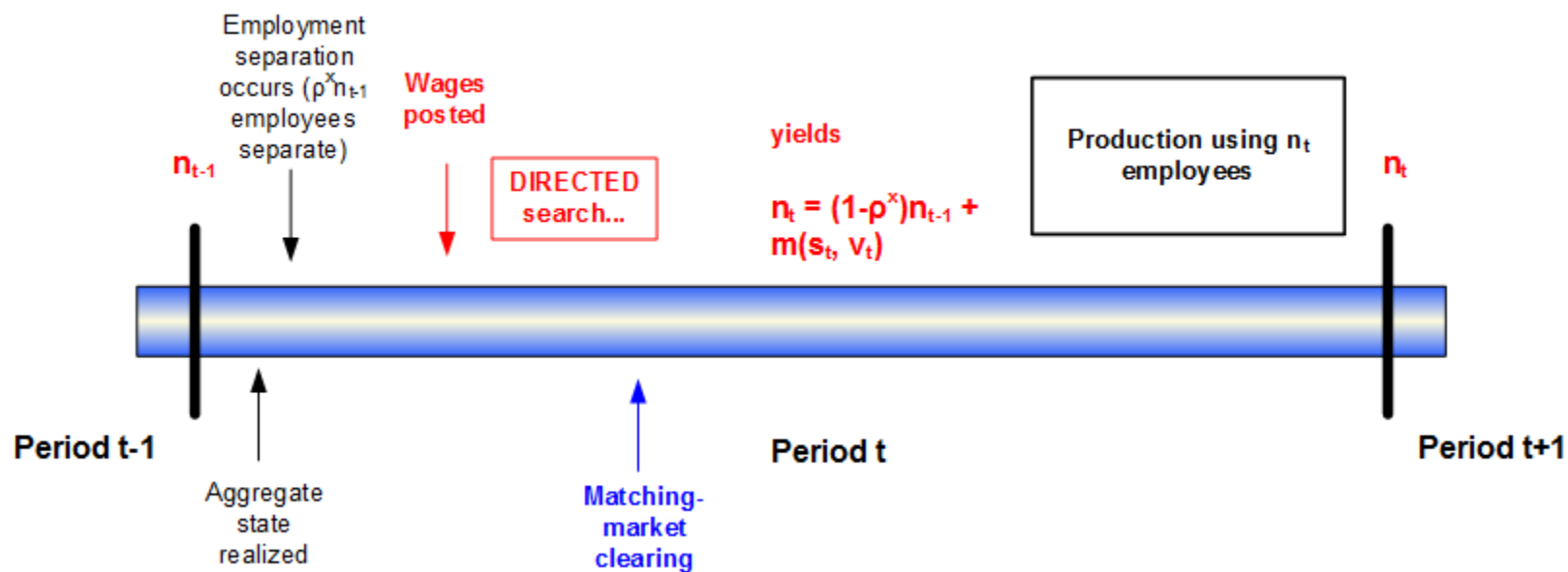

**LABOR MATCHING MODELS:
COMPETITIVE SEARCH EQUILIBRIUM
(CSE)**

JANUARY 23, 2017

COMPETITIVE SEARCH EQUILIBRIUM

- Wage determination ...
- ... then matching market clearing



COMPETITIVE SEARCH EQUILIBRIUM (CSE)

- ❑ Question: can a **“competitive”** notion of wage-setting be entertained in a search and matching model?
 - ❑ Would get away from the non-genericity of the Hosios bargaining parameterization

- ❑ May be apriori an appealing way of describing labor markets
 - ❑ Locating a firm or a worker is costly and time-consuming...
 - ❑ ...but once matched, wages are more or less determined by “market forces,” perhaps with little/no room for “bargaining”

- ❑ Moen (1997 *JPE*) and Shimer (1996) the original implementations of CSE
 - ❑ Static partial equilibrium labor matching models

CSE – BASICS OF ENVIRONMENT

- ❑ Need “many markets” and “many firms”
 - ❑ To rationalize “**competition**”
- ❑ Index continuum of labor “submarkets” by j – e.g., local labor markets
- ❑ Within a submarket j , many firms looking to hire workers
 - ❑ Even within a “local” labor market, coordination frictions in finding workers may exist
 - ❑ Index by i
- ❑ Several equivalent ways to implement
- ❑ Perfectly-competitive “recruiting” sector
- ❑ Firms post wages before individuals search for jobs
- ❑ Individuals announce wages before firms search for workers

