STICKY NOMINAL WAGES AND THE LINEAR-QUADRATIC APPROACH

FEBRUARY 13, 2017

THE THREE-EQUATION MODEL

❒ **The basic framework**

Phillips Curve / Aggregate Supply

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + u_t$$

Absent a "cost-push" shock, no tradeoff between stabilizing inflation and stabilizing output (Blanchard and Gali 2007 *JMCB*, 2010 *AEJ:Macro* "divine coincidence")

IS Curve / Aggregate Demand

$$\hat{y}_t = -\frac{1}{\sigma}\left[\hat{R}_t - E_t \hat{\pi}_{t+1}\right] + E_t \hat{y}_{t+1} + g_t$$

Monetary Policy (interest rate) Rule

$$\hat{R}_t = \delta_\pi \hat{\pi}_t + \delta_y \hat{y}_t$$

THE THREE-EQUATION MODEL

- **The basic framework**

Phillips Curve / Aggregate Supply

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + u_t$$

Absent a "cost-push" shock, no tradeoff between stabilizing inflation and stabilizing output (Blanchard and Gali 2007 *JMCB*, 2010 *AEJ:Macro* "divine coincidence")

IS Curve / Aggregate Demand

$$\hat{y}_t = -\frac{1}{\sigma}\left[\hat{R}_t - E_t \hat{\pi}_{t+1}\right] + E_t \hat{y}_{t+1} + g_t$$

Monetary Policy (interest rate) Rule

$$\hat{R}_t = \delta_\pi \hat{\pi}_t + \delta_y \hat{y}_t$$

- **Optimal policy problem**
  - **Optimize criterion function subject to Phillips Curve and IS curve**

- **What is the correct / natural / interesting / relevant criterion function?**
  - **Typical NK criterion: expected lifetime utility of the representative agent expressed as a second-order approximation → yields a central bank loss function quadratic in inflation gaps and output gaps**

DERIVING CENTRAL BANK OBJECTIVE

- **Lifetime household utility**

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(n_t) \right]$$

DERIVING CENTRAL BANK OBJECTIVE

❐ **Lifetime household utility**

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(n_t) \right]$$

Expand around PARETO-EFFICIENT steady-state (*c, n*)

Pareto-efficient steady-state "achieved" by assuming

- **Long-run inflation = 0**
- **Sufficient fiscal instruments exist to correct long-run distortions due to monopoly power (in particular, a proportional subsidy to labor income to undo the effects of an inefficient real wage, financed with lump-sum tax)**

DERIVING CENTRAL BANK OBJECTIVE

- **Lifetime household utility**

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(n_t) \right]$$

Expand around PARETO-EFFICIENT steady-state (*c, n*)

Pareto-efficient steady-state "achieved" by assuming

- Long-run inflation = 0

- Sufficient fiscal instruments exist to correct long-run distortions due to monopoly power (in particular, a proportional subsidy to labor income to undo the effects of an inefficient real wage, financed with lump-sum tax)

SECOND-order Taylor expansion

Some terms drop out due to assumption of long-run Pareto efficiency

…many many many more steps… (see Woodford and Rotemberg 1998 *NBER* Appendix)

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \hat{y}_t^2 \right]$$

Central Bank Loss Function – depends on only output gaps and inflation gaps

Unconstrained optimum clearly involves setting all inflation gaps and output gaps to zero: achieves zero loss

OPTIMAL POLICY PROBLEM

- ❐ **Minimize**

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \hat{y}_t^2 \right]$$

subject to implementation as a (sticky-price) private-sector equilibrium

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + u_t$$

$$\hat{y}_t = -\frac{1}{\sigma}\left[\hat{R}_t - E_t \hat{\pi}_{t+1} \right] + E_t \hat{y}_{t+1} + g_t$$

A LINEAR-QUADRATIC PROBLEM

OPTIMAL POLICY PROBLEM

- **Minimize**

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \hat{y}_t^2 \right]$$

subject to implementation as a (sticky-price) private-sector equilibrium

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + u_t$$

$$\hat{y}_t = -\frac{1}{\sigma}\left[\hat{R}_t - E_t \hat{\pi}_{t+1} \right] + E_t \hat{y}_{t+1} + g_t$$

