
LABOR MATCHING MODELS: FURTHER EQUILIBRIUM CONCEPTS

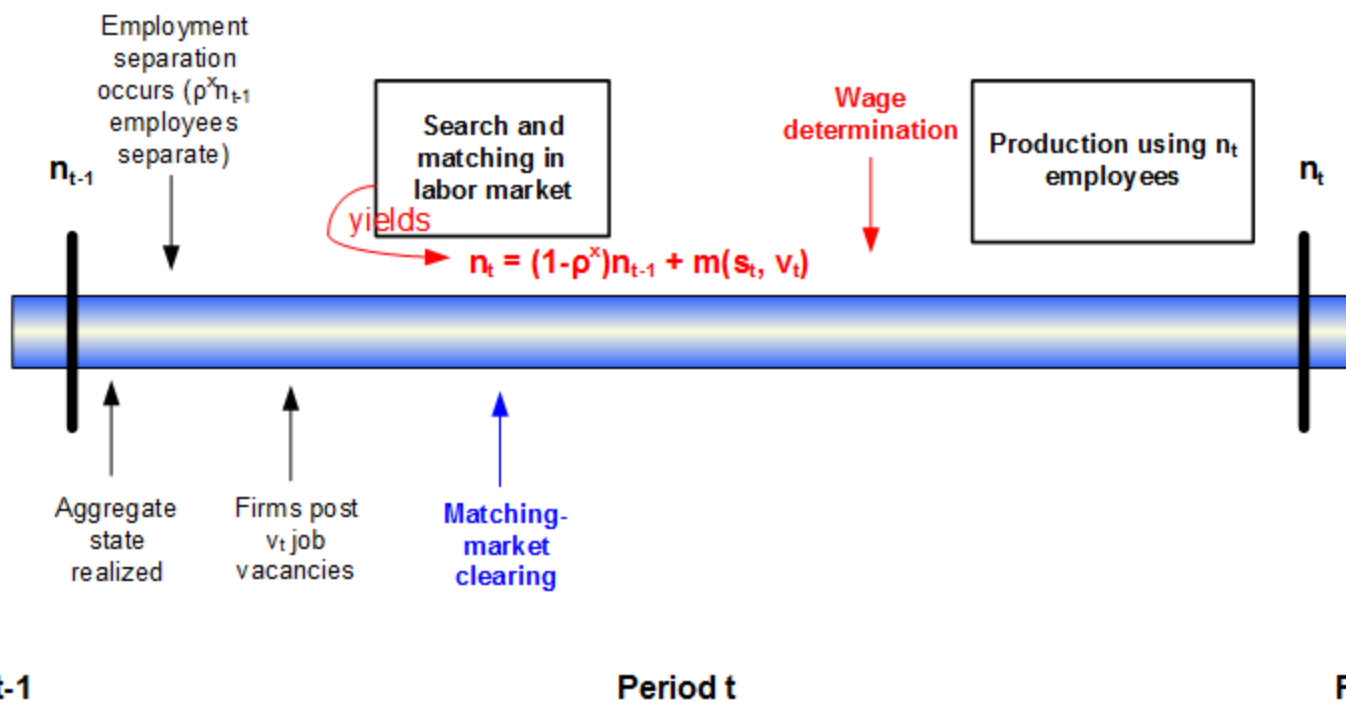
JANUARY 25, 2017

GENERALIZE MATCHING MODEL

- ❑ Equilibrium concepts
- ❑ **Search Equilibrium** (undirected search + post-match wage determination)
 - ❑ Extensive margin generically inefficient

