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# **SIMPLE DSGE MODELS OF "WORK"**

## **PART II**

**JANUARY 11, 2017**

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# BUSINESS CYCLE IMPLICATIONS

- ❑ Embed quasi-linear preferences into standard RBC model
- ❑ Approximate and simulate
  - ❑ Hansen uses LQ (linear-quadratic) approximation
  
- ❑ Hansen results

Table 1

Standard deviations in percent (a) and correlations with output (b) for U.S. and artificial economies.

Series	Quarterly U.S. time series <sup>a</sup> (55,3–84,1)		Economy with divisible labor <sup>b</sup>		Economy with indivisible labor <sup>b</sup>	
	(a)	(b)	(a)	(b)	(a)	(b)
Output	1.76	1.00	1.35 (0.16)	1.00 (0.00)	1.76 (0.21)	1.00 (0.00)
Consumption	1.29	0.85	0.42 (0.06)	0.89 (0.03)	0.51 (0.08)	0.87 (0.04)
Investment	8.60	0.92	4.24 (0.51)	0.99 (0.00)	5.71 (0.70)	0.99 (0.00)
Capital stock	0.63	0.04	0.36 (0.07)	0.06 (0.07)	0.47 (0.10)	0.05 (0.07)
Hours	1.66	0.76	0.70 (0.08)	0.98 (0.01)	1.35 (0.16)	0.98 (0.01)
Productivity	1.18	0.42	0.68 (0.08)	0.98 (0.01)	0.50 (0.07)	0.87 (0.03)

**The main successes claimed:** in particular, RATIO of S.D. much higher than basic RBC model; but, at 2.7, TOO high!

## BOTH EXTENSIVE AND INTENSIVE ADJUSTMENT

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- ❑ **A major challenge for DGE modeling: elasticity of “labor supply”**
  - ❑ **Intensive margin** (“hours supply”)
  - ❑ **Extensive margin** (“labor force participation”)
  
- ❑ **Cho and Cooley (1994)**
  - ❑ **Study RBC economy with both margins operating**
  - ❑ **Extensive margin:** “number of days” worked within a period
    - ❑ Household pays a cost for each “day” it chooses to work
  - ❑ **Intensive margin:** hours worked per day worked
  - ❑ **NOTE:** No “frictions” in **finding** jobs
  
- ❑ **Percent of total hours fluctuations accounted for by extensive fluctuations vs. intensive fluctuations**
  - ❑ **Cho and Cooley (1994): 75% extensive, 25% intensive**
  - ❑ **Hansen (1985): 55% extensive, 20% intensive (rest from cov term)**

# STATIC EXAMPLE

- General utility function**

$$u(c) - \frac{a}{1+\gamma} n^{1+\gamma} e - \psi(e)e$$

$\uparrow$   
 intensive

$\uparrow$   
 extensive

Description of Economy	$\psi(e)$	Elasticity of <b>equilibrium</b> total hours
Both intensive and extensive margins		
Only extensive margin (Hansen-Rogerson)		
Only intensive margin ("typical" RBC model)		

- e denotes "employment rate"** – fraction of days worked
- n denotes hours worked per day**

# STATIC EXAMPLE

**General utility function**  $u(c) - \frac{a}{1+\gamma} n^{1+\gamma} e - \psi(e)e$

↑
↑

intensive
extensive

Description of Economy	$\psi(e)$	Elasticity of <b>equilibrium</b> total hours
Both intensive and extensive margins	$(b/(1+\tau))e^\tau$	<b>Intermediate</b>
Only extensive margin (Hansen-Rogerson)	$b$	<b>High</b>
Only intensive margin ("typical" RBC model)	0 (also fix $e = 1$ )	<b>Low</b>

- e denotes "employment rate"** – fraction of days worked
- n denotes hours worked per day**

## STATIC EXAMPLE

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- **Consumer optimization**

$$\max_{c,n,e} u(c) - \frac{a}{1+\gamma} n^{1+\gamma} e - \frac{b}{1+\tau} e^{1+\tau}$$

$$\text{s.t } c \leq wne$$

- **Combine with firm optimization and market clearing!**
- **Examining EQUILIBRIUM aggregate hours (“effective  $L^S$ ”)**
  - **Not “labor supply” (“notional  $L^S$ ”)**

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- **Combine with firm optimization and market clearing!**
- **Examining EQUILIBRIUM aggregate hours (“effective  $L^S$ ”)**
  - Not “labor supply” (“notional  $L^S$ ”)
- **Impose parameter values to capture three different cases**
- **Elasticity equilibrium total hours**
  - Adjustment only at extensive margin: 4
  - Adjustment only at intensive margin: 0.36
  - Adjustment at both margins: 1.29
  - Recall common compromise value in macro models:  $\geq 1$

# BUSINESS CYCLE IMPLICATIONS

- ❑ Embed in standard RBC model
- ❑ Approximate and simulate
  - ❑ Cho and Cooley use LQ (linear-quadratic) approximation
- ❑ Cho and Cooley results

Table 2  
Calibration results, first parameterization.<sup>a</sup>

Series	U.S.		Model	
	Std. dev.	Corr. with output	Std. dev.	Corr. with output
Output	1.76	1.00	1.76 (0.17)	1.00 (0.00)
Consumption	1.29	0.85	0.53 (0.06)	0.88 (2.49)
Investment	8.60	0.92	5.63 (0.57)	0.98 (0.40)
Capital stock	0.63	0.04	0.47 (0.08)	0.07 (6.73)
Aggregate hours	1.74	0.77	1.06 (0.12)	0.98 (0.56)
Hours	0.46	0.76	0.25 (0.02)	0.98 (1.24)
Employment	1.50	0.81	0.81 (0.08)	0.98 (1.04)
Productivity	1.18	0.35	0.75 (0.08)	0.96 (0.81)
Agg. hrs/Productivity				
in physical units		1.47		1.42
in efficiency units		1.42		1.42