A LINEAR-QUADRATIC PROBLEM

Monetary Policy (interest rate) Rule

$$\hat{R}_t = \delta_\pi \hat{\pi}_t + \delta_y \hat{y}_t$$

- **Assume**
  - **Commitment**
  - **Timeless perspective**

- **Choose δ_π and δ_y to minimize monetary-authority loss function**
  - **For given processes for the u_t and g_t shocks**
  - **Analytical results sometimes feasible – because LQ structure**
  - **Bigger models rely on computational methods**

OPTIMAL POLICY: FURTHER ISSUES

- ❐ **Richer interest-rate rules**

Data suggests high persistence of monetary-policy interest rates – thus allow for (optimal) "interest-rate smoothing"

$$\hat{R}_t = \delta_r \hat{R}_{t-1} + (1-\delta_r)\left[\delta_\pi \hat{\pi}_t + \delta_y \hat{y}_t\right]$$

Allow for lagged output and inflation gaps in policy rule?

$$\hat{R}_t = \delta_r \hat{R}_{t-1} + (1-\delta_r)\left[\sum_{i=0}^{K_\pi} \delta_{\pi_i} \hat{\pi}_{t-i} + \sum_{i=0}^{K_y} \delta_{y_i} \hat{y}_{t-i}\right]$$

And/or allow for expected/forecasted output and inflation gaps?

OPTIMAL POLICY: FURTHER ISSUES

- **Richer interest-rate rules**

Data suggests high persistence of monetary-policy interest rates – thus allow for (optimal) “interest-rate smoothing”

$$\hat{R}_t = \delta_r \hat{R}_{t-1} + (1-\delta_r)\left[\delta_\pi \hat{\pi}_t + \delta_y \hat{y}_t\right]$$

Allow for lagged output and inflation gaps in policy rule?

$$\hat{R}_t = \delta_r \hat{R}_{t-1} + (1-\delta_r)\left[\sum_{i=0}^{K_\pi} \delta_{\pi_i} \hat{\pi}_{t-i} + \sum_{i=0}^{K_y} \delta_{y_i} \hat{y}_{t-i}\right]$$

And / or allow for expected / forecasted output and inflation gaps?

- **Relevant “long-run targets?” – i.e., how to define $\hat{x}_t$?**
  - **Target a constant (steady-state)?**
  - **Target the “flexible-price” time-varying level of inflation / output?**
    - **Underlying RBC model provides natural benchmark time-varying inflation / output – then define gaps relative to these**

NOMINAL WAGE RIGIDITY

- **Nominal rigidities originally discussed in terms of wages, not prices**
- **But somehow dismissed as modern macro literature evolved**
    - **Basic reason (roughly): are wage payments allocative?**
        - **Goodfriend and King (2001): "…potential allocative inefficiencies from infrequent setting of nominal wages are likely to be offset in the context of long-term employment relationships.." and "…unlikely to influence recommendations for policy."**

NOMINAL WAGE RIGIDITY

- **Nominal rigidities originally discussed in terms of wages, not prices**
- **But somehow dismissed as modern macro literature evolved**
  - **Basic reason (roughly): are wage payments allocative?**
    - **Goodfriend and King (2001): "…potential allocative inefficiencies from infrequent setting of nominal wages are likely to be offset in the context of long-term employment relationships.." and "…unlikely to influence recommendations for policy."**
    - **House and Basu (2016, *Handbook of Macroeconomics*): "Modern monetary business-cycle models rely heavily on price and wage rigidity. While there is substantial evidence that prices do not adjust frequently, there is much less evidence on whether wage rigidity is an important feature of real world labor markets….the wages for newly-hired workers […] respond strongly to identified monetary policy innovations….[and] a model with implicit contracts between workers and firms and a flexible allocative wage replicates these patterns well."**