SEARCH EQUILIBRIUM

- Matching market clearing ...
- ... then wage determination

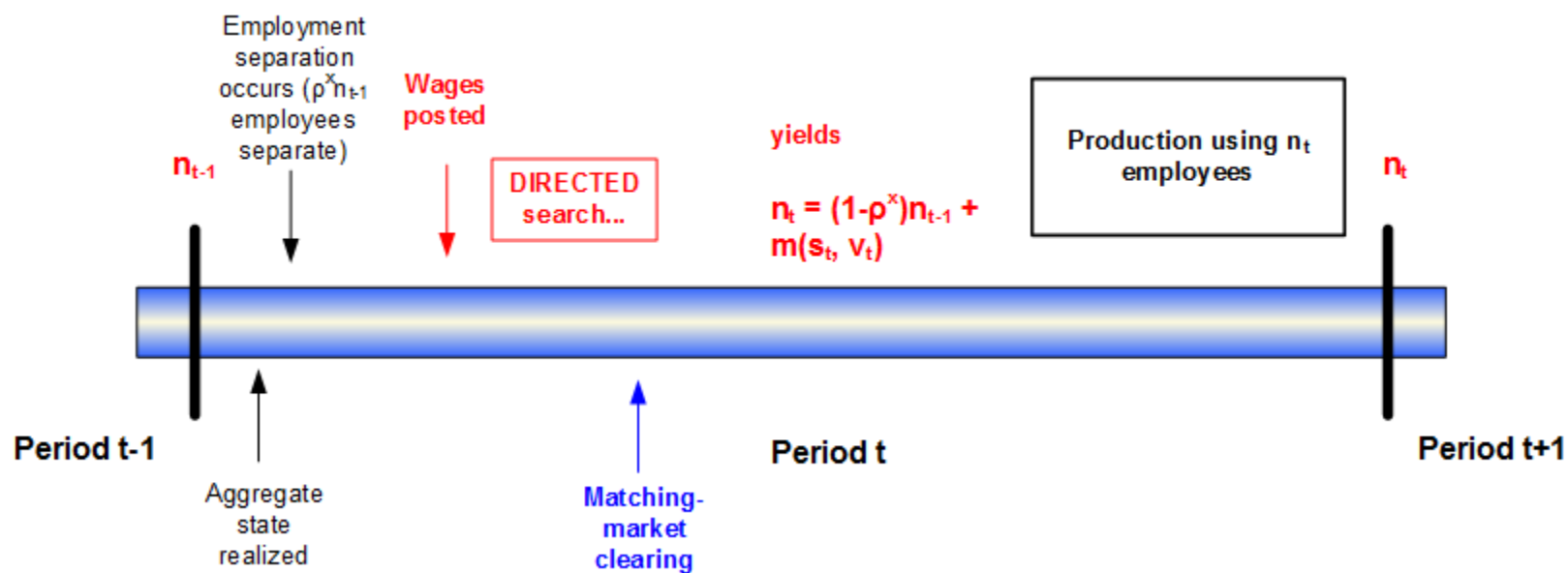


GENERALIZE MATCHING MODEL

- ❑ Equilibrium concepts
- ❑ **Search Equilibrium** (undirected search + post-match wage determination)
 - ❑ Extensive margin generically inefficient
- ❑ **Competitive Search Equilibrium** (wage posting + directed search)
 - ❑ Extensive margin efficient

