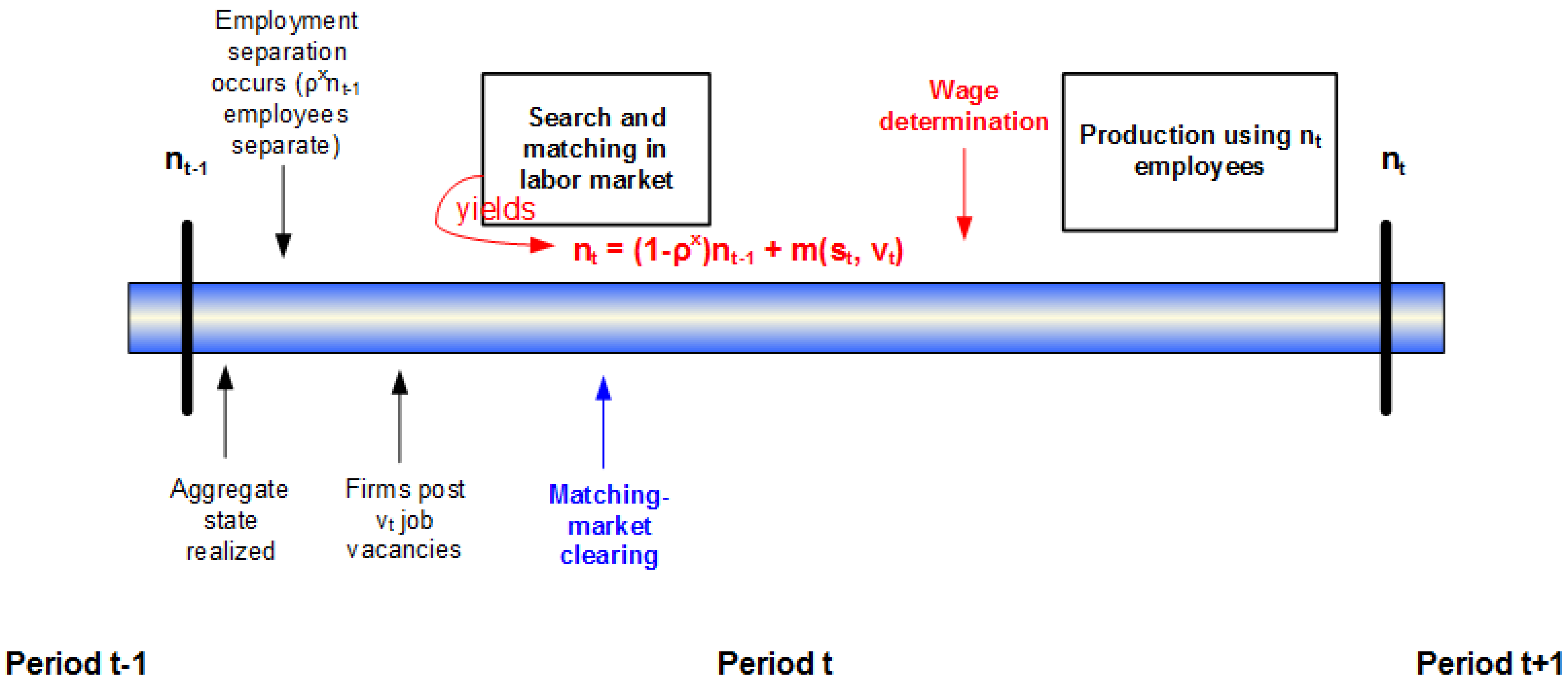
MATCHING MARKET CLEARING

JANUARY 25, 2017

TIMING OF EVENTS

- **Matching market clearing …**
- **… then wage determination**
- **Focus just on extensive margin**



Labor Supply (LFP)

- **Representative household**

$$\max_{c_t, n_t^s, s_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\Big(\underbrace{\left(1-k_t^h\right) s_t + n_t^s}_{\equiv lfp_t} \Big) \right]$$

$$c_t = (1-\tau_t^n) w_t n_t^s + (1-k_t^h) s_t b$$

$$n_t^s = (1-\rho_x) n_{t-1}^s + s_t \cdot k_t^h$$

- $lfp_t = (1-k_t^h)s_t + n_t^s$

- **Consumption-LFP optimality condition**

$$\frac{h'(lfp_t)}{u'(c_t)} = k_t^h (1-\tau_t^n) w_t + (1-k_t^h)\cdot b + k_t^h (1-\rho_x) E_t \left\{ \Xi_{t+1} \left(\frac{h'(lfp_{t+1}) - u'(c_{t+1}) b}{u'(c_{t+1})} \right) \cdot \left(\frac{1-k_{t+1}^h}{k_{t+1}^h} \right) \right\}$$