Erceg, Henderson, Levin (2000 JME)

NOMINAL WAGE RIGIDITY

- Nominal rigidities originally discussed in terms of wages, not prices
- But somehow dismissed as modern macro literature evolved
  - Basic reason (roughly): are wage payments allocative?
    - Goodfriend and King (2001): "…potential allocative inefficiencies from infrequent setting of nominal wages are likely to be offset in the context of long-term employment relationships.." and "…unlikely to influence recommendations for policy."
- EHL 2001: introduce sticky nominal wages in DSGE NK model
  - Adapt Calvo framework
  - Main results
    - Sticky nominal wages introduce an endogenous shifter in the price-Phillips curve (i.e., a source of the "cost-push" shock u_t)
    - Optimal policy
      - Stabilize a weighted average of output gap, nominal price inflation, and nominal wage inflation
      - Complete price inflation stability no longer optimal
- EHL formulation of sticky wages has become a standard in "medium-scale" DSGE models
  - Christiano, Eichenbaum, and Evans (2005 *JPE*), Smets and Wouters (2007 *AER*)

NOMINAL WAGE RIGIDITY

- Dixit-Stiglitz sticky-price structure
    - Continuum of differentiated goods
        - Each goods producer has monopoly power
        - Each goods producer faces exogenous probability of re-setting nominal price
    - Final goods producer “packages” intermediate goods and sells composite final good

“Retailer”

NOMINAL WAGE RIGIDITY

- Dixit-Stiglitz sticky-price structure
  - Continuum of differentiated goods
    - Each goods producer has monopoly power
    - Each goods producer faces exogenous probability of re-setting nominal price
  - Final goods producer “packages” intermediate goods and sells composite final good

“Retailer”

- Dixit-Stiglitz sticky-wage structure (as first formulated by EHL)
  - Continuum of differentiated labors
    - Each labor supplier has monopoly power
    - Each labor supplier faces exogenous probability of re-setting nominal wage
  - Final labor supplier “packages” intermediate labors and sells composite final labor

“Employment agency”

- Dixit-Stiglitz-Calvo machinery adapted to the neoclassical labor market

EHL MODEL – EMPLOYMENT AGENCY

- **Representative employment agency (perfectly-competitive)**
  - **Aggregates individual households' labors**

$$N_t = \left[\int_0^1 n_{it}^{\frac{\varepsilon^w - 1}{\varepsilon^w}} di \right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}}$$

ε^w the elasticity of substitution between different types of labor

  - **Sells N_t to intermediate-goods firms (i.e., composite labor needed for goods production)**

EHL MODEL – EMPLOYMENT AGENCY

- **Representative employment agency (perfectly-competitive)**
  - **Aggregates individual households' labors**

$$N_t = \left[\int_0^1 n_{it}^{\frac{\varepsilon^w - 1}{\varepsilon^w}} di\right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}}$$

ε^w the elasticity of substitution between different types of labor

  - **Sells N_t to intermediate-goods firms (i.e., composite labor needed for goods production)**

- **Employment agency profit-maximization problem**

$$\max_{n_{it}} W_t N_t - \int_0^1 W_{it} n_{it} di$$

Revenues: aggregate wage payments from firms

Costs: individual wage payments to differentiated households

$\downarrow$ profit maximization

$$n_{it} = \left[\frac{W_{it}}{W_t}\right]^{-\varepsilon^w} N_t$$

Usual Dixit-Stiglitz demand functions (for each type of labor *i*)

EHL MODEL – HOUSEHOLDS

***Exogenous probability* of *not* being able to (re-)set wage**

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left[u(c_{it+s}) + v(n_{it+s})\right]$$

Full set of state-contingent securities allows each hh *i* to insure against its own idiosyncratic wage – i.e., full consumption insurance

Households *i*'s individual labor income

Time *t* budget constraint

$$P_t c_{it} + M_{it} - M_{it-1} + B_{it} - R_t B_{it-1} = (1+\tau^w) W_{it} n_{it} + P_t pr_t + T_t$$

