Simple DSGE Models of "Work"

January 11, 2017

TIME USE?

- "Labor" and "leisure" the only time uses in standard macro models
- Aguiar and Hurst's (2007 QJE) and Aguiar, Hurst, and Karabarbounis (2013 *AER*) time-use classifications
- Are other uses of time important for macro issues?
- Diamond's (1982) monograph *On Time*

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- Rogerson (1988 *JME*) and Hansen (1985 *JME*) issue:
  - Can "indivisibility" in labor (i.e., binary individual labor/leisure outcome) be tractably modeled?
  - ANSWER: YES – through "randomization" over WHO actually works
  - Market structure: complete Arrow securities in the cross section of agents, to insure away chance of not working
  - Aggregation result: representative-agent preferences quasi-linear in labor

LABOR AGGREGATION – MAIN RESULTS

- **Total equilibrium labor**

$$n = eh$$

Number (measure) of individuals working (e); Total hours (n); Hours per worker (h)

- **Normalize $h = 1$ for employed individuals (so $n = e$)**

LABOR AGGREGATION – MAIN RESULTS

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Number (measure) of individuals working

$$n = eh$$

Total hours (n) — Hours per worker (h)

- Normalize $h = 1$ for employed individuals (so $n = e$)

- Suppose non-convexity
  - Measure e of individuals work $h = 1$ hours
  - Measure 1-e of individuals work $h = 0$ hours
  - An INDIVIDUAL'S decision problem is NOT convex
    - Must choose {0, 1} (work/don't work)

Terminology:

Indivisible labor (work zero hours or fixed hours)

No intensive margin

- Economy populated by individuals each with $u(c)+\begin{cases} v(1-n=0) \text{ if work} \\ v(1-n=1) \text{ if don't work} \end{cases}$

equivalent (in terms of aggregate outcomes) to a representative-agent economy with quasi-linear preferences

$$u(c)-An$$

LABOR AGGREGATION – MAIN RESULTS

- **Theoretical attraction:** makes "low substitution" (of labor across time periods) economy at the micro level a "high substitution" (of labor across time periods) economy at the macro level
  - Helps make aggregate hours more volatile over time in a DSGE model – can illustrate with two-period model

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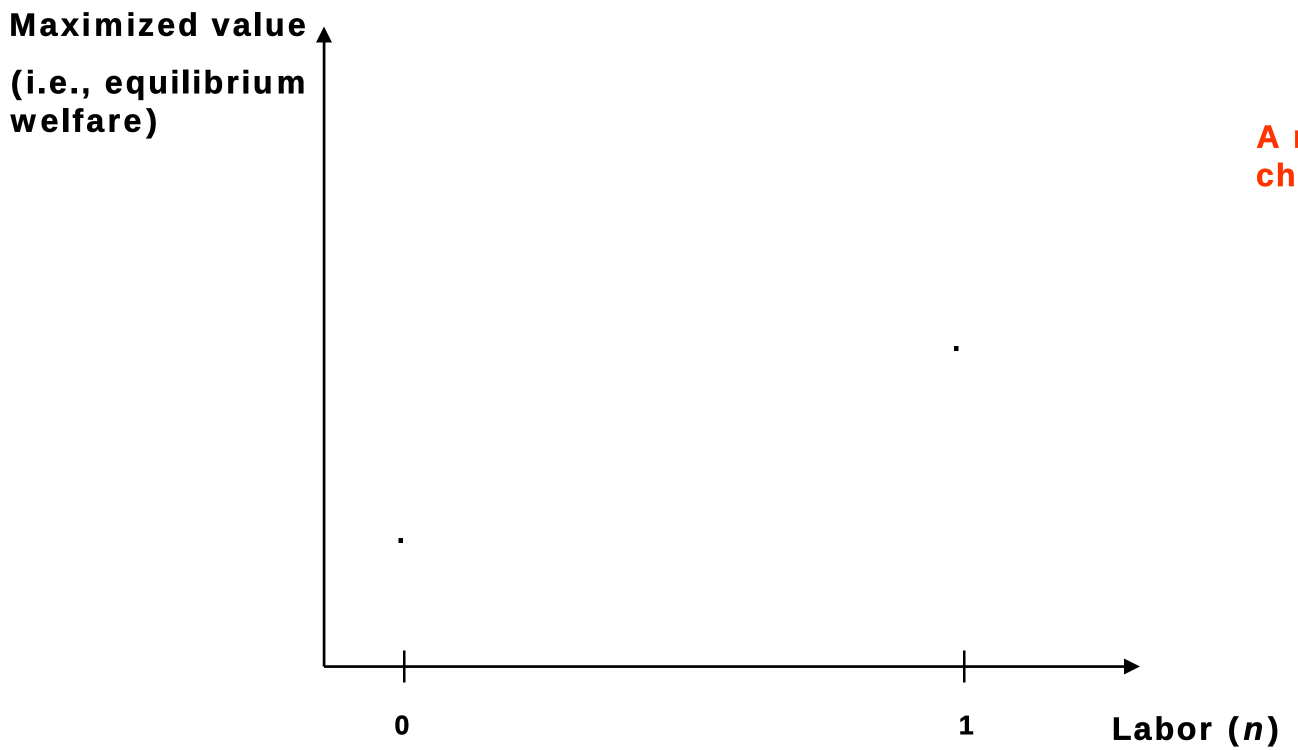
- Interpretation(s)
  - **Underlying market structure:** individuals choose "lotteries" over possibility of being employed, rather than whether or not to work
  - **Insurance:** individuals can (and do) purchase (actuarially fair) full insurance against employment risk
  - **"Risk-neutrality":** representative consumer "doesn't care" how many hours he works in a given period – because of full risk sharing!