Labor Supply (LFP)

- **Definition: Household optimality is a set of state-contingent functions for $\left\{c_t, s_t, n_t^s\right\}$ that satisfy**

  - **Consumption-LFP optimality condition**

$$\frac{h'(lfp_t)}{u'(c_t)} = k_t^h(1-\tau_t^n)w_t + (1-k_t^h)\cdot b + k_t^h(1-\rho_x)E_t\left\{\Xi_{t+1}\left(\frac{h'(lfp_{t+1}) - u'(c_{t+1})b}{u'(c_{t+1})}\right)\cdot\left(\frac{1-k_{t+1}^h}{k_{t+1}^h}\right)\right\}$$

  - **Household budget constraint**

$$c_t = (1-\tau_t^n)w_t n_t^s + (1-k_t^h)s_t b$$

  - **Perceived law of motion of employment (aka job-finding constraint)**

$$n_t^s = (1-\rho_x)n_{t-1}^s + s_t \cdot k_t^h$$

taking as given exogenous processes $\left\{k_t^h, w_t, \tau_t^n\right\}$ and n_{-1}

Labor Demand (JC)

- **Representative firm**

$$\max_{v_t, n_t^D} E_0\left[\sum_{t=0}^{\infty}\Xi_{t|0}\left(z_t f(n_t^D) - w_t n_t^f - \gamma v_t\right)\right]$$

$$\text{s.t.}\;\; n_t^D = (1-\rho_x)n_{t-1}^D + v_t k_t^f$$

- **Job-creation condition**

$$\frac{\gamma}{k_t^f} = z_t f'(n_t^D) - w_t n_t^D + (1-\rho_x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k_{t+1}^f}\right\}$$

LABOR DEMAND (JC)

- **Definition: Firm optimality is a set of state-contingent functions $\left\{v_t, n_t^D\right\}$ that satisfy**

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$$\frac{\gamma}{k_t^f} = z_t f'(n_t^D) - w_t n_t^D + (1-\rho_x) E_t \left\{ \Xi_{t+1|t} \frac{\gamma}{k_{t+1}^f} \right\}$$

  - **Perceived law of motion of employment (aka job hiring constraint)**

$$n_t^D = (1-\rho_x) n_{t-1}^D + v_t \cdot k_t^f$$

taking as given exogenous processes $\left\{k_t^f, w_t\right\}$ and n_{-1}

MATCHING MARKET CLEARING

- **Definition: Equilibrium optimality is a set of state-contingent functions** $\{\theta_t, v_t, s_t, n_t, c_t\}$ **that satisfies aggregate RC,**
  - **Consumption-LFP optimality condition**

$$\frac{h'\left((1-k^h(\theta_t))s_t + n_t^s\right)}{u'(c_t)} = k^h(\theta_t)(1-\tau_t^n)w_t + (1-k^h(\theta_t))\cdot b$$

$$+k^h(\theta_t)(1-\rho_x)E_t\left\{\Xi_{t+1}\left(\frac{h'(lfp_{t+1})-u'(c_{t+1})b}{u'(c_{t+1})}\right)\cdot\left(\frac{1-k^h(\theta_{t+1})}{k^h(\theta_{t+1})}\right)\right\}$$

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$$\frac{\gamma}{k^f(\theta_t)} = z_t f'(n_t^D) - w_t n_t^D + (1-\rho_x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}$$

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  - **Aggregate LOM for employment**

$$n_t = (1-\rho_x)n_{t-1} + m(s_t, v_t)$$

  - **AND…**

Matching Market Clearing

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  - **Aggregate LOM for employment**

$$n_t = (1-\rho_x)n_{t-1} + m(s_t, v_t)$$

  - **Matching-market clearing** $k^h(\theta_t)\cdot s_t = k^f(\theta_t)\cdot v_t = m(s_t, v_t)$

LABOR SUPPLY (LFP)

- Consider ρ_x = 1, b = 0
- Definition: Household optimality is a set of state-contingent functions for $\{c_t, s_t, n_t^s\}$ that satisfy
  - Consumption-LFP optimality condition

$$\frac{h'\left((1-k_t^h)s_t + n_t^s\right)}{u'(c_t)} = k_t^h(1-\tau_t^n)w_t$$

  - Household budget constraint

$$c_t = (1-\tau_t^n)w_t n_t^s$$

  - Perceived "law of motion" of employment (aka job-finding constraint)

$$n_t^s = s_t \cdot k_t^h$$

taking as given exogenous processes $\{k_t^h, w_t, \tau_t^n\}$ and n_{-1}

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$$n_t^s = s_t \cdot k_t^h \quad \Leftrightarrow \quad s_t = \frac{n_t^s}{k_t^h}$$

taking as given exogenous processes $\{k_t^h, w_t, \tau_t^n\}$ and n_{-1}

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$$\frac{h'\left(\frac{n_t^s}{k_t^h} - n_t^s + n_t^s\right)}{u'(c_t)} = k_t^h(1-\tau_t^n)w_t$$