EHL MODEL – HOUSEHOLDS

***Exogenous probability* of *not* being able to (re-)set wage**

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left[u(c_{it+s}) + v(n_{it+s}) \right]$$

Full set of state-contingent securities allows each hh *i* to insure against its own idiosyncratic wage – i.e., full consumption insurance

Households *i*'s individual labor income

Time *t* budget constraint

$$P_t c_{it} + M_{it} - M_{it-1} + B_{it} - R_t B_{it-1} = (1+\tau^w) W_{it} n_{it} + P_t pr_t + T_t$$

Proportional subsidy to labor income financed by lump-sum tax corrects long-run distortions due to BOTH labor monopoly power AND goods monopoly power

EHL MODEL – HOUSEHOLDS

Exogenous probability **of *not* being able to (re-)set wage**

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left[u(c_{it+s}) + v(n_{it+s}) \right]$$

Full set of state-contingent securities allows each hh *i* to insure against its own idiosyncratic wage – i.e., full consumption insurance

Households *i*'s individual labor income

Time *t* budget constraint

$$P_t c_{it} + M_{it} - M_{it-1} + B_{it} - R_t B_{it-1} = (1+\tau^w) W_{it} n_{it} + P_t pr_t + T_t$$

Proportional subsidy to labor income financed by lump-sum tax corrects long-run distortions due to BOTH labor monopoly power AND goods monopoly power

hh *i* also takes as given demand function for *its* labor

$$n_{it} = \left[\frac{W_{it}}{W_t} \right]^{-\varepsilon^w} N_t$$

↓ **Substitute using demand function**

EHL MODEL – HOUSEHOLDS

Note time subscripts!

***Exogenous probability* of *not* being able to (re-)set wage**

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left[u(c_{it+s}) + v\left(\left[\frac{W_{it}}{W_{t+s}} \right]^{-\varepsilon^w} N_{t+s} \right) \right]$$

$$P_t c_{it} + M_{it} - M_{it-1} + B_{it} - R_t B_{it-1} = (1+\tau^w) W_{it} \left[\frac{W_{it}}{W_t} \right]^{-\varepsilon^w} N_t + P_t pr_t + T_t$$

EHL MODEL – HOUSEHOLDS

***Exogenous probability* of *not* being able to (re-)set wage**

Note time subscripts!

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left[u(c_{it+s}) + v\left(\left[\frac{W_{it}}{W_{t+s}} \right]^{-\varepsilon^w} N_{t+s} \right) \right]$$

$$P_t c_{it} + M_{it} - M_{it-1} + B_{it} - R_t B_{it-1} = (1+\tau^w) W_{it} \left[\frac{W_{it}}{W_t} \right]^{-\varepsilon^w} N_t + P_t pr_t + T_t$$

- **FOCs with respect to**
  - ***c_{it}***
  - ***W_{it}***
  - **And assets**

EHL MODEL – OPTIMAL WAGE-SETTING

FOC wrt W_{it}

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon} N_{t+s} \left[-\varepsilon^w v'\left(n_{it+s}\right) \left(\frac{W_{it}}{W_{t+s}} \right)^{-1} + (1-\varepsilon^w)(1+\tau^w) u'(c_{it+s}) \right] \right\} = 0$$

EHL MODEL – OPTIMAL WAGE-SETTING

FOC wrt W_{it}

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon} N_{t+s} \left[-\varepsilon^w v'\left(n_{it+s}\right) \left(\frac{W_{it}}{W_{t+s}} \right)^{-1} + (1-\varepsilon^w)(1+\tau^w) u'(c_{it+s}) \right] \right\} = 0$$

Rewrite: multiply by $1/(\varepsilon^w\text{-}1)$

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon} N_{t+s} \left[\frac{-\varepsilon^w}{\varepsilon^w - 1} v'\left(n_{it+s}\right) \left(\frac{W_{it}}{W_{t+s}} \right)^{-1} - (1+\tau^w) u'(c_{it+s}) \right] \right\} = 0$$