COMPETITIVE SEARCH EQUILIBRIUM

- Wage determination ...
- ... then matching market clearing

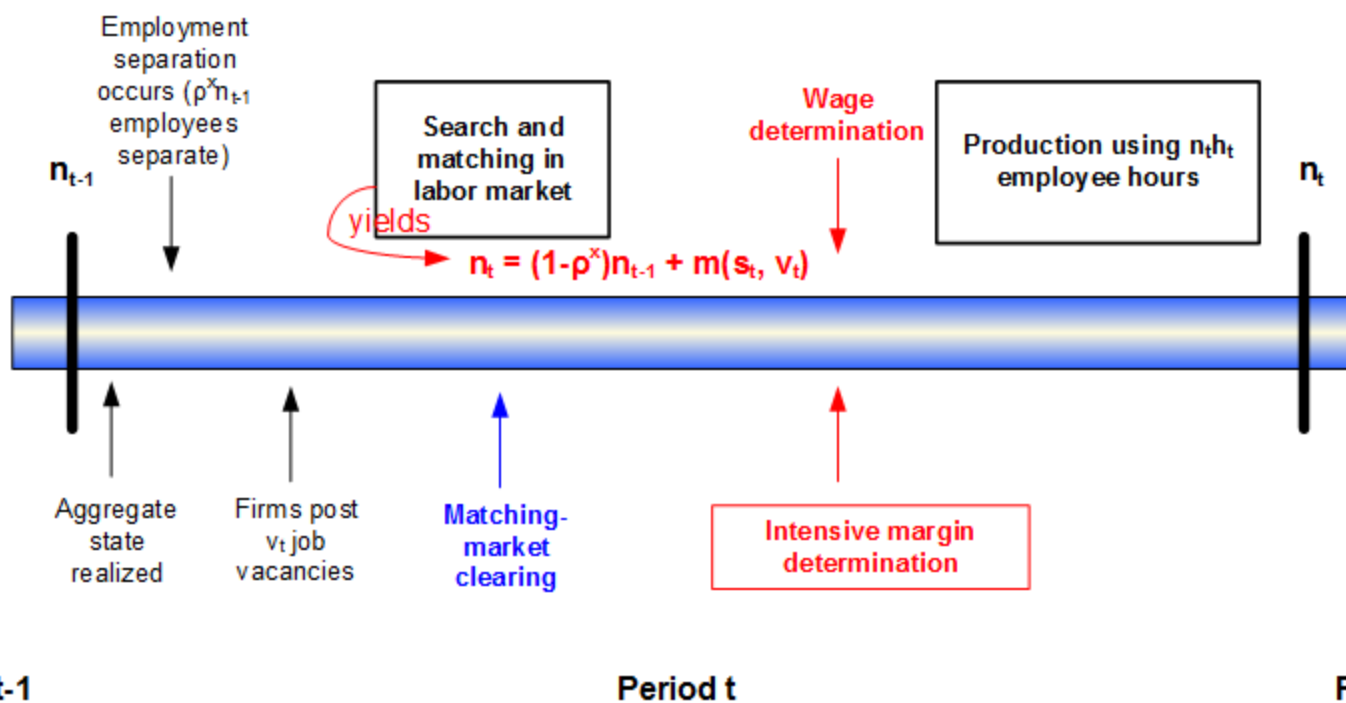


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- ❑ **Competitive Equilibrium**
 - ❑ Introduce intensive margin

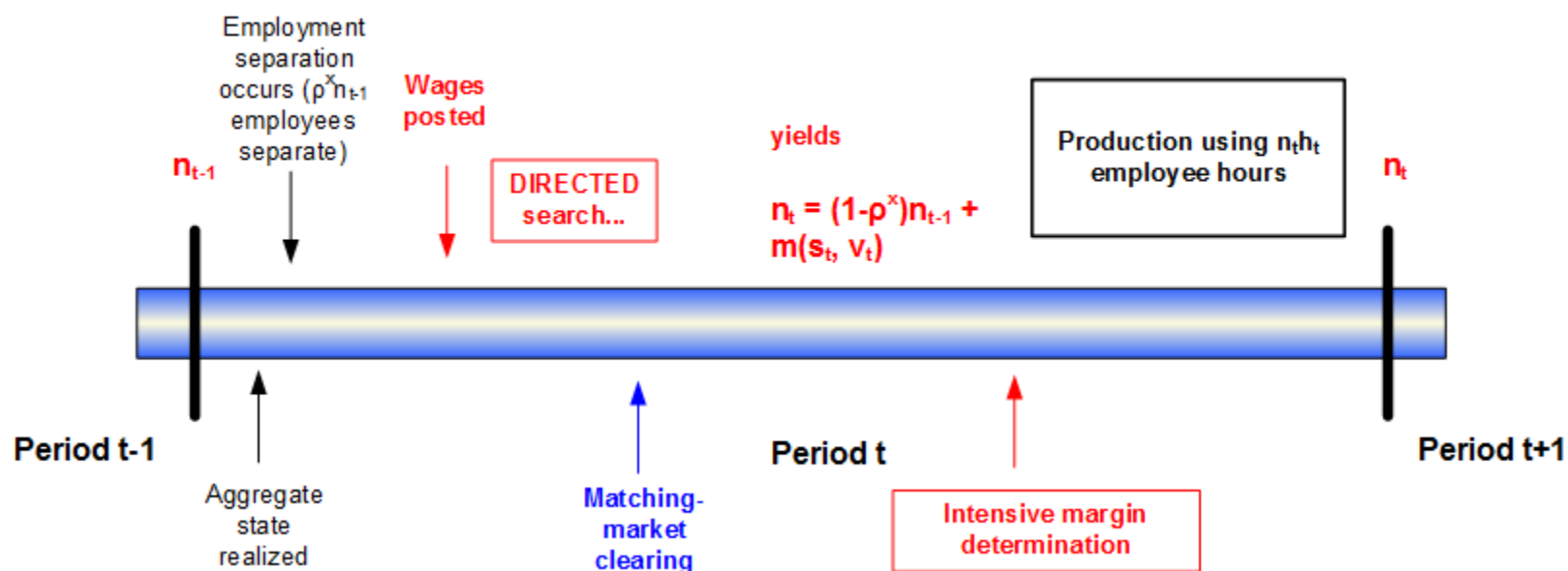
COMPETITIVE EQUILIBRIUM

- Matching market clearing ...
- ... then wage determination
- Intensive margin determination



