Labor Matching Models: Efficiency Properties

January 23, 2017

LABOR-MATCHING EFFICIENCY

- **Social Planning problem**
  - **Social Planner also subject to matching *TECHNOLOGY***

$$\max_{c_t, v_t, n_{t+1}} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$c_t + \gamma v_t = z_t n_t f(h_t) + (1-n_t)b$$ **Fix *h* = 1**

$$n_{t+1} = (1-\rho_x)n_t + m(1-n_t, v_t)$$ **And *n* = 1 – *u***

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Multipliers

$$c_t + \gamma v_t = z_t n_t f(h_t) + (1-n_t) b$$ Fix $h = 1$ — λ_t

$$n_{t+1} = (1-\rho_x) n_t + m(1-n_t, v_t)$$ And $n = 1 - u$ — μ_t

- **FOCs**

$$u'(c_t) - \lambda_t = 0$$

$$-\lambda_t \gamma + \mu_t m_2(1-n_t, v_t) = 0$$

$$-\mu_t + \beta E_t \left\{ \lambda_{t+1} \left[z_{t+1} - b \right] \right\} + \beta E_t \left\{ \mu_{t+1} \left[(1-\rho_x) - m_1(1-n_t, v_t) \right] \right\} = 0$$

↓ Eliminate multipliers

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$$\frac{\gamma}{m_2(1-n_t,v_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - b - \frac{\gamma m_1(1-n_t,v_t)}{m_2(1-n_t,v_t)} + \frac{(1-\rho_x)\gamma}{m_2(1-n_t,v_t)}\right]\right\}$$

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Cobb-Douglas matching

$$m(u,v) = u^{\alpha}v^{1-\alpha}$$

$$m_1(u,v) = \alpha u^{\alpha-1}v^{1-\alpha} = \alpha\theta^{1-\alpha}$$

$$m_2(u,v) = (1-\alpha)u^{\alpha}v^{-\alpha} = (1-\alpha)\theta^{-\alpha}$$

AND

$$k^h(\theta) = \frac{m(u,v)}{u} = m(1,\theta) = \theta^{1-\alpha}$$

$$k^f(\theta) = \frac{m(u,v)}{v} = m(\theta^{-1},1) = \theta^{-\alpha}$$

AND $m_1(u,v) = \alpha k^h(\theta)$ $\quad m_2(u,v) = (1-\alpha)k^f(\theta)$

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Combine and rearrange

AND $m_1(u,v) = \alpha k^h(\theta)$ $\quad m_2(u,v) = (1-\alpha)k^f(\theta)$

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - \left(\alpha\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\alpha)b\right) + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})}\right]\right\}$$

KEY IDEAS

Taking the pricing kernel as given, the only unknown process here is θ_t

Efficiency in job-postings is governed by efficient market tightness

LABOR-MATCHING EFFICIENCY

- Socially-efficient vacancy posting characterized by

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- Decentralized vacancy posting characterized by

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$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

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↓

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(z_{t+1} - \left(\eta\left[z_{t+1}+\gamma\theta_{t+1}\right]+(1-\eta)b\right) + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})}\right)\right\}$$

- **Efficiency in vacancy posting requires $\eta = \alpha$!**

MORTENSEN-HOSIOS CONDITION

- Cobb-Douglas matching technology + Nash bargaining
  - Efficient level of job-creation requires $\eta = \alpha$
  - Mortensen (1982 *AER*), Hosios (1990 *ReStud*)

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- Intuition: search activity generates externalities
  - One extra individual (firm) searching for a job (worker) ***LOWERS*** the probability that *all other* individuals (firms) will find a match…
  - …but ***RAISES*** the probability that *all other* firms (individuals) will find a match
  - Congestion externality – search imposes both positive and negative externalities (on opposite sides of the market)

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- Nash bargaining: η governs the private returns to search
  - Share of total match surplus kept by individual
- Cobb-Douglas matching: α governs the social returns to search
  - Elasticity of aggregate number of matches with respect to u

- Efficiency requires equating private and social returns: $\eta = \alpha$

HOSIOS CONDITION

- Also holds under some more general conditions
  - Endogenous search intensity
  - Endogenous "vacancy posting intensity" (Pissarides Chapter 5)
- Pissarides (2000, p. 198): "..we are not likely to find intuition for it…"
- RSW (2005 *JEL* p. 982): "…genuinely surprising result…"
- Is the Hosios condition empirically relevant?
  - Who knows?...it's a nongeneric parameterization…
  - …but valuable because eliminates wage-determination frictions but retains matching technology
- Is Nash bargaining empirically relevant?

How are Wages Determined?

- Nash bargaining
  - Underlying alternating offers bargaining game
  - The relevant outside option as bargaining is occurring?
    - Value of outside market opportunities?
    - Value of continuing negotiations? (Hall and Milgrom 2008 *AER*)

- Proportional bargaining

- Rigid real wages (completely rigid or partially rigid)

HOW ARE WAGES DETERMINED?

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  - ❒ Underlying alternating offers bargaining game
  - ❒ The relevant outside option as bargaining is occurring?
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- ❒ Proportional bargaining
- ❒ Rigid real wages (completely rigid or partially rigid)
- ❒ Bargaining theoretic wages are ex-post of match formation
- ❒ Seems very different from taking wages as given (ex-ante of match formation)
- ❒ Competitive search equilibrium
  - ❒ Moen (1997 *JPE*): basic static partial labor search model