MONOPOLISTIC COMPETITION IN A DSGE MODEL

FEBRUARY 1, 2017

EMPIRICAL AND THEORETICAL CONSIDERATIONS

- **Evidence supports existence of markups in goods markets (i.e., $p > mc$)**
  - **Basu and Fernald (1997 *JPE*) often-cited source**
- **Evidence also supports positive (but small?...) pure economic profits**
- **Are firms always price-takers?**
  - **If not, must endow them with market power**
- **If increasing returns in production exist, a model without market power does not admit an equilibrium with increasing returns**
- **Introduce imperfect competition**
  - **Typically monopolistic competition…**
  - **…a building block of sticky nominal price monetary models**

WORKHORSE MODEL

- Dixit-Stiglitz (1977 *AER*) model
  - Common specification of imperfect competition in macro models
  - Typical building block of sticky price monetary models
  - Basic idea: imperfectly-substitutable goods combined yield an aggregate good

CES function: ε the constant elasticity of substitution between any pair of differentiated goods

$$c_t = \left[\sum_{i=1}^{N} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Discrete number of differentiated goods

$$c_t = \left[\int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Continuum of differentiated goods

- Important properties of aggregator
  - Symmetric in all arguments ← Drives efficiency/optimal policy results
  - Strictly increasing in all arguments
  - Strictly concave in all arguments
  - Homogenous of degree one

WORKHORSE MODEL

- Dixit-Stiglitz (1977 *AER*) model
  - Common specification of imperfect competition in macro models
  - Typical building block of sticky price monetary models
  - Basic idea: imperfectly-substitutable goods combined yield an aggregate good

CES function: ε the constant elasticity of substitution between any pair of differentiated goods

In some applications, ε can be time-varying (either endogenously or exogenously)

$$c_t = \left[\sum_{i=1}^{N} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Discrete number of differentiated goods

In some applications, make this endogenous and time-varying, N_t

$$c_t = \left[\int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Continuum of differentiated goods

- Important properties of aggregator
  - Symmetric in all arguments ← **Drives efficiency/optimal policy results**
  - Strictly increasing in all arguments
  - Strictly concave in all arguments
  - Homogenous of degree one

TWO EQUIVALENT IMPLEMENTATIONS

- **Consumption aggregator** $c_t = \left[\int_0^{N_t} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$

"First-stage" problem

- **Consumer chooses c_t…** ← Utility-maximization problem

"Second-stage" problem

- **…then chooses each of the c_{it}** ← Cost-minimization problem (dual)
- **Each differentiated good *i* produced by a unique producer**
  - **KEY: takes as given the demand function it faces**

Two Equivalent Implementations

- **Consumption aggregator** $c_t = \left[\int_0^{N_t} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$

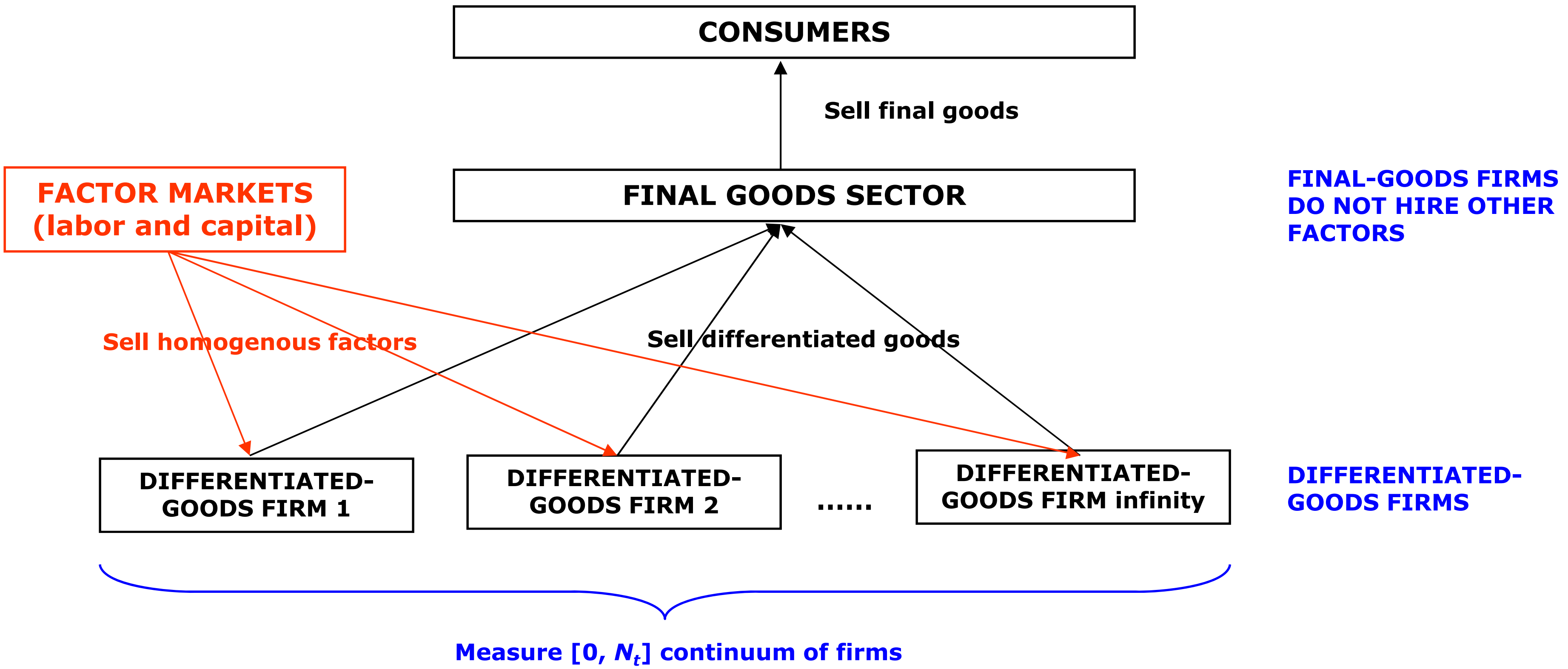
"First-stage" problem
- Consumer chooses c_t… ← Utility-maximization problem