COMPETITIVE SEARCH EQUILIBRIUM

- ❑ Wage determination ...
- ❑ ... then matching market clearing
- ❑ Intensive margin determination



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- ❑ Equilibrium concepts
- ❑ **Search Equilibrium** (undirected search + post-match wage determination)
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- ❑ **Competitive Search Equilibrium** (wage posting + directed search)
 - ❑ Extensive margin efficient
- ❑ **Competitive Equilibrium**
 - ❑ Introduce intensive margin
 - ❑ Intensive margin determined ex-post of match
 - ❑ Whether or not extensive margin is efficient
- ❑ **Matching Market Equilibrium**

INTENSIVE MARGIN

- Dynamic firm profit-maximization problem

$$\max_{v_t, n_t^f} E_0 \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f f(h_t) - w_t n_t^f h_t - \gamma v_t \right) \right]$$

$$\text{s.t. } n_t^f = (1 - \rho^x) n_{t-1}^f + v_t k^f(\theta_t)$$

- Total output produced by all employees = $\mathbf{z} \cdot \mathbf{n} \cdot \mathbf{f}(\mathbf{h})$
- Vacancy posting condition

$$\frac{\gamma}{k^f(\theta_t)} = z_t f(h_t) - w_t h_t + (1 - \rho^x) E_t \left\{ \Xi_{t+1|t} \frac{\gamma}{k^f(\theta_{t+1})} \right\}$$

- How is \mathbf{h} determined?

INTENSIVE MARGIN

- **Dynamic household utility maximization problem**

$$\max_{c_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_t) - h(n_t^h \cdot e(h_t)) \right) \right]$$

$$\text{s.t. } c_t = w_t n_t^h h_t + (1 - k^h(\theta_t)) \cdot (1 - n_t^h) \cdot b$$

$$n_t^h = (1 - \rho_x) n_{t-1}^h + (1 - n_t^h) k^h(\theta_t)$$

INTENSIVE MARGIN

- Dynamic household utility maximization problem

$$\max_{c_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_t) - n_t^h \cdot e(h_t) \right) \right]$$

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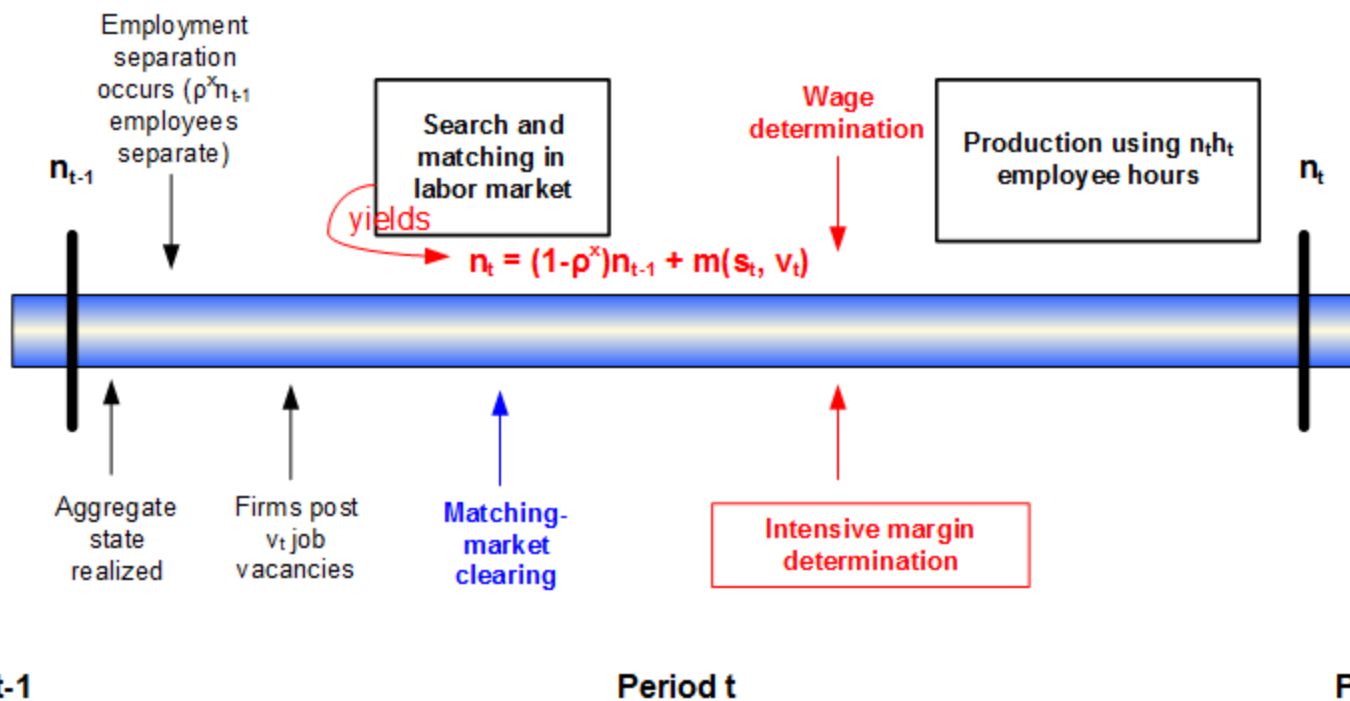
- Suppose **household** $h(\cdot)$ is quasi-linear
- How is h determined?