  - **Household budget constraint**

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  - Consumption-LFP optimality condition

$$\frac{h'\left(\frac{n_t^s}{k^h(\theta_{nt})}\right)}{u'(c_t)} = k(\theta_{nt})(1-\tau_t^n)w_t$$

  - Household budget constraint

$$c_t = (1-\tau_t^n)w_t n_t^s$$

taking as given exogenous processes $\{k^h(\theta_{nt}), w_t, \tau_t^n\}$ and n_{-1}

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- **Definition: Firm optimality is a set of state-contingent functions $\{v_t, n_t^D\}$ that satisfy**
  - **Job-creation condition**

$$\frac{\gamma}{k_t^f} = z_t f'(n_t^D) - w_t$$

  - **Perceived law of motion of employment (aka job hiring constraint)**

$$n_t^D = v_t \cdot k_t^f$$

taking as given exogenous processes $\{k_t^f, w_t\}$ and n_{-1}

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- **Definition: Firm optimality is a set of state-contingent functions** $\left\{v_t, n_t^D\right\}$ **that satisfy**
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$$\frac{\gamma}{k^f(\theta_{nt})} = z_t f'(n_t^D) - w_t$$

  - **Perceived law of motion of employment (aka job hiring constraint)**

$$n_t^D = v_t \cdot k^f(\theta_{nt})$$

taking as given exogenous processes $\left\{k^f(\theta_{nt}), w_t\right\}$ **and** $\boldsymbol{n_{-1}}$

MATCHING MARKET CLEARING

- **Definition: Equilibrium optimality is a set of state-contingent functions $\{\theta_t, v_t, s_t, n_t, c_t\}$ that satisfies aggregate RC,**
  - **Aggregate RC** $c_t + \gamma \cdot v_t = z_t f(n_t)$
  - **Consumption-LFP optimality condition**

$$\frac{h'\left(\frac{n_t}{k^h(\theta_{nt})}\right)}{u'(c_t)} = k^h(\theta_{nt})(1-\tau_t^n)w_t$$

Solve for θ using hh job finding constraint

  - **Job-creation condition**

$$\frac{\gamma}{k^f(\theta_{nt})} = z_t f'(n_t) - w_t$$

Solve for θ using firm job hiring constraint

  - **Aggregate LOM for employment**

$$n_t = m(s_t, v_t)$$

  - **Matching-market clearing** $k^h(\theta_{nt}) \cdot s_t = k^f(\theta_{nt}) \cdot v_t = m(s_t, v_t)$

MATCHING MARKET CLEARING

- **Functional forms**

$$m(s_t, v_t) = s_t^{\alpha_n} v_t^{1-\alpha_n} \quad \Rightarrow \quad m_s(s_t, v_t) = \alpha_n s_t^{\alpha_n - 1} v_t^{1-\alpha_n} = \alpha_n \theta_{nt}^{1-\alpha_n} \quad m_v(s_t, v_t) = (1-\alpha_n) s_t^{\alpha_n} v_t^{-\alpha_n} = (1-\alpha_n)\theta_{nt}^{-\alpha_n}$$

$$k^h(\theta_{nt}) = \theta_{nt}^{1-\alpha_n} \qquad k^f(\theta_{nt}) = \theta_{nt}^{-\alpha_n}$$

$$f(n_t) = n_t^{\alpha} \quad \Rightarrow \quad f(n_t) = \alpha n_t^{\alpha-1}$$

$$h(lfp_t) = \frac{\psi_n}{1+1/\varphi_n} lfp_t^{1+1/\varphi_n} \quad \Rightarrow \quad h'(lfp_t) = \psi_n lfp_t^{1/\varphi_n} \qquad u(c_t) = c_t \quad \Rightarrow \quad u'(c_t) = 1$$

$$\frac{h'\left(\dfrac{n_t}{k^h(\theta_{nt})}\right)}{u'(c_t)} = k^h(\theta_{nt}) w_t \qquad \frac{\gamma}{k^f(\theta_{nt})} = z_t f'(n_t) - w_t$$