- Gain in DSGE model performance?
- Intuitive plausibility?
- Empirical relevance?

THE SIMPLE MICROECONOMICS



A non-convex choice set…

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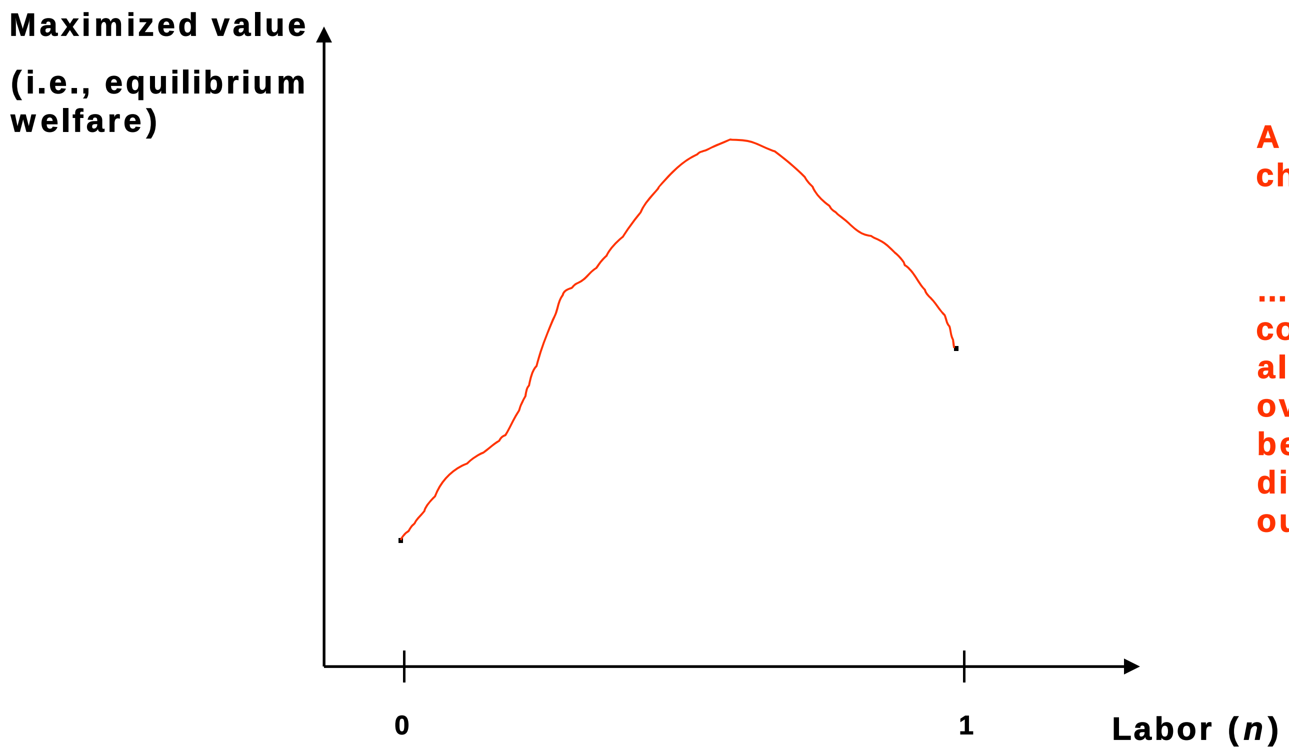

A non-convex choice set…

…can be made convex by allowing choice over lotteries between the discrete outcomes

Introduce lotteries by allowing individuals to purchase insurance (i.e., complete AD assets) against the risk of being unemployed

THE SIMPLE MICROECONOMICS

- **Consider simple static problem**

$$\max_{c_1,c_2,y} e\left[u(c_1)+v(1-n=0)\right]+(1-e)\left[u(c_2)+v(1-n=1)\right]$$

Expected utility

subject to

$$c_1 + py = w$$

State 1: work (probability *e*)

$$c_2 + py = y$$

State 2: don't work (probability 1-*e*)

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$$n = e$$

Subject to exogenous randomness

Utility in principle depends on *n*, so would like to optimize on *n*…

…but *n* is trivially exogenous at the optimal choice!

