Economics 8861.01

Project 2 – (Partial/Sketch of) Suggested Solutions

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The deterministic steady state at the initial parameter set, which corresponds to the Hagedorn and Manovskii (2008 *AER*) style calibration:

[nss/lfpss, uess/lfpss, wss, thetass, vss] = [0.7713, 0.2287, 0.9588, 0.2525, 0.0578]

The deterministic steady state at the final parameter set, which corresponds to the Shimer (2005 *AER*) style calibration:

[nss/lfpss, uess/lfpss, wss, thetass, vss] = [0.8659, 0.1341, 0.9365, 0.7459, 0.1000]

In what follows, the setup of the period-t state vector and period-t costate vector is (using the notation of Schmitt-Grohe and Uribe (2004 *JECD*),

$$x_t = \left[n_t, z_t \right]' \qquad y_t = [w_t, \theta_t, ue_t]'$$

(other setups are possible for the costate vector y_t, but the state vector x_t **MUST** be declared as above).

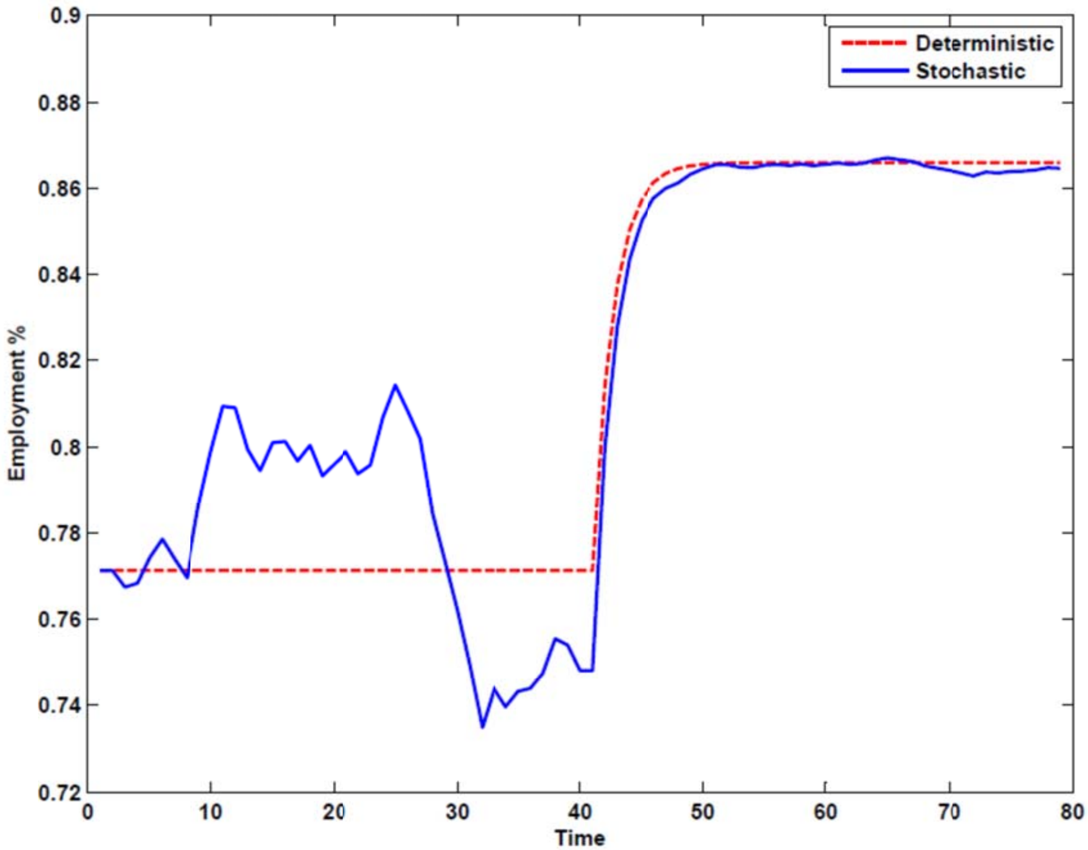
The linearized (in levels) decision rules around the initial steady state are

$$g_x = \begin{bmatrix} 0 & 0.2450 \\ 0 & 7.8007 \\ -0.6731 & -1.0692 \end{bmatrix}, \qquad h_x = \begin{bmatrix} 0.6731 & 1.0692 \\ 0 & 0.95 \end{bmatrix}$$

In the period in which the once-and-for-all shock occurs, and forever thereafter, the (linearized) decision rules are computed around the final deterministic steady state. The deterministic transition, **as well as the "stochastic" transition (i.e., with stochastic TFP shocks occurring),** from the initial steady state to the final steady state is governed by

$$g_x = \begin{bmatrix} 0 & 0.9159 \\ 0 & 1.6637 \\ -0.5468 & -0.0704 \end{bmatrix}, \qquad h_x = \begin{bmatrix} 0.5468 & 0.0704 \\ 0 & 0.95 \end{bmatrix}.$$

A graphical representation of the deterministic transition for the employment rate appears in blue, **and one particular (randomly chosen) stochastic transition appears in red,** below:



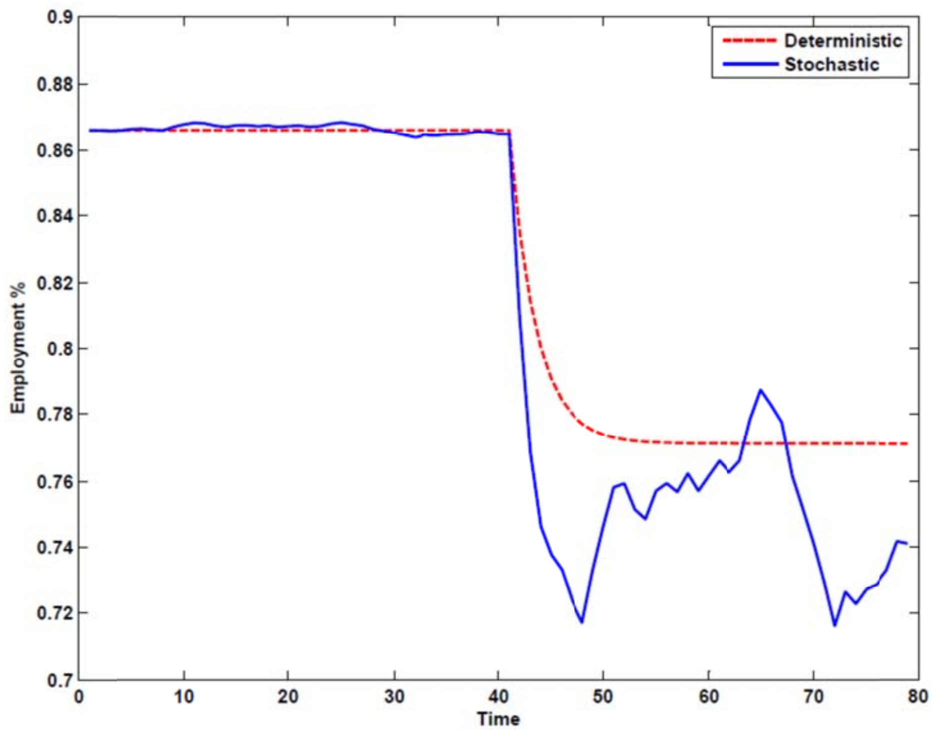
Simple observation reveals that:

- The transition is **quick** – within four periods (four quarters), employment is within a tolerance of 10e-6 of the final deterministic steady state level (nss = 0.8921); within eight periods (eight quarters), employment is within a tolerance of 10e-10 of the final deterministic steady state. **Depending on which criteria you choose, the transition occurs within four quarters (*S* = 4) or within eight quarters (*S* = 8) – which is "quick."**

- The employment rate (and hence the unemployment rate, given the definition in this setup of $u + n = 1$) is more volatile under the Hagedorn and Manovskii-type calibration than under the Shimer-type calibration.

Given how quick the transition dynamics are, the number of periods T that are simulated around each deterministic steady state need not be long. In the graphical example above, $T < 20$.

However, conducting the transition dynamics in the **OTHER** direction (i.e., reversing the initial and final sets of parameters) leads to a **SLOWER** transition (as a couple of papers helpfully experimented with and reported to shed more light on the model and its results):



In this case,

- Within **twenty** periods (twenty quarters), employment is within a tolerance of 1e-6 of the final deterministic steady state level (nss = 0.7711); within **forty** periods (forty quarters), employment is within a tolerance of 1e-10 of the final deterministic steady state. **Depending on which criteria you choose, the transition occurs within twenty quarters (*S* = 20) or within forty quarters (*S* = 40) – regardless of which criteria, the transition is clearly not as "quick" as the transition in the other direction.**

Given the slower transition dynamics in this case, the number of periods T that are simulated around each deterministic steady state needs to longer than above. In this example, T around 50 is reasonable.

Finally, as in the baseline case, employment and thus unemployment is more volatile under the HM calibration than under the Shimer calibration. This is the main point of comparing and contrasting fluctuations of the two calibrations; the rest is in some "details" – e.g., what is the precise matching function, what is the timing of the LOM for employment (the Project description provided one specific LOM), what is the solution method (linear vs. nonlinear), etc.

Note: the deterministic steady state using Shimer's (2005 *AER*) parameter values of $\eta = 0.72$ and $b = 0.40$ is

[nss/lfpss, uess/lfpss, wss, thetass, vss] = [0.8535, 0.1465, 0.9651, 0.3697, 0.0401],

and the linearized (in levels) decision rules around this steady state are

$$g_x = \begin{bmatrix} 0 & 0.9388 \\ 0 & 0.6077 \\ -0.5686 & -0.0248 \end{bmatrix}, \quad h_x = \begin{bmatrix} 0.5686 & 0.0248 \\ 0 & 0.95 \end{bmatrix}.$$

	ue	***vac***	***θ***	***w***
Volatility	5.95	2.47	15.18	4.83

Table 1. Simulated business-cycle moments using HM parameters, unfiltered.

	ue	***vac***	***θ***	***w***
Volatility	0.69	0.55	4.91	5.03

Table 2. Simulated business-cycle moments using α = 0.50, η = 0.50, and b = 0.50, unfiltered.

	ue	***vac***	***θ***	***w***
Volatility	0.74	0.30	2.20	5.18

Table 3. Simulated business-cycle moments using α = 0.72, η = 0.72, and b = 0.40.