NOMINAL RIGIDITIES IN A DSGE MODEL: THE CANONICAL NEW KEYNESIAN MODEL

FEBRUARY 13, 2017

THE AGGREGATE SUPPLY BLOCK

❒ **Optimal pricing and aggregate inflation described by**

$$x_t^1 = p_{it}^{*1-\varepsilon} y_t \frac{\varepsilon - 1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} \left(\frac{p_{it}^*}{p_{it+1}^*} \right)^{1-\varepsilon} x_{t+1}^1 \right\}$$

$$x_t^2 = p_{it}^{*-\varepsilon} y_t mc_t + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{1+\varepsilon} \left(\frac{p_{it}^*}{p_{it+1}^*} \right)^{-\varepsilon} x_{t+1}^2 \right\}$$

$$x_t^1 = x_t^2$$

$$1 = \alpha \pi_t^{\varepsilon - 1} + (1 - \alpha) p_t^{*1-\varepsilon}$$

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Note DROPPED law of motion for price dispersion…around a zero-inflation steady-state, average dispersion = 0…see Yun (2005 *AER*) for NK model with LOM for s_t

Condense and log-linearize around zero-inflation (π = 1) steady state

See Woodford text (2003), Walsh text (2003, Chapter 5) for details

Measures sensitivity of inflation to marginal cost

All measured in log (percent) deviations from steady state

$$\hat{z}_t \equiv \ln(z_t / \bar{z})$$

$$\boxed{\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa m\hat{c}_t}$$

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$$

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- **Reminiscent of "old" Keynesian Phillips curve**
  - **Relation between current inflation and current measure of activity**
  - **Here couched in language of "gaps:" inflation gap and output gap**
  - **Different from "old" Phillips curve because of presence of expected future inflation: expectations-augmented Phillips Curve**

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  - **"Divine Coincidence" (Blanchard and Gali 2007 *JMCB*, 2010 *AEJ:Macro*)**

- **Introduce a "cost-push" shock:** $\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + u_t$
  - **Idea: captures variation in *mc* due to factors other than output gap (aka "excess demand")**

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  THE NEW KEYNESIAN PHILLIPS CURVE

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Some sources of u_t variation: exogenous markup variation; sticky nominal *wages*; pricing "complementarities;" fin. frictions

THE AGGREGATE DEMAND BLOCK

- Completely ignore transactions role of money
  - No money demand…a "cashless" environment
  - Money plays only a unit-of-account role
  - Justification: cash balances a tiny fraction of wealth

Technical Aside: as long as money enters household preferences additively-separably, *could* have money, but wouldn't affect the *real* equilibrium at all

- Critical household optimality condition: consumption-savings

$$u_{c_t} = \beta R_t E_t \left[u_{c_{t+1}} \cdot \frac{1}{\pi_{t+1}} \right]$$

Derived off a riskless one-period NOMINAL asset

Gross nominal interest rate

In closed-economy equilibrium, same as investment

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With CRRA utility (risk-aversion σ), log-linearization yields

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THE NEW KEYNESIAN "IS" CURVE – aka AGGREGATE DEMAND CURVE

$$\boxed{\hat{y}_t = -\frac{1}{\sigma}\underbrace{\left[\hat{R}_t - E_t\hat{\pi}_{t+1}\right]}_{\text{Ex-ante real interest rate}} + E_t\hat{y}_{t+1} + g_t}$$

Force in "aggregate demand shock" (i.e., govt spending)

With trivial resource constraint: $y_t = c_t$

SOME SIMPLE ANALYSIS

- ❐ **New Keynesian Phillips Curve/Aggregate Supply Curve**

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + u_t$$

- ❐ **AS slopes upwards** $\partial \hat{\pi}_t / \partial \hat{y}_t > 0$
- ❐ **Higher expected future inflation shifts AS outwards**
  - ❐ **Interpretation: the expectations of future inflation that policy-makers induce have an effect on the period-*t* equilibrium**
  - ❐ **Expectations missing in "classical" Phillips Curve/AS-AD frameworks**
- ❐ **Current inflation a function of all future output gaps and cost-push shocks (recursively substitute for π_t)**

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- **Current inflation a function of all future output gaps and cost-push shocks (recursively substitute for** π_t**)**

- **Reference empirical value(s) of** κ**: anywhere from ≈ 0.05 to ≈ 0.3**
  - **Sensitive to whether** $mc \approx y$**…**
  - **Sensitive to precise empirical measure of** ***mc***
    - **"Unit labor cost" (i.e.,** ***w/mpn*****)**
    - **"Employment Cost Index" (ECI)**
    - **Data: what does "unit labor cost" or "ECI" measure?…**

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- ❐ **AD slopes downwards with respect to ex-ante real interest rate**

- ❐ **Higher expected future output shifts AD outwards**
  - ❐ **Consumption-smoothing foundation**

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- **AD slopes downwards with respect to ex-ante real interest rate**

- **Higher expected future output shifts AD outwards**
  - **Consumption-smoothing foundation**

- **Current output (gap) a function of all future real interest rates, which depend on expected policy and expected inflation (recursively substitute for y_t)**

- **Reference value of σ: 1 (based on RBC models)**

- **What "determines" policy?**

MONETARY POLICY

- **To "close" the model, need to specify how policy is set**
- **Taylor Rule (or variants of) the most common**

$$\hat{R}_t = \delta_\pi \hat{\pi}_t + \delta_y \hat{y}_t$$

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- ❐ **Taylor's coefficients: $\delta_\pi = 1.5$, $\delta_y = 0.5$**

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The subject of MUCH theoretical and empirical interest

- Taylor Principle: $\delta_\pi > 1$ required for "good policy"
  - Basic idea: $\delta_\pi > 1$ ensures real interest rate rises more than one-for-one with a rise in inflation – the "dampening" effect on AD lowers inflation
- Empirical evidence
  - $\delta_\pi < 1$ pre-Volcker; $\delta_\pi > 1$ Volcker/Greenspan; δ_π ?.... Bernanke....

MONETARY POLICY

- **Why an interest rate rule and not a money-supply rule?**

- **Historical/Practical Reasons**
  - **Experience with monetarism (in U.S. and U.K.) not very successful**
  - **Fed's original mandate was to smooth out the seasonality of interest rates – and hence (explicitly or implicitly…) likely operated in terms of interest rates**
  - **Hard to communicate: "We are going to increase the base money supply, *M0*, by 0.72 percent this week…"**

- **Theoretical Reasons**
  - **Money demand "shifts" not well understood**
  - **Interest rates govern basic decision margins**
    - **Intertemporal margin**
    - **(Cash-credit margin in a non-cashless model)**

THE THREE-EQUATION MODEL

- **The basic framework**

Phillips Curve/Aggregate Supply

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Monetary Policy (interest rate) Rule

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- **(A couple of…) critical questions**
  - **How to define the "target" variables (underneath the hatted variables…)?**
  - **How to allow more "flexibility" in the monetary policy rule?**
- **A constant avalanche of analysis using this basic framework**
  - **See Woodford text (2003) or Gali (2010) text for a *start* on this literature…**