"Second-stage" problem
- …then chooses each of the c_{it} ← Cost-minimization problem (dual)
- Each differentiated good *i* produced by a unique producer
  - KEY: takes as given the demand function it faces

- **Production aggregator** $y_t = \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$
  - Final-goods producer chooses y_{it}… ← Profit-maximization problem
  - …to sell a composite final good y_t to consumers ← Expenditure-minimization problem (dual)
  - Each differentiated good *i* produced by a unique intermediate-goods producer
    - KEY: takes as given the demand function it faces

- **Isomorphic results**

MARKET STRUCTURE



MARKET ORGANIZATION

- **Differentiated producer *i* production technology** $y_{it} = \underbrace{\overbrace{z_t f\left(k_{it}, n_{it}\right)}^{\text{Usual CRS}} - \Phi}$

  "Net-of-fixed-factor production technology" exhibits IRS (i.e., marginal cost < average cost)

  Fixed production factor: a) sometimes useful for calibrating profit share b) often set to zero

  - **Rotemberg and Woodford (*Frontiers* chapter): "materials cost" foundations**

MARKET ORGANIZATION

- **Differentiated producer *i* production technology** $y_{it} = \underbrace{\overbrace{z_t f\left(k_{it}, n_{it}\right)}^{\text{Usual CRS}} - \Phi}$

  "Net-of-fixed-factor production technology" exhibits IRS (i.e., marginal cost < average cost)

  Fixed production factor: a) sometimes useful for calibrating profit share b) often set to zero

  - **Rotemberg and Woodford (*Frontiers* chapter): "materials cost" foundations**

- **Differentiated producer *i* hires inputs on perfectly-competitive markets…**

- **…and sells its output on its own *monopolistically-competitive* market**
  - **Sells "directly" to consumers…**
  - **…or to final-goods firms**

- **For starters, suppose $\Phi = 0$ (→ mc = ac assuming CRS)**

Final-Goods Firms

- **Production Model**
  - **(Representative) final goods producer**

$$\max_{y_{it}\rangle_{i=0}^{N_t}} y_t - \int_0^{N_t} p_{it} y_{it} di$$

Substitute in CES final-goods aggregator

$$\max_{y_{it}\rangle_{i=0}^{N_t}} \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^{N_t} p_{it} y_{it} di$$

FINAL-GOODS FIRMS

- **Production Model**
  - **(Representative) final goods producer**

$$\max_{y_{it}\rangle_{i=0}^{N_t}} y_t - \int_0^{N_t} p_{it} y_{it} di$$

Substitute in CES final-goods aggregator

$$\max_{y_{it}\rangle_{i=0}^{N_t}} \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^{N_t} p_{it} y_{it} di$$

    - **Takes as given all p_{it}**

  - **Profit-maximization leads to demand functions for each underlying differentiated good *i***