EHL MODEL – OPTIMAL WAGE-SETTING

FOC wrt W_{it}

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon} N_{t+s} \left[-\varepsilon^w v'\left(n_{it+s}\right) \left(\frac{W_{it}}{W_{t+s}} \right)^{-1} + (1-\varepsilon^w)(1+\tau^w) u'(c_{it+s}) \right] \right\} = 0$$

Rewrite: multiply by 1 / (ε^w-1)

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon} N_{t+s} \underbrace{\left[\frac{-\varepsilon^w}{\varepsilon^w - 1} v'\left(n_{it+s}\right) \left(\frac{W_{it}}{W_{t+s}} \right)^{-1} - (1+\tau^w) u'(c_{it+s}) \right]} \right\} = 0$$

If wages are completely flexible (i.e., if α_w = 0):

$$-\frac{\varepsilon^w}{\varepsilon^w - 1} \underbrace{\frac{v'(n_{it})}{u'(c_{it})}}_{\textit{mrs} \text{ (between consumption and leisure) of hh } i} = (1+\tau^w) \underbrace{\left(\frac{W_{it}}{W_t} \right)}_{\text{relative wage set by hh } i}$$

EHL MODEL – OPTIMAL WAGE-SETTING

FOC wrt W_{it}

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}}\right)^{-\varepsilon} N_{t+s} \left[-\varepsilon^w v'\left(n_{it+s}\right) \left(\frac{W_{it}}{W_{t+s}}\right)^{-1} + (1-\varepsilon^w)(1+\tau^w) u'(c_{it+s}) \right] \right\} = 0$$

↓ **Rewrite: multiply by 1 / (ε^w-1)**

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}}\right)^{-\varepsilon} N_{t+s} \underbrace{\left[\frac{-\varepsilon^w}{\varepsilon^w-1} v'\left(n_{it+s}\right) \left(\frac{W_{it}}{W_{t+s}}\right)^{-1} - (1+\tau^w) u'(c_{it+s}) \right]}_{} \right\} = 0$$

If wages are completely flexible (i.e., if α_w = 0):

$$-\frac{\varepsilon^w}{\varepsilon^w-1} \underbrace{\frac{v'(n_{it})}{u'(c_{it})}}_{\text{mrs (between consumption and leisure) of hh } i} = (1+\underset{= 0}{\cancel{\tau^w}}) \underbrace{\left(\frac{W_{it}}{W_t}\right)}_{\text{relative wage set by hh } i}$$

A Dixit-Stiglitz pricing result

$$\boxed{-\frac{\varepsilon^w}{\varepsilon^w-1} mrs_{it} = \left(\frac{W_{it}}{W_t}\right)}$$

Relative wage is a markup over mrs

EHL MODEL – OPTIMAL WAGE-SETTING

FOC wrt W_{it}

$$E_t\sum_{s=0}^{\infty}\beta^s\alpha_w^s\left\{\left(\frac{W_{it}}{W_{t+s}}\right)^{-\varepsilon}N_{t+s}\left[-\varepsilon^w v'\left(n_{it+s}\right)\left(\frac{W_{it}}{W_{t+s}}\right)^{-1}+(1-\varepsilon^w)(1+\tau^w)u'(c_{it+s})\right]\right\}=0$$

Rewrite: multiply by 1 / (ε^w-1)

$$E_t\sum_{s=0}^{\infty}\beta^s\alpha_w^s\left\{\left(\frac{W_{it}}{W_{t+s}}\right)^{-\varepsilon}N_{t+s}\underbrace{\left[\frac{-\varepsilon^w}{\varepsilon^w-1}v'\left(n_{it+s}\right)\left(\frac{W_{it}}{W_{t+s}}\right)^{-1}-(1+\tau^w)u'(c_{it+s})\right]}\right\}=0$$