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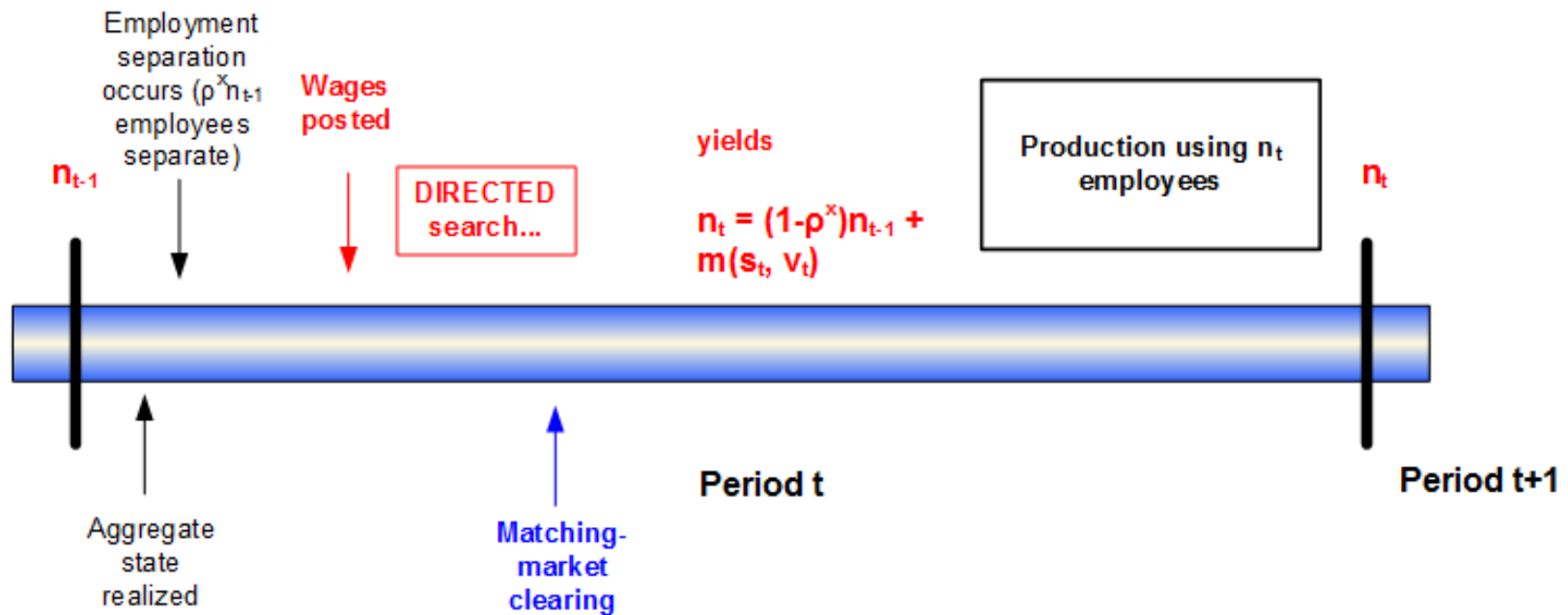
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- ❑ Individuals announce wages before firms search for workers

CSE – BASICS OF ENVIRONMENT

- ❑ Unemployed individuals **direct** their job search (“send an application”) to a particular submarket
 - ❑ Based on **wages** announced by firms in that submarket
 - ❑ And on **probability** of getting a job in that submarket
 - ❑ **Directed search** a key component of CSE...
 - ❑ ...but match formation still subject to probabilities

- ❑ Ordering of events
 - ❑ Wages determined **before** search...
 - ❑ ...all parties **direct search** according to posted wages...
 - ❑ ...then **probability** of landing a match resolved

TIMELINE



Competitive Search

CSE – BASICS OF ENVIRONMENT

- ❑ Idea of firm wage-posting/wage-announcement implementation
 - ❑ Define (expected) payoff function to firm ij of finding an additional worker
 - ❑ Define (expected) payoff function to individual searching for/applying to a job at firm ij
 - ❑ Firm ij maximizes its payoff subject to the directed search (aka reaction) function defined by the individual's payoff function
 - ❑ i.e., firm *internalizes* the effect of wages on the other side of the market...
 - ❑ ...can already see how congestion *externality* issues will be taken care of...

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- ❑ **Internalizing congestion externalities would also be achieved by...**
 - ❑ Individuals announcing wages taking into account reactions by firms
 - ❑ "Market maker" calling out wages taking into account reactions by both sides of market

CSE – IMPLEMENTATION

- Firm ij payoff function described by vacancy-posting decision

$$\gamma = k^f(\theta_{ijt}) \left[z_t - w_{ijt} + (1 - \rho_x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]$$

↑
Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of matching with a worker) x (contemporaneous payoff + continuation payoff)

Note ij subscripts:

Matching probability depends on tightness of "applications" at firm ij ...

...but future asset value of employee depends on market j conditions (i.e., replacement value depends on (sub-)market conditions)

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- Value equations for an individual searching for a match at firm ij

$$W(w_{ijt}) = w_{ijt} + E_t \left\{ \Xi_{t+1|t} \left[(1 - \rho_x) W(w_{j,t+1}) + \rho_x U_{t+1} \right] \right\}$$

With probability $k^h(\theta_{ijt})$, individual gets this payoff

$$U_{it} = b_i + E_t \left\{ \Xi_{t+1|t} \left[k^h(\theta_t) W(w_{t+1}) + (1 - k^h(\theta_t)) U_{t+1} \right] \right\}$$