Each differentiated firm *i* chooses its p_i to maximize profit

$$y_{it} = p_{it}^{-\varepsilon} \cdot y_t$$

TAKEN AS GIVEN BY DIFFERENTIATED FIRM *i*

Relative price of firm *i*'s output

Aggregate output a *shifter* of firm *i*'s demand function

Differentiated-Goods Firms

- **Production Model**
  - **Differentiated goods producer *i***

$$\max_{p_{it}} p_{it} y_{it} - w_t n_{it} - r_t k_{it}$$

↓ Substitute in demand function

$$\max_{p_{it}} p_{it} p_{it}^{-\varepsilon} y_t - w_t n_{it} - r_t k_{it}$$

DIFFERENTIATED-GOODS FIRMS

- **Production Model**
  - **Differentiated goods producer *i***

$$\max_{p_{it}} p_{it} y_{it} - w_t n_{it} - r_t k_{it}$$

Substitute in demand function

$$\max_{p_{it}} p_{it} p_{it}^{-\varepsilon} y_t - w_t n_{it} - r_t k_{it}$$

- **A "two-stage" optimization problem**
  - **Stage 1: Choose optimal p_i**
  - **(Intermediate "stage"): "choose" to produce the y_i corresponding to the optimal choice of p_i**
  - **Stage 2: Choose factor inputs to produce y_i at minimum cost**

i.e., production y_i pinned down from downward-sloping demand curve →

GIVEN 1) CRS $f(k, n)$ and 2) $\Phi = 0$

→ mc = ac = CONSTANT (with respect to quantity)

DIFFERENTIATED-GOODS FIRMS

- **Production Model**
  - **Differentiated goods producer *i***

$$\max_{p_{it}} p_{it}y_{it} - w_t n_{it} - r_t k_{it}$$

Substitute in demand function

$$\max_{p_{it}} p_{it}p_{it}^{-\varepsilon}y_t - w_t n_{it} - r_t k_{it}$$

- **A "two-stage" optimization problem**
  - **Stage 1: Choose optimal p_i**
  - **(Intermediate "stage"): "choose" to produce the y_i corresponding to the optimal choice of p_i**
  - **Stage 2: Choose factor inputs to produce y_i at minimum cost**

i.e., production y_i pinned down from downward-sloping demand curve

GIVEN 1) CRS $f(k, n)$ and 2) $\Phi = 0$

→ mc = ac = CONSTANT (with respect to quantity)

STAGE-1 PROBLEM

$$\max_{p_{it}} p_{it}p_{it}^{-\varepsilon}y_t - mc_t y_{it}$$

Substitute in demand function

$$\max_{p_{it}} p_{it}p_{it}^{-\varepsilon}y_t - mc_t p_{it}^{-\varepsilon}y_t$$

DIFFERENTIATED-GOODS FIRMS

- **Production Model or Consumption Model**
  - **Differentiated goods producer *i* optimal choice of p_i**

$$p_{it} = \underbrace{\frac{\varepsilon}{\varepsilon - 1}} \cdot mc_t$$

Gross product-market markup

Linked *only* to degree of substitutability

RBC model: ε = infinity (perf. comp.)

Monopoly model requires $\varepsilon > 1$ and $\varepsilon <$ infinity

DIFFERENTIATED-GOODS FIRMS

- **Production Model or Consumption Model**
  - **Differentiated goods producer *i* optimal choice of p_i**

$$p_{it} = \underbrace{\frac{\varepsilon}{\varepsilon - 1}} \cdot mc_t$$

Gross product-market markup

Linked *only* to degree of substitutability

RBC model: ε = infinity (perf. comp.)

Monopoly model requires $\varepsilon > 1$ and $\varepsilon <$ infinity

  - **Stage 2: cost-minimization**
    - **Given optimal (p_i, y_i)**

$$\max_{k_{it}, n_{it}} p_{it} z_t f(k_{it}, n_{it}) - w_t n_{it} - r_t k_{it}$$

NOTE: cost-minimization equivalent to profit-maximization *GIVEN* (p_i, y_i) -- i.e., DUAL PROBLEM

DIFFERENTIATED-GOODS FIRMS

- **Production Model or Consumption Model**
  - **Differentiated goods producer *i* optimal choice of p_i**

$$p_{it} = \frac{\varepsilon}{\varepsilon - 1} \cdot mc_t$$

Gross product-market markup

Linked *only* to degree of substitutability

RBC model: ε = infinity (perf. comp.)