INTENSIVE MARGIN

- How is h determined?
 - Could depend on when wage determination occurs (pre-match? post-match?)

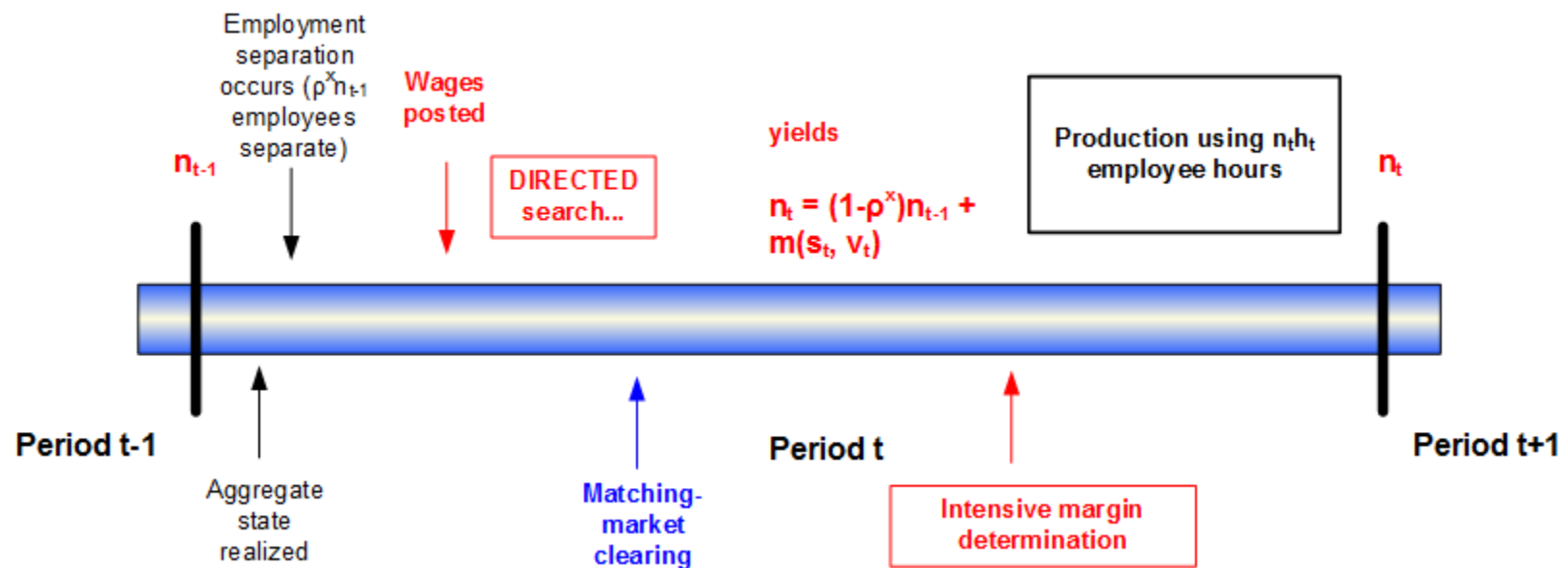
INTENSIVE MARGIN

- Matching market clearing ...
- ... then wage determination
- Intensive margin determination



INTENSIVE MARGIN

- ❑ Wage determination ...
- ❑ ... then matching market clearing
- ❑ Intensive margin determination



INTENSIVE MARGIN

- How is h determined?
 - Could depend on when wage determination occurs (pre-match? post-match?)