With probability $1 - k^h(\theta_{ijt})$, individual gets this payoff

- With individuals (households) optimally directing their search, the expected payoff of searching for/applying to a job at firm ij is

$$k^h(\theta_{ijt}) W(w_{ijt}) + (1 - k^h(\theta_{ijt})) U_t = X$$

Payoff of searching at another firm or another submarket independent of ij

CSE – IMPLEMENTATION

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- First-order conditions

$$1) \quad -k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) \mathbf{W}'(w_{ijt}) = 0$$

$$2) \quad \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \left[z_t - w_{ijt} + (1 - \rho_x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{j,t+1})} \right) \right\} \right] - \varphi_{ijt} \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} [\mathbf{W}(w_{ijt}) - \mathbf{U}_{it}] = 0$$

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Taking into account how matching probabilities are affected by tightness is the central idea

CSE – IMPLEMENTATION

□ **First-order conditions**

$$1) \quad -k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) \mathbf{W}'(w_{ijt}) = 0 \quad \xrightarrow{w'(\cdot) = 1} \quad \varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}$$

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Cobb-Douglas matching

$$m(u, v) = u^\alpha v^{1-\alpha}$$

Combine and rearrange

$$k^h(\theta) = \frac{m(u, v)}{u} = m(1, \theta) = \theta^{1-\alpha}$$

$$k^f(\theta) = \frac{m(u, v)}{v} = m(\theta^{-1}, 1) = \theta^{-\alpha}$$

AND

$$\frac{\partial k^h(\theta)}{\partial \theta} = (1 - \alpha)\theta^{-\alpha}$$

$$\frac{\partial k^f(\theta)}{\partial \theta} = -\alpha\theta^{-\alpha-1}$$

$$(1 - \alpha)(\mathbf{W}(w_t) - \mathbf{U}_t) = \alpha \mathbf{J}(w_t)$$

Exactly the Nash-bargaining sharing rule with **endogenous emergence** of Hosios condition ($\eta = \alpha$)...

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(Competition within submarket j and symmetry across submarkets: drop ij indices)

$(1 - \alpha)(\mathbf{W}(w_t) - \mathbf{U}_t) = \alpha \mathbf{J}(w_t)$

Inserting value equations and solving explicitly for wage obviously gives same outcome as Nash-bargained wage with $\eta = \alpha$...

CSE – INTERPRETATIONS

- ❑ Mortensen and Pissarides (1999 *Handbook Chapter* p. 2589-2592)
 - ❑ “Price of time” priced efficiently by markets **in ex-ante** CSE
 - ❑ “Price of time” generically mispriced **if ex-post wage determination**
 - ❑ (“Price of time” = matching probabilities, which reflect congestion externalities)

- ❑ Bargaining equilibrium features a particular type of market incompleteness: workers and firms cannot contract on efficient surplus sharing before meeting

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- ❑ CSE effectively fills in this missing market...
 - ❑ ...provided we’re willing to assume/believe the strong degree of **commitment** built into CSE model
 - ❑ (i.e., each side of a job-match would have an incentive to try to “renegotiate” the “posted” wage once they actually meet)

 - ❑ An open (?) question in search theory

RELEVANCE OF HOSIOS CONDITION IN DSGE

- Optimal policy (monetary and/or fiscal) will depend on whether or not $\eta = \alpha$
 - Yet another distortion (if $\eta = \alpha$ not satisfied) for policy to respond to
 - **New Keynesian models:** monetary policy (**absent Pigouvian fiscal policy...**) can be used to correct search externalities (Cooley and Quadrini (2004 *JET*), Arseneau and Chugh (2008 *JME*), Faia (2008 *JEDC*), Blanchard and Gali (2010 *AEJ:Macro*), Ravenna and Walsh (2011 *AEJ:Macro*))

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- Model dynamics can depend (noticeably) on whether or not $\eta = \alpha$
 - Positive analysis: Walsh (2005 *RED*) the first to demonstrate this, many others since
 - Optimal policy analysis: Arseneau and Chugh (2012 *JPE*), Chugh, Merkl, and Lechthaler (2016)
- Hosios issues arise in any DGE model with **any** type of search and matching market
 - New monetarist models (aka, “money search” models)
 - Rocheteau and Wright (2005 *Econometrica*)
 - Aruoba and Chugh (2010 *JET*)
 - Product search models
 - Gourio and Rudanko (2014 *ReStud*)
 - Arseneau, Chahrour, Chugh, and Finkelstein (2015 *JMCB*)

DSGE (LABOR) SEARCH MODELS

- ❑ Search models articulate trading frictions – cannot instantaneously or costlessly find trading partners
 - ❑ An appealing description of labor markets
 - ❑ Maybe of other markets also

- ❑ Tractable to incorporate in DSGE models because of assumption of aggregate matching function

- ❑ Too ad-hoc or “reduced-form” because of assumption of (black box) aggregate matching friction?

- ❑ The Shimer Puzzle and attempted answers continue(?)...
- ❑ ...as do New Keynesian modelers’ incorporation of labor matching structure
 - ❑ Perhaps enables talking meaningfully about the tradeoffs between inflation and unemployment...
 - ❑ ...i.e., seemingly resuscitates the original Phillips Curve, not the NK Phillips Curve (which links inflation to marginal costs...)