RISK AVERSION

JANUARY 18, 2017

RISK AVERSION – STATIC CASE

- **CRRA (two-period model)**

  Note: $V(c^*_1, c^*_2)$ is value function, aka indirect utility function

$$V(c_1,c_2)=\underbrace{\frac{c_1^{1-\sigma}-1}{1-\sigma}}_{\equiv u(c_1)}+\underbrace{\frac{c_2^{1-\sigma}-1}{1-\sigma}}_{\equiv u(c_2)} \qquad \sigma>0$$

Utility additively-separable over time

- **Attitude of consumers toward smoothing consumption between time periods**
  - **IES = 1 / σ**

- **Attitude of consumers toward risky outcomes within a given time period**

$$RRA(c)=-\frac{cu''(c)}{u'(c)}=\sigma \qquad ARA(c)=-\frac{u''(c)}{u'(c)}=\frac{\sigma}{c}$$

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- **CRRA utility: σ governs both intertemporal attitudes and intratemporal (relative) risk attitudes**
  - **Inverses of each other!**

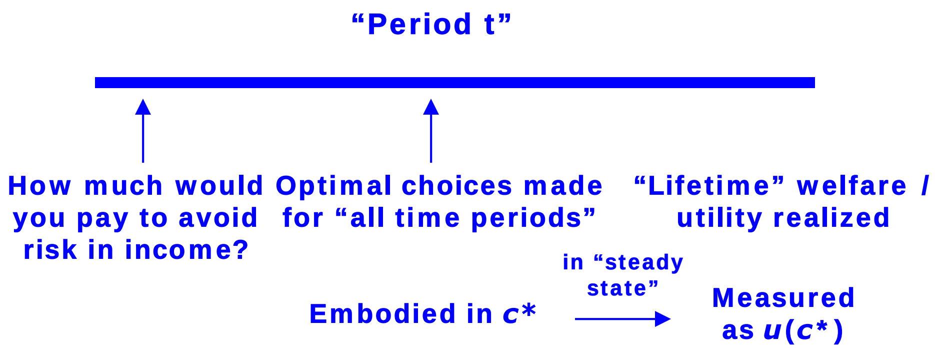
- **Must / should IES and RRA be so directly related in reality?**
  - **Not at all…Epstein-Zin (EZ) utility function disentangles the two concepts (basic reason: moves away from vNM expected utility)**

Risk Aversion – Dynamic Case

- Notion of value function different in one-good vs. multi-good model
- Basic intuition
  - Individuals will smooth shocks over all possible margins of adjustment
  - Across consumption-leisure margin
  - Across consumption-savings margin

RISK AVERSION – DYNAMIC CASE

- **Notion of value function different in one-good vs. multi-good model**
- **Static**



RISK AVERSION – DYNAMIC CASE

- **Notion of value function different in one-good vs. multi-good model**
- **Static**

"Period t"

How much would you pay to avoid risk in income?

Optimal choices made for "all time periods"

"Lifetime" welfare / utility realized

Embodied in c^* → in "steady state" → Measured as $u(c^*)$

- **Dynamic**

Period t

Period t+1

a_{t-1}

$a^*_t = a(a_{t-1})$

How much would you pay to avoid risk in income?

Optimal choices made for "all time periods"

Embodied in a^*_t → in "steady state" → Measured as $V(a^*)$

PDV of lifetime utility function

Risk Aversion – Dynamic Case

- Notion of value function different in one-good vs. multi-good model
- Static multi-good environment ($u(c_1, c_2, c_3, \ldots)$ – no time indexes)
  - Stiglitz (1969 *Econometrica*)
  - Using indirect utility function (aka value function)
- Dynamic one-good environment ($V(a_{t-1}\,;\,.) = u(c_t) + \beta u(c_{t+1}) + \ldots$)
  - Constantinides (1990 *JPE*)
  - Using indirect utility function (aka value function)
- Dynamic multi-good environment
  - Swanson (2012 *AER*)
  - Using indirect utility function aka value function

$$V(a_{t-1};\cdot) \equiv \max_{\{c_t, n_t, a_t\}} \left\{ u(c_t, n_t) + \beta V(a_t;\cdot) \right\}$$

$$c_t + a_t = (1 + r_t) a_{t-1} + w_t n_t + \underbrace{\sigma \varepsilon_{t+1}}$$