If wages are completely flexible (i.e., if α_w = 0):

$$-\frac{\varepsilon^w}{\varepsilon^w-1}\underbrace{\frac{v'(n_{it})}{u'(c_{it})}}=(1+\cancelto{= 0}{\tau^w})\underbrace{\left(\frac{W_{it}}{W_t}\right)}$$

mrs **(between consumption and leisure) of hh *i***

relative wage set by hh *i*

A Dixit-Stiglitz pricing result

$$\boxed{-\frac{\varepsilon^w}{\varepsilon^w-1}mrs_{it}=\left(\frac{W_{it}}{W_t}\right)}$$

Relative wage is a markup over mrs

mrs is the household's "marginal cost" of producing / supplying labor

EHL MODEL – OPTIMAL WAGE-SETTING

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon} N_{t+s} \left[\frac{-\varepsilon^w}{\varepsilon^w - 1} v'(n_{it+s}) \left(\frac{W_{it}}{W_{t+s}} \right)^{-1} - (1+\tau^w) u'(c_{it+s}) \right] \right\} = 0$$

Rewrite: multiply each term by W_{t+s}/W_{t+s} ,

multiply each term by $u'(c_{it+s})$ / $u'(c_{it+s})$, and

multiply entire expression by W_{it}

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ u'(c_{it+s}) W_{t+s} \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon} N_{t+s} \left[\frac{-\varepsilon^w}{\varepsilon^w - 1} mrs_{it+s} - (1+\tau^w) \frac{W_{it}}{W_{t+s}} \right] \right\} = 0$$

EHL MODEL – OPTIMAL WAGE-SETTING

$$E_t\sum_{s=0}^{\infty}\beta^s\alpha_w^s\left\{\left(\frac{W_{it}}{W_{t+s}}\right)^{-\varepsilon}N_{t+s}\left[\frac{-\varepsilon^w}{\varepsilon^w-1}v'\left(n_{it+s}\right)\left(\frac{W_{it}}{W_{t+s}}\right)^{-1}-(1+\tau^w)u'(c_{it+s})\right]\right\}=0$$

Rewrite: multiply each term by W_{t+s}/W_{t+s} ,

multiply each term by $u'(c_{it+s})$ / $u'(c_{it+s})$, and

multiply entire expression by W_{it}

$$E_t\sum_{s=0}^{\infty}\beta^s\alpha_w^s\left\{u'(c_{it+s})W_{t+s}\left(\frac{W_{it}}{W_{t+s}}\right)^{-\varepsilon}N_{t+s}\left[\frac{-\varepsilon^w}{\varepsilon^w-1}mrs_{it+s}-(1+\tau^w)\frac{W_{it}}{W_{t+s}}\right]\right\}=0$$

$$P_tx_t^1\equiv E_t\sum_{s=0}^{\infty}\beta^s\alpha_w^s\left\{u'(c_{it+s})W_{t+s}\left(\frac{W_{it}}{W_{t+s}}\right)^{-\varepsilon}N_{t+s}\left[\frac{-\varepsilon^w}{\varepsilon^w-1}mrs_{it+s}\right]\right\}$$

PDV of nominal marginal cost (ie, mrs) until next wage change

$$P_tx_t^2\equiv E_t\sum_{s=0}^{\infty}\beta^s\alpha_w^s\left\{u'(c_{it+s})W_{t+s}\left(\frac{W_{it}}{W_{t+s}}\right)^{-\varepsilon}N_{t+s}\left[-(1+\tau^w)\frac{W_{it}}{W_{t+s}}\right]\right\}$$

PDV of nominal marginal revenue (ie, labor income) until next wage change

Next step: express x^1 and x^2 recursively -> WAGE PHILLIPS CURVE expressed compactly as $x^1 = x^2$

EHL MODEL – EQUILIBRIUM

- **Price Phillips Curve (i.e., FOC of sticky-price firm)**
- **Household Euler equation**
- **Aggregate resource constraint**
    - **Deadweight loss from both sticky prices and sticky wages**

EHL set τ^w such that no deadweight loss

- **Wage Phillips Curve (aka consumption-leisure optimality condition)**
- **Law of motion for real wage**