Substitute…(and set tax rate = 0)

MATCHING MARKET CLEARING

- In (θ, n) space

$$\psi_n \cdot \left(n_t \cdot \theta_{nt}^{\alpha_n - 1}\right)^{1/\varphi_n} = \theta_{nt}^{1-\alpha_n} w_t$$

$$\gamma = \theta_{nt}^{-\alpha_n}\left(z_t \alpha n_t^{\alpha-1} - w_t\right)$$

...and solve for θ^{LFP} and θ^{JC}

$$\theta_{nt}^{LFP} = \left[\frac{w_t}{\psi_n \cdot n_t^{1/\varphi_n}}\right]^{-\frac{1}{(1-\alpha_n)(1+1/\varphi_n)}}$$

$$\theta_{nt}^{JC} = \left[\frac{z_t \alpha n_t^{\alpha-1} - w_t}{\gamma}\right]^{\frac{1}{\alpha_n}}$$

Matching Market Clearing

- **In (θ, n) space**

$$\psi_n \cdot \left(n_t \cdot \theta_{nt}^{\alpha_n - 1}\right)^{1/\varphi_n} = \theta_{nt}^{1-\alpha_n} w_t \qquad \gamma = \theta_{nt}^{-\alpha_n}\left(z_t \alpha n_t^{\alpha-1} - w_t\right)$$

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- **Arseneau and Chugh (2012 *JPE*, Proposition 1 (p. 950))**
  - **Efficient allocations characterized by**

$$\left(MRS_{c_t, lfp_t} \equiv\right)\frac{h'(lfp_t)}{u'(c_t)} = \frac{\gamma \cdot m_s(s_t, v_t)}{m_v(s_t, v_t)}\left(\equiv MRT_{c_t, lfp_t}\right)$$

- **Given functional forms, stated in (θ, n) space?...**

MATCHING MARKET CLEARING

- In (θ, n) space

$$\psi_n \cdot \left(n_t \cdot \theta_{nt}^{\alpha_n - 1}\right)^{1/\varphi_n} = \theta_{nt}^{1-\alpha_n} w_t \qquad \gamma = \theta_{nt}^{-\alpha_n}\left(z_t \alpha n_t^{\alpha-1} - w_t\right)$$

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$$\theta_{nt}^{LFP} = \left[\frac{w_t}{\psi_n \cdot n_t^{1/\varphi_n}}\right]^{-\frac{1}{(1-\alpha_n)(1+1/\varphi_n)}} \qquad \theta_{nt}^{JC} = \left[\frac{z_t \alpha n_t^{\alpha-1} - w_t}{\gamma}\right]^{\frac{1}{\alpha_n}}$$

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- Given functional forms, stated in (θ, n) space

$$\theta_{nt}^{LFP} = \theta_{nt}^{JC}$$

Obvious in (decentralized equilibrium that supports…) efficient allocations….

MATCHING MARKET CLEARING

- In (θ, n) space

$$\psi_n \cdot \left(n_t \cdot \theta_{nt}^{\alpha_n - 1}\right)^{1/\varphi_n} = \theta_{nt}^{1-\alpha_n} w_t \qquad \gamma = \theta_{nt}^{-\alpha_n}\left(z_t \alpha n_t^{\alpha-1} - w_t\right)$$

...and solve for θ^{LFP} and θ^{JC}

$$\theta_{nt}^{LFP} = \left[\frac{w_t}{\psi_n \cdot n_t^{1/\varphi_n}}\right]^{-\frac{1}{(1-\alpha_n)(1+1/\varphi_n)}} \qquad \theta_{nt}^{JC} = \left[\frac{z_t \alpha n_t^{\alpha-1} - w_t}{\gamma}\right]^{\frac{1}{\alpha_n}}$$

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  - Efficient allocations characterized by

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- Given functional forms, stated in (θ, n) space

$$\theta_{nt}^{LFP,EFF} = \theta_{nt}^{JC,EFF}$$

Obvious in (decentralized equilibrium that supports...) efficient allocations....