$\sigma > 0$ intuitively measures risk in the asset payoff r

RISK AVERSION – DYNAMIC CASE

- **Dynamic multi-good environment**

$$V(a_{t-1};\cdot) \equiv \max_{\{c_t, n_t, a_t\}} \left\{ u(c_t, n_t) + \beta V(a_t;\cdot) \right\}$$

$$c_t + a_t = (1+r_t)a_{t-1} + w_t n_t - \mu$$

- **Define μ as fee to avoid the risk $\sigma\varepsilon_{t+1}$ (as in Arrow (1971) and Pratt (1964))**

Risk Aversion – Dynamic Case

- **Dynamic multi-good environment**

$$V(a_{t-1};\cdot) \equiv \max_{\{c_t, n_t, a_t\}} \left\{ u(c_t, n_t) + \beta V(a_t;\cdot) \right\}$$

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- **Define μ as fee to avoid the risk $\sigma\varepsilon_{t+1}$ (as in Arrow (1971) and Pratt (1964))**

- **Proposition 1 (Swanson 2012), absolute risk aversion**

  - **At steady state** $-\dfrac{cu'''(c)}{u''(c)}$

  - **Outside steady state** $ARA(a_{t-1};.) = -\dfrac{E_t V''(a(a_{t-1});.)}{E_t V'(a(a_{t-1});.)}$

Benveniste and Scheinkman (1979 *EC*) proved differentiability of *V*(.)

- **Steady-state ARA for "standard" RBC-BGP model (Proposition 2)**

$$ARA(\bar{a};.) = \frac{-u_{cc} + \lambda u_{cn}}{u_c} \cdot \frac{r}{1 + w \cdot (\text{fct. of } w, u_{cn}, u_{nn})}$$

Risk Aversion – Dynamic Case

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$$ARA(\bar{a};.) = \frac{-u_{cc} + \lambda u_{cn}}{u_c} \cdot \frac{r}{1 + w \cdot (\text{fct. of } w, u_{cn}, u_{nn})}$$

- **If no labor supply margin, $u_n = \underline{u}_{cn} = u_{nn} = 0$**

$$\Rightarrow ARA(\bar{a};.) = \frac{-u_{cc}}{u_c} \cdot r$$

ARA **measure identical to basic Arrow and Pratt measures, adjusted for the annuity value of asset returns**

ARA **<u>WITH</u> labor supply margin < *ARA* <u>WITHOUT</u> labor supply margin**

$$\frac{-u_{cc} + \lambda u_{cn}}{u_c} \cdot \frac{r}{1 + w \cdot (\text{fct. of } w, u_{cn}, u_{nn})} < \frac{-u_{cc} + \lambda u_{cn}}{u_c} \cdot r$$

Risk Aversion – Dynamic Case

- How to measure relative risk aversion?

$$V(a_{t-1};\cdot) \equiv \max_{\{c_t,n_t,a_t\}} \left\{u(c_t,n_t) + \beta V(a_t;\cdot)\right\}$$

$$c_t + a_t = (1+r_t)a_{t-1} + w_t n_t + A_t \sigma \varepsilon_{t+1}$$

A_t = PDV of optimal consumption

- Individual can pay $A_t \mu$ to avoid the risk $A_t \sigma \varepsilon_{t+1}$

RISK AVERSION – DYNAMIC CASE

- **How to measure relative risk aversion?**

$$V(a_{t-1};\cdot) \equiv \max_{\{c_t, n_t, a_t\}} \left\{ u(c_t, n_t) + \beta V(a_t;\cdot) \right\}$$

$$c_t + a_t = (1+r_t)a_{t-1} + w_t n_t + A_t \sigma \varepsilon_{t+1}$$

***A_t* = PDV of optimal consumption**

  - **Individual can pay $A_t\mu$ to avoid the risk $A_t\sigma\varepsilon_{t+1}$**

- **If allowing both *c* and *n* adjustment**
  - **At steady state** $RRA(\bar{a};.) = -\frac{\bar{A}V''(\bar{a};.)}{V'(\bar{a};.)}$
  - **Outside steady state** $RRA(a_{t-1};.) = -\frac{A_t E_t V''(a(a_{t-1});.)}{E_t V'(a(a_{t-1});.)}$