EHL MODEL – EQUILIBRIUM

- Price Phillips Curve (i.e., FOC of sticky-price firm)
- Household Euler equation
- Aggregate resource constraint
  - Deadweight loss from both sticky prices and sticky wages

EHL set τ^w such that no deadweight loss

- Wage Phillips Curve (aka consumption-leisure optimality condition)
- Law of motion for real wage
  - A non-trivial equilibrium condition in models with sticky nominal wages + sticky nominal prices *and/or* explicit money demand

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}$$

$$\text{Growth of real wage} = \frac{\text{Nominal wage inflation}}{\text{Nominal price inflation}}$$

EHL MODEL – EQUILIBRIUM

- **Price Phillips Curve (i.e., FOC of sticky-price firm)**
- **Household Euler equation**
- **Aggregate resource constraint**
  - **Deadweight loss from both sticky prices and sticky wages**

EHL set τ^w such that no deadweight loss

- **Wage Phillips Curve (aka consumption-leisure optimality condition)**
- **Law of motion for real wage**
  - **A non-trivial equilibrium condition in models with sticky nominal wages + sticky nominal prices *and/or* explicit money demand**

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}$$

$$\text{Growth of real wage} = \frac{\text{Nominal wage inflation}}{\text{Nominal price inflation}}$$

  - **Intuition**
    - **Sticky nominal price-setting and/or money demand influences price inflation**
    - **Sticky nominal wage-setting influences wage inflation**
    - **Technology influences real wage growth**
    - **No reason that all three of these are compatible with each other (simple example: w_t = TFP every period)**

EHL MODEL – EQUILIBRIUM

- Price Phillips Curve (i.e., FOC of sticky-price firm)
- Household Euler equation
- Aggregate resource constraint
  - Deadweight loss from both sticky prices and sticky wages

EHL set τ^w such that no deadweight loss

- Wage Phillips Curve (aka consumption-leisure optimality condition)
- Law of motion for real wage
  - A non-trivial equilibrium condition in models with sticky nominal wages + sticky nominal prices *and/or* explicit money demand

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}$$

In sticky-wage economy, this condition is an allocative condition, not simply an identity – hence part of the definition of equilibrium

$$\text{Growth of real wage} = \frac{\text{Nominal wage inflation}}{\text{Nominal price inflation}}$$

See Chugh (2006 *RED* p. 691-692)

- Intuition
  - Sticky nominal price-setting and/or money demand influences price inflation
  - Sticky nominal wage-setting influences wage inflation
  - Technology influences real wage growth
  - No reason that all three of these are compatible with each other (simple example: w_t = TFP every period)

EHL MODEL – OPTIMAL POLICY

- **Optimal policy problem – maximize representative household's lifetime utility subject to all private-sector equilibrium conditions**

- **In LQ form (see EHL Appendix B), minimize**

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \hat{y}_t^2 + \theta \hat{\pi}_t^{w2} \right]$$

Relative importance of stabilizing *wage* inflation versus stabilizing *price* inflation depends on relative stickiness of prices and wages – NO divine coincidence here

subject to (linearized) private-sector equilibrium conditions

EHL MODEL – OPTIMAL POLICY

- **Optimal policy problem – maximize representative household’s lifetime utility subject to all private-sector equilibrium conditions**

- **In LQ form (see EHL Appendix B), minimize**

$$E_0\sum_{t=0}^{\infty}\beta^t\left[\hat{\pi}_t^2+\hat{y}_t^2+\theta\hat{\pi}_t^{w2}\right]$$

Relative importance of stabilizing *wage* inflation versus stabilizing *price* inflation depends on relative stickiness of prices and wages – NO divine coincidence here

subject to (linearized) private-sector equilibrium conditions

- **Can represent (linearized) price Phillips curve as including an endogenously-time-varying term arising from sticky nominal wages**
  - **An *endogenous* cost-push term**
  - **EHL p. 298**

- **Main Result**
  - **Fully stabilizing price inflation NOT optimal**
  - **Stabilize a weighted average of price inflation and wage inflation**