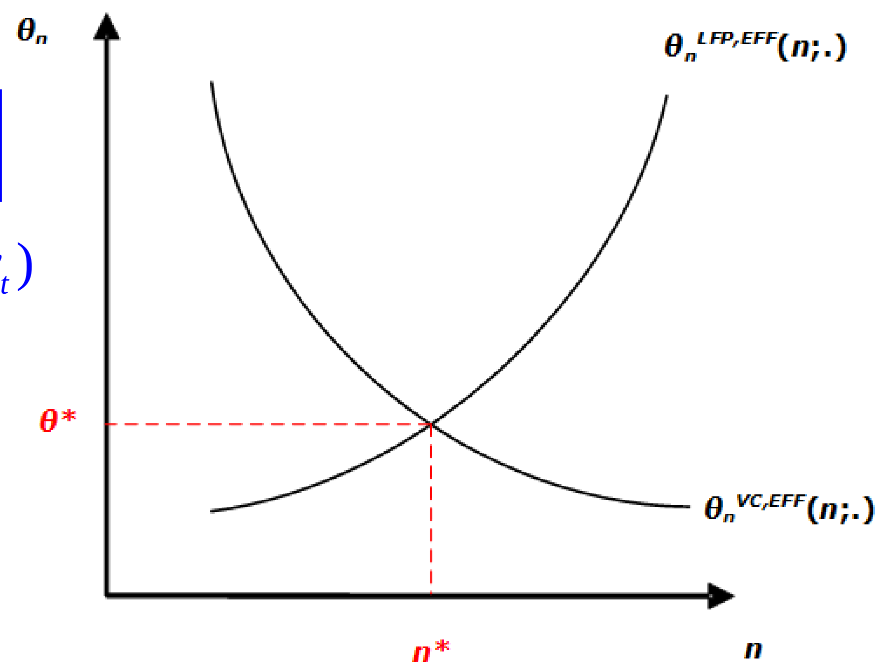
Matching Market Clearing

- Aggregate matching-market clearing

Aggregate matching market clearing

$$k_t^h \cdot s_t = k_t^f \cdot v_t = m(s_t, v_t)$$

$$\equiv \text{new employees}_t$$



MATCHING MARKET CLEARING

- Aggregate matching-market clearing
- Experiment 1: labor income tax rate $\tau^n > 0$

$$WEDGE_{n,\tau^n} \equiv \left| \int_{n^{LOW}}^{n^*} \left[\theta_n^{LFP,EFF}(n_i;\cdot) - \theta_n^{VC,EFF}(n_i;\cdot) \right] dn_i \right|$$

Aggregate matching market clearing

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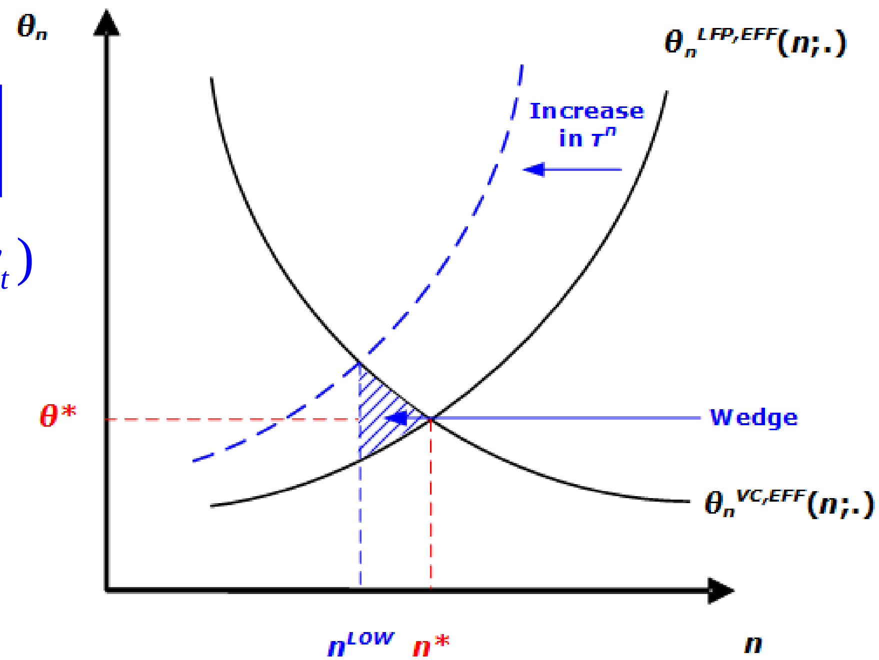

MATCHING MARKET CLEARING

- ❒ **Aggregate matching-market clearing**
- ❒ **Experiment 2: unemployment benefit *b* > 0**

$$WEDGE_{n,b} \equiv \left| \int_{n^*}^{n^{HIGH}} \left[\theta_n^{LFP,EFF}(n_i;\cdot) - \theta_n^{VC,EFF}(n_i;\cdot) \right] dn_i \right|$$

Aggregate matching market clearing

$$k_t^h \cdot s_t = k_t^f \cdot v_t = m(s_t, v_t)$$

$$\equiv \text{new employees}_t$$