- **Steady-state RRA for “standard” RBC-BGP model**

$$RRA(\bar{a};.) = \frac{-u_{cc} + \lambda u_{cn}}{u_c} \cdot \frac{r \cdot \bar{A}}{1 + w \cdot (\text{fct. of } w, u_{cn}, u_{nn})}$$

Risk Aversion – Dynamic Case

- **Steady-state RRA for "standard" RBC-BGP model**

$$RRA(\bar{a};.) = \frac{-u_{cc} + \lambda u_{cn}}{u_c} \cdot \frac{r \cdot \bar{A}}{1 + w \cdot (\text{fct. of } w, u_{cn}, u_{nn})}$$

- **If no labor supply margin, $u_n = u_{cn} = u_{nn} = 0$**

= at steady-state optimal choice!
Consistent with PIH

$$\Rightarrow RRA(\bar{a};.) = \frac{-u_{cc}}{u_c} \cdot \bar{c}$$

RRA **measure identical to Arrow and Pratt measures**

- **Cannot compare *RRA* <u>WITH</u> labor supply margin to *RRA* <u>WITHOUT</u> labor supply margin**
  - **Unlike *ARA***

- **How to measure *RRA* using BGP-consistent utility functions?**

Risk Aversion – Dynamic Case

- **How to measure *RRA* using BGP-consistent utility functions?**
  - **Example 1: multiplicatively separable KPR**

$$u(c_t, n_t) = \frac{\left[c_t(1-n_t)^{\chi}\right]^{1-\sigma}}{1-\sigma} \quad \Rightarrow RRA(\bar{a};.) = \sigma - \chi(1-\sigma) < \sigma$$

RISK AVERSION – DYNAMIC CASE

- **How to measure *RRA* using BGP-consistent utility functions?**
  - **Example 1: multiplicatively separable KPR**

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  - **Example 2: additively separable, Frisch**

$$u(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \eta \frac{n_t^{1+\chi}}{1+\chi} \quad \Rightarrow RRA(\bar{a};.) = \frac{\sigma}{1 + \frac{\sigma}{\chi}\frac{wn}{c}}$$

RISK AVERSION – DYNAMIC CASE

- **How to measure *RRA* using BGP-consistent utility functions?**
  - **Example 1: multiplicatively separable KPR**

$$u(c_t,n_t)=\frac{\left[c_t(1-n_t)^{\chi}\right]^{1-\sigma}}{1-\sigma} \quad \Rightarrow RRA(\bar{a};.)=\sigma-\chi(1-\sigma)<\sigma$$

  - **Example 2: additively separable, Frisch**

$$u(c_t,n_t)=\frac{c_t^{1-\sigma}}{1-\sigma}-\eta\frac{n_t^{1+\chi}}{1+\chi} \quad \Rightarrow RRA(\bar{a};.)=\frac{\sigma}{1+\frac{\sigma}{\chi}\frac{wn}{c}}$$

  - **Example 3: $\lim_{\chi\to 0}$ = 0, i.e., quasi-linear utility**
    - **The "risk neutral" case (Hansen 1985, Rogerson 1988)**

$$u(c_t,n_t)=\frac{c_t^{1-\sigma}}{1-\sigma}-\eta\cdot n_t \quad \Rightarrow RRA(\bar{a};.)=0$$

RISK AVERSION – DYNAMIC CASE

- Swanson (2012 *AER*) analysis foundational
  - Search and matching models
  - New Keynesian models
  - ANY DSGE model

- Risk aversion measures away from deterministic steady state must be approximated numerically
  - Quantitatively not far from deterministic steady state

- How does it change the performance (e.g., simulations) of standard RBC-BGP models?