Pareto-efficient fluctuations upon a one-time exogenous TFP shock.

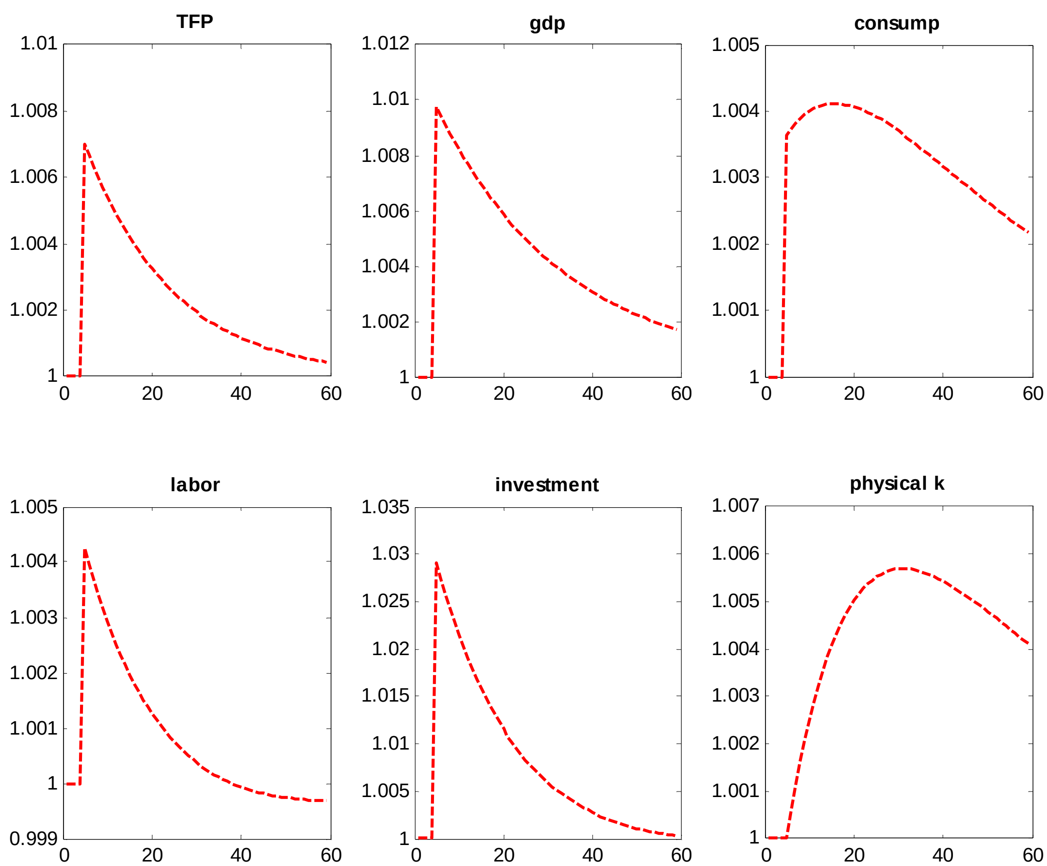

Figure 1. Impulse responses of several variables after one-time TFP shock (impulse occurs in period five).

Table 1 presents unfiltered simulated business-cycle moments, and Table 2 presents HP-filtered (with quarterly smoothing parameter $\lambda = 1600$) simulated business-cycle moments.

	GDP	**C**	**I**	**N**	**TFP**
Volatility	3.01	1.78	7.65	1.06	1.92
Rel. vol. wrt GDP	1	0.59	2.53	0.35	0.64
Autocorr.	0.93	0.97	0.92	0.91	0.92
Corr. w/GDP	1	0.92	0.96	0.89	0.98

Table 1. Simulated business-cycle moments, non-filtered.

	GDP	**C**	**I**	**N**	
Volatility	1.22	0.47	3.67	0.54	
Rel. vol. wrt GDP	1	0.38	3.00	0.44	
Autocorr.	0.69	0.72	0.69	0.68	
Corr. w/GDP	1	0.98	0.99	0.99	

Table 2. Simulated business-cycle moments, HP-filtered.