- Common setup for post-match wages
 - Simultaneous Nash bargaining over w and h

$$\max_{w_t, h_t} (\mathbf{W}_t - \mathbf{U}_t)^\eta \mathbf{J}_t^{1-\eta}$$

INTENSIVE MARGIN

- How is h determined?
 - Could depend on when wage determination occurs (pre-match? post-match?)
- Common setup for post-match wages
 - Simultaneous Nash bargaining over w and h

$$\max_{w_t, h_t} (\mathbf{W}_t - \mathbf{U}_t)^\eta \mathbf{J}_t^{1-\eta}$$

- Value expressions for an atomistic individual's potential new job/potential new employee

$$\mathbf{W}_t = w_t h_t - \frac{e(h_t)}{u'(c_t)} + E_t \left\{ \Xi_{t+1|t} \left[(1 - \rho_x) \mathbf{W}_{t+1} + \rho_x \mathbf{U}_{t+1} \right] \right\}$$

$$\mathbf{U}_t = b + E_t \left\{ \Xi_{t+1|t} \left[k^h(\theta_t) \mathbf{W}_{t+1} + (1 - k^h(\theta_t)) \mathbf{U}_{t+1} \right] \right\}$$

$$\mathbf{J}_t = z_t f(h_t) - w_t h_t + E_t \left\{ \Xi_{t+1|t} (1 - \rho_x) \mathbf{J}_{t+1} \right\}$$

INTENSIVE MARGIN

- Compute FOCs wrt w and h
- FOC wrt w yields

$$w_t h_t = \eta \left[z_t f(h_t) + \gamma \theta_t \right] + (1 - \eta)b$$

Identical algebra to the $h = 1$ case

- FOC wrt h yields

$$\eta \mathbf{J}_t \left(\frac{\partial \mathbf{W}_t}{\partial h_t} - \frac{\partial \mathbf{U}_t}{\partial h_t} \right) = (1 - \eta)(-1)(\mathbf{W}_t - \mathbf{U}_t) \frac{\partial \mathbf{J}_t}{\partial h_t} \quad (\text{VERIFY THE DERIVATION})$$

↓
Insert marginal values and rearrange
(a key observation is that....)

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

INTENSIVE MARGIN

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PRIVATE BILATERAL
EFFICIENCY

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

(...given simultaneous bargaining over w and h)

- Interpretation: $mrs_t = mpn_t$ for each given worker
 - Private bilateral efficiency on the hours margin
 - Whether or not Hosios efficiency holds on extensive margin

COMPETITIVE EQUILIBRIUM

- ❑ **Competitive Equilibrium**

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

- ❑ **Intensive margin is (bilaterally) efficient**
- ❑ **Whether or not extensive margin is efficient**

- ❑ **Where?**
 - ❑ **In a particular industry...**
 - ❑ **In a particular submarket...**
 - ❑ **In a particular “island”...**

COMPETITIVE EQUILIBRIUM

❑ Competitive Equilibrium

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

- ❑ Intensive margin is (bilaterally) efficient
- ❑ Whether or not extensive margin is efficient
- ❑ Where?
 - ❑ In a particular industry...
 - ❑ In a particular submarket...
 - ❑ In a particular “island”...
- ❑ For whom?...
- ❑ Those who have already **entered** the submarket
- ❑ But **entering** a submarket / **finding** a match could be (temporarily?) inefficient due to congestion externalities in market *ij*
 - ❑ Inefficient θ_{ij}

FREE-ENTRY CONDITIONS

- Set $h = 1$
- Free entry into matching market for firms?
- Job-creation condition

FREE-ENTRY CONDITIONS

- ❑ Set $h = 1$
- ❑ Free entry into matching market for firms?
- ❑ Job-creation condition (now with **diminishing marg. product in n**)

$$\frac{\gamma}{k^f(\theta_t)} = z_t f'(n_t) - w_t n_t + (1 - \rho_x) E_t \left\{ \Xi_{t+1|t} \frac{\gamma}{k^f(\theta_{t+1})} \right\}$$

- ❑ Free entry into matching market for workers?
- ❑ Not required given CRS matching and free entry by firms...
- ❑ ...but will lead to neat results
- ❑ **Endogenize labor force participation (aka "labor supply")**

LABOR FORCE PARTICIPATION