Monopoly model requires $\varepsilon > 1$ and $\varepsilon <$ infinity

  - **Stage 2: cost-minimization**
    - Given optimal (p_i, y_i)

NOTE: cost-minimization equivalent to profit-maximization *GIVEN* (p_i, y_i) -- i.e., DUAL PROBLEM

$$\max_{k_{it}, n_{it}} p_{it} z_t f(k_{it}, n_{it}) - w_t n_{it} - r_t k_{it}$$

substitute $p_{it} = \left[z_t f(k_{it}, n_{it})\right]^{-1/\varepsilon} y_t^{1/\varepsilon}$ from dmd. fct.

$$\max_{k_{it}, n_{it}} \left[z_t f(k_{it}, n_{it})\right]^{1-1/\varepsilon} y_t^{1/\varepsilon} - w_t n_{it} - r_t k_{it}$$

  - **Factor demands (k_i, n_i) solve**

$$\frac{\varepsilon - 1}{\varepsilon} p_{it} z_t f_k(k_{it}, n_{it}) = r_t \qquad \frac{\varepsilon - 1}{\varepsilon} p_{it} z_t f_n(k_{it}, n_{it}) = w_t$$

BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Symmetric equilibrium across all *i***

$$\frac{\varepsilon-1}{\varepsilon} p_t z_t f_k(k_t, n_t) = r_t \quad \& \quad \frac{\varepsilon-1}{\varepsilon} p_t z_t f_n(k_t, n_t) = w_t \quad \& \quad p_t = \frac{\varepsilon}{\varepsilon-1} \cdot mc_t$$

↓ **implies**

$$mc_t = \frac{w_t}{z_t f_n(k_t, n_t)} = \frac{r_t}{z_t f_k(k_t, n_t)}$$

BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Symmetric equilibrium across all *i***

$$\frac{\varepsilon-1}{\varepsilon} p_t z_t f_k(k_t, n_t) = r_t \quad \& \quad \frac{\varepsilon-1}{\varepsilon} p_t z_t f_n(k_t, n_t) = w_t \quad \& \quad p_t = \frac{\varepsilon}{\varepsilon-1} \cdot mc_t$$

↓ **implies**

$$mc_t = \frac{w_t}{z_t f_n(k_t, n_t)} = \frac{r_t}{z_t f_k(k_t, n_t)}$$

Symmetric equilibrium price of good? Substitute demand functions into DS aggregator and compute…