Both intensive and extensive adjustment:  $1.06/0.75 = 1.42$



# HOME PRODUCTION MODELS

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- **What else do individuals/households do with their time?**
  - **Aguiar and Hurst (2007 *QJE*) and Aguiar, Hurst, and Karabarbounis (2013 *AER*): over 2 hours per day of **nonmarket work (i.e., nonmarket LABOR)****
  - **Shopping**
  - **Cooking**
  - **Cleaning**
  - **Etc...**

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  - ❑ Shopping
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  - ❑ Cleaning
  - ❑ Etc...
  
- ❑ **“Household capital” expenditures also sizable**
  - ❑ **Investment in consumer durables and residential investment at least as large as investment in market capital**
  
- ❑ **“Home production” in RBC model**
  - ❑ **Overview by Greenwood, Rogerson, and Wright (1995)**
  - ❑ **Allow households to accumulate “home capital” and “work” at home (cleaning, cooking, etc.) in order to produce and consume “home goods” (distinct from “market goods”)**

# BASIC HOME PRODUCTION MODEL

- Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{Mt}, c_{Ht}, n_{Mt}, n_{Ht})$$

“Usual” market consumption    “Usual” market labor  
“Home good” consumption    “Home” labor

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"Home good" consumption      "Home" labor

□ **Technology**

$$f(n_{Mt}, k_{Mt}, z_{Mt}) (= k_{Mt}^{\alpha} (z_{Mt} n_{Mt})^{1-\alpha})$$

"Usual" market productivity

"Usual" market production function

"Usual" market/business capital

$$g(n_{Ht}, k_{Ht}, z_{Ht}) (= k_{Ht}^{\gamma} (z_{Ht} n_{Ht})^{1-\gamma})$$

"Home" production function

- Home output can ONLY be used for consumption

"Home" capital

"Home" productivity

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“Usual” market productivity

“Usual” market production function

“Usual” market/business capital

$$g(n_{Ht}, k_{Ht}, z_{Ht}) \left( = k_{Ht}^{\gamma} (z_{Ht} n_{Ht})^{1-\gamma} \right)$$

“Home” production function

- Home output can ONLY be used for consumption

“Home” capital

“Home” productivity

□ **Household Budget Constraint**

$$c_{Mt} + [k_{Mt+1} - (1 - \delta_M)k_{Mt}] + [k_{Ht+1} - (1 - \delta_H)k_{Ht}] = w_t n_{Mt} + r_t k_{Mt}$$

- **Unit relative price between market capital and home capital**
- **All income earned through market-factor rental**
- **Home consumption not “purchased” – produced at home!**

# BASIC HOME PRODUCTION MODEL

## □ Other model details

- (Constant) labor income and capital income taxation included (for calibration purposes)
- Capital freely-allocatable every period between home and market/business uses

$$k_t = k_{Mt} + k_{Ht} \quad \forall t$$

- Representative (market) firm:  $\max_{n_{Mt}, k_{Mt}} f(n_{Mt}, k_{Mt}, z_{Mt}) - w_t n_{Mt} - r_t k_{Mt}$

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## □ Business cycle implications

- Approximate and simulate using “usual” methods
- **Main Results**
  - SD(hours)/SD(productivity) matches data better than basic RBC
  - Corr(hours, real wage) matches data ( $\approx 0$ ) better than basic RBC

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## Business cycle implications

- Approximate and simulate using “usual” methods

### Main Results

- SD(hours)/SD(productivity) matches data better than basic RBC
- Corr(hours, real wage) matches data ( $\approx 0$ ) better than basic RBC
- Results rely on ability to substitute between  $c_{Mt}$  and  $c_{Ht}$  and incentive to do so

Governed by correlation between  $z_{Mt}$  and  $z_{Ht}$

Governed by CES elasticity over  $c_{Mt}$  and  $c_{Ht}$



# RBC MODELS AND LABOR MARKET FLUCTUATIONS

- Can interpret as micro-foundation for Greenwood-Hercowitz-Huffman (1988) preferences (GHH preferences)

$$u(c_t, n_t) = \ln \left( c_t - \frac{\psi}{1+\nu} n_t^{1+\nu} \right)$$

- Exhibits zero income effect on market hours  $n_t$
- **Seems inconsistent with balanced-growth facts...**
- ...unless  $z_M$  and  $z_H$  are growing at the same long-run rates, in which case there is **no reason to substitute between home and mrkt work**

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- 
- Can interpret as micro-foundation for a **preference shifter**

$$u(c_t, n_t) = \ln c_t - \frac{a_t}{1+\nu} n_t^{1+\nu}$$

**Exogenous, time-varying process affects (shifts) MRS between consumption and leisure – a mechanism emphasized by Hall (1997)**

- Change in (endogenous) home outcomes → **shift** in individual's labor supply schedule in a "reduced-form" model with preference shock

# RBC MODELS AND LABOR MARKET FLUCTUATIONS

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- ❑ **Standard model (intensive adjustment)**
- ❑ **Indivisible labor model (extensive adjustment)**
- ❑ **Home production model**
- ❑ **Alternative preference specifications**
- ❑ **Consequences of government spending fluctuations**
- ❑ **Overview by Hansen and Wright (1992 *Minneapolis Fed Review*)**
- ❑ **Labor search and matching frictions**