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- **Insurance**
  - **Quantity *y* purchased by consumer**
  - **(Competitive) price *p***
  - **Only pays off (*y* units) in the event the consumer doesn't work**

- **FOCs yield** $u'(c_1)=\lambda_1$ **and** $u'(c_2)=\lambda_2$ **along with** $-\lambda_1 ep-\lambda_2(1-e)p+\lambda_2(1-e)=0$

- **Conjecture** $\lambda=\lambda_1=\lambda_2 \rightarrow \quad c=c_1=c_2$

THE SIMPLE MICROECONOMICS

- Perfectly competitive representative insurance firm

$$\max_{y} py-(1-e)y$$

→ $p = (1-e)$

Actuarially fair: competitive price of one unit of insurance that pays off in the event “don’t work” = probability of event

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- **Theorem: Risk-averse consumer + actuarially fair insurance contract → consumer will choose to fully insure against loss (*y* = *w* in this case) (See, e.g., Varian (1992) text)**

→ Consumer's TOTAL INCOME/WEALTH (including any insurance payoff) not a function of his employment status

$$\therefore \lambda=\lambda_1=\lambda_2 \quad \rightarrow \quad c=c_1=c_2$$

Verifies conjecture

A consequence of complete markets:
consumption equated across states
(intratemporal consumption smoothing)

The Simple Microeconomics

- **Perfect competition in output production → *w* = *f'*(*E*)**
- **Labor-market clearing** $E = \int_0^1 h(i)di$
  - **Recall either *h* = 0 or *h* = 1**

Aggregate labor hired by firm

THE SIMPLE MICROECONOMICS

- **Perfect competition in output production → $w = f'(E)$**
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  - **Recall either $h = 0$ or $h = 1$**

Aggregate labor hired by firm

- **Equilibrium: $E = e$ and resource constraint $c = f(e)$**

Use equilibrium results to construct equivalent alternative problem that yields same aggregates

$$\max_{c,e} e\left[u(c)+v(0)\right]+(1-e)\left[u(c)+v(1)\right]$$

$$\text{s.t. } ec+(1-e)c+epy+(1-e)py = ew+(1-e)y$$

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Use equilibrium results to construct equivalent alternative problem that yields same aggregates

In particular, use *n* = *e*

$$\max_{c,n} n\left[u(c)+v(0)\right]+(1-n)\left[u(c)+v(1)\right]$$

$$\text{s.t. } nc+(1-n)c+npy+(1-n)py=nw+(1-n)y$$

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$y = w$ (fully insure) ↓ $p = 1 - e$ ($= 1 - n$) (competitive insurance price)

$$\max_{c,n} u(c)+nv(0)+(1-n)v(1)$$

$$\text{s.t. } c = wn$$

THE SIMPLE MICROECONOMICS

$$\max_{c,n} u(c) + \left[v(0) - v(1)\right]n + v(1)$$
$$\text{s.t. } c = wn$$

Drop $v(1)$ because constant

Define $A = v(1) - v(0)$ (simply a constant!)

$$\max_{c,n} u(c) - An$$
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Quasi-linear utility:

"risk-neutral" in labor *because* of full insurance!

- **Equilibrium: $w = f'(E)$, $E = n$, and $c = f(n)$**
- **Rogerson Result**
  - **Aggregates (c, n) in this economy identical to those from the indivisible labor economy with lotteries/full insurance**
  - **An application of perfect risk sharing / representative consumer results**
  - **If embedded in dynamic model (Hansen 1985), individuals do not care (i.e., are risk neutral with respect to) whether they work more in the present or the future (more to come on this soon….)**

BUSINESS CYCLE IMPLICATIONS

- **Embed quasi-linear preferences into standard RBC model**
- **Approximate and simulate**
  - **Hansen uses LQ (linear-quadratic) approximation**

- **Hansen results**

Table 1

Standard deviations in percent (a) and correlations with output (b) for U.S. and artificial economies.

Series	Quarterly U.S. time series[a] (55,3–84,1) (a)	(b)	Economy with divisible labor[b] (a)	(b)	Economy with indivisible labor[b] (a)	(b)
Output	1.76	1.00	1.35 (0.16)	1.00 (0.00)	1.76 (0.21)	1.00 (0.00)
Consumption	1.29	0.85	0.42 (0.06)	0.89 (0.03)	0.51 (0.08)	0.87 (0.04)
Investment	8.60	0.92	4.24 (0.51)	0.99 (0.00)	5.71 (0.70)	0.99 (0.00)
Capital stock	0.63	0.04	0.36 (0.07)	0.06 (0.07)	0.47 (0.10)	0.05 (0.07)
Hours	1.66	0.76	0.70 (0.08)	0.98 (0.01)	1.35 (0.16)	0.98 (0.01)
Productivity	1.18	0.42	0.68 (0.08)	0.98 (0.01)	0.50 (0.07)	0.87 (0.03)

<u>The main successes claimed:</u> in particular, RATIO of S.D. much higher than basic RBC model; but, at 2.7, TOO high!

THE VERDICT?

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Browning, Hansen, and Heckman (1999 *Macro Handbook* p. 602)

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Lotteries model predicts an individual's employment status is *iid* over time.

Micro evidence shows it is highly persistent over time.

"Rogerson's aggregation result is every bit as important as the one giving rise to the aggregate production function."

Prescott (2004 *Nobel Lecture* p. 385)

- **"[Not so fast….]"**

Mulligan (2001 *B.E. Journal of Macroeconomics*)