$$p_t = ?...$$

BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Price index**

Final good

$$y_t = \int_0^{N_t} p_{it} \cdot y_{it} di$$

Substitute demand functions $y_{it} = p_{it}^{-\varepsilon} \cdot y_t$ →

$$y_t = \int_0^{N_t} p_{it} \cdot p_{it}^{-\varepsilon} y_t di$$

→ **y_t independent of index of integration *i***

$$y_t = y_t \cdot \int_0^{N_t} p_{it}^{1-\varepsilon} di$$

→ **Cancel y_t terms**

$$1 = \int_0^{N_t} p_{it}^{1-\varepsilon} di$$

BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Price index**

Final good

$$y_t = \int_0^{N_t} p_{it} \cdot y_{it} di$$

Substitute demand functions $y_{it} = p_{it}^{-\varepsilon} \cdot y_t$ →

$$y_t = \int_0^{N_t} p_{it} \cdot p_{it}^{-\varepsilon} y_t di$$

y_t independent of index of integration i →

$$y_t = y_t \cdot \int_0^{N_t} p_{it}^{1-\varepsilon} di$$

Cancel y_t terms →

$$1 = \int_0^{N_t} p_{it}^{1-\varepsilon} di$$

Substitute $p_{it} = p_t$ for all $i \in [0, N_t]$ →

$$1 = \int_0^{N_t} p_t^{1-\varepsilon} di$$

p_t independent of index of integration i →

$$1 = p_t^{1-\varepsilon} \cdot \int_0^{N_t} 1 \cdot di$$

Rewrite →

$$1 = p_t^{1-\varepsilon} \cdot N_t$$

Solve for p_t →

$$p_t = N_t^{\frac{1}{\varepsilon-1}}$$

BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Price index**

Final good

$$y_t = \int_0^{N_t} p_{it} \cdot y_{it} di$$

Substitute demand functions $y_{it} = p_{it}^{-\varepsilon} \cdot y_t$ →

$$y_t = \int_0^{N_t} p_{it} \cdot p_{it}^{-\varepsilon} y_t di$$

y_t independent of index of integration i →

$$y_t = y_t \cdot \int_0^{N_t} p_{it}^{1-\varepsilon} di$$

Cancel y_t terms →

$$1 = \int_0^{N_t} p_{it}^{1-\varepsilon} di$$

Substitute $p_{it} = p_t$ for all $i \in [0, N_t]$ →

$$1 = \int_0^{N_t} p_t^{1-\varepsilon} di$$

p_t independent of index of integration i →

$$1 = p_t^{1-\varepsilon} \cdot \int_0^{N_t} 1 \cdot di$$

Rewrite →

$$1 = p_t^{1-\varepsilon} \cdot N_t$$

Solve for p_t →

$$p_t = N_t^{\frac{1}{\varepsilon - 1}}$$

- **Symmetric equilibrium price depends on measure [0, N_t] of monopolistically-competitive firms**
  - **If $N_t = 1 \rightarrow p_t = 1$**

BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Symmetric equilibrium across all *i***

$$\frac{\varepsilon-1}{\varepsilon} p_t z_t f_k(k_t, n_t) = r_t \quad \& \quad \frac{\varepsilon-1}{\varepsilon} p_t z_t f_n(k_t, n_t) = w_t \quad \& \quad p_t = \frac{\varepsilon}{\varepsilon-1} \cdot mc_t$$

↓ **implies**

$$mc_t = \frac{w_t}{z_t f_n(k_t, n_t)} = \frac{r_t}{z_t f_k(k_t, n_t)}$$

Symmetric equilibrium price of good? Substitute demand functions into DS aggregator and compute…

$$p_t = N_t^{\frac{1}{\varepsilon-1}}$$

BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Symmetric equilibrium across all *i***

$$\frac{\varepsilon-1}{\varepsilon} p_t z_t f_k(k_t, n_t) = r_t \quad \& \quad \frac{\varepsilon-1}{\varepsilon} p_t z_t f_n(k_t, n_t) = w_t \quad \& \quad p_t = \frac{\varepsilon}{\varepsilon-1} \cdot mc_t$$

↓ **implies**

$$mc_t = \frac{w_t}{z_t f_n(k_t, n_t)} = \frac{r_t}{z_t f_k(k_t, n_t)}$$

Symmetric equilibrium price of good? Substitute demand functions into DS aggregator and compute…

$$p_t = N_t^{\frac{1}{\varepsilon-1}}$$

With measure [0, N_t] of intermediate firms… what if measure [0, 1] of firms?

BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Symmetric equilibrium across all *i***

$$\frac{\varepsilon-1}{\varepsilon} p_t z_t f_k(k_t, n_t) = r_t \quad \& \quad \frac{\varepsilon-1}{\varepsilon} p_t z_t f_n(k_t, n_t) = w_t \quad \& \quad p_t = \frac{\varepsilon}{\varepsilon-1} \cdot mc_t$$

↓ **implies**

$$mc_t = \frac{w_t}{z_t f_n(k_t, n_t)} = \frac{r_t}{z_t f_k(k_t, n_t)}$$

Symmetric equilibrium price of good? Substitute demand functions into DS aggregator and compute…

$$p_t = 1$$

With measure [0, N_t] of intermediate firms… what if measure [0, 1] of firms?

BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Symmetric equilibrium across all *i***

$$\frac{\varepsilon-1}{\varepsilon} p_t z_t f_k(k_t, n_t) = r_t \quad \& \quad \frac{\varepsilon-1}{\varepsilon} p_t z_t f_n(k_t, n_t) = w_t \quad \& \quad p_t = \frac{\varepsilon}{\varepsilon-1} \cdot mc_t$$

↓ **implies**

$$mc_t = \frac{w_t}{z_t f_n(k_t, n_t)} = \frac{r_t}{z_t f_k(k_t, n_t)}$$

Symmetric equilibrium price of good? Substitute demand functions into DS aggregator and compute…

$$p_t = 1$$

With measure [0, N_t] of intermediate firms… what if measure [0, 1] of firms?

↓

$$mc_t = \frac{\varepsilon-1}{\varepsilon}$$

< 1 with ε > 1 and ε < infinity

Monopoly power causes factor prices to fall below marginal products…hence inefficiently low equilibrium factor use…hence inefficiently low total output

MONOPOLISTICALLY-COMPETITIVE EQUILIBRIUM

- **Equilibrium Conditions (symmetric across all differentiated goods)**
  - **Consumption-leisure optimality condition**
  - **Consumption-savings optimality condition**
  - **Aggregate resource constraint**

$$c_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t)$$

  - **(Market clearing in labor, capital, and goods markets)**

MONOPOLISTICALLY-COMPETITIVE EQUILIBRIUM

- **Equilibrium Conditions (symmetric across all differentiated goods)**

  - **Consumption-leisure optimality condition**
  - **Consumption-savings optimality condition**
  - **Aggregate resource constraint**

$$c_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t)$$

  - **(Market clearing in labor, capital, and goods markets)**

  - $mc_t = \frac{\varepsilon - 1}{\varepsilon} \quad \forall t$ **(< 1 with ε > 1)**

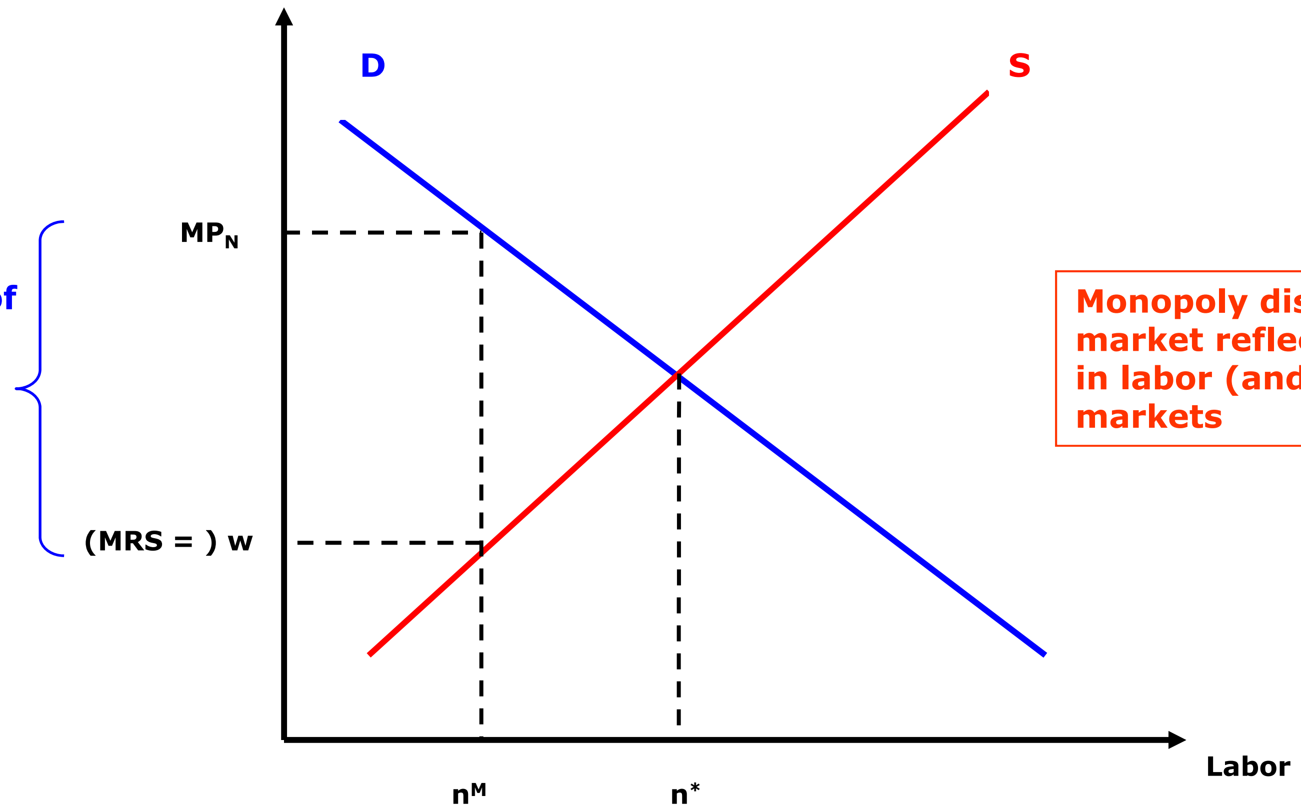
  - **Factor prices a mark*down* of marginal products**

$$w_t = \frac{\varepsilon - 1}{\varepsilon} \cdot z_t f_n(k_t, n_t), \quad r_t = \frac{\varepsilon - 1}{\varepsilon} \cdot z_t f_k(k_t, n_t)$$

THE LABOR WEDGE

markdown of real wage from marginal product of labor

"Labor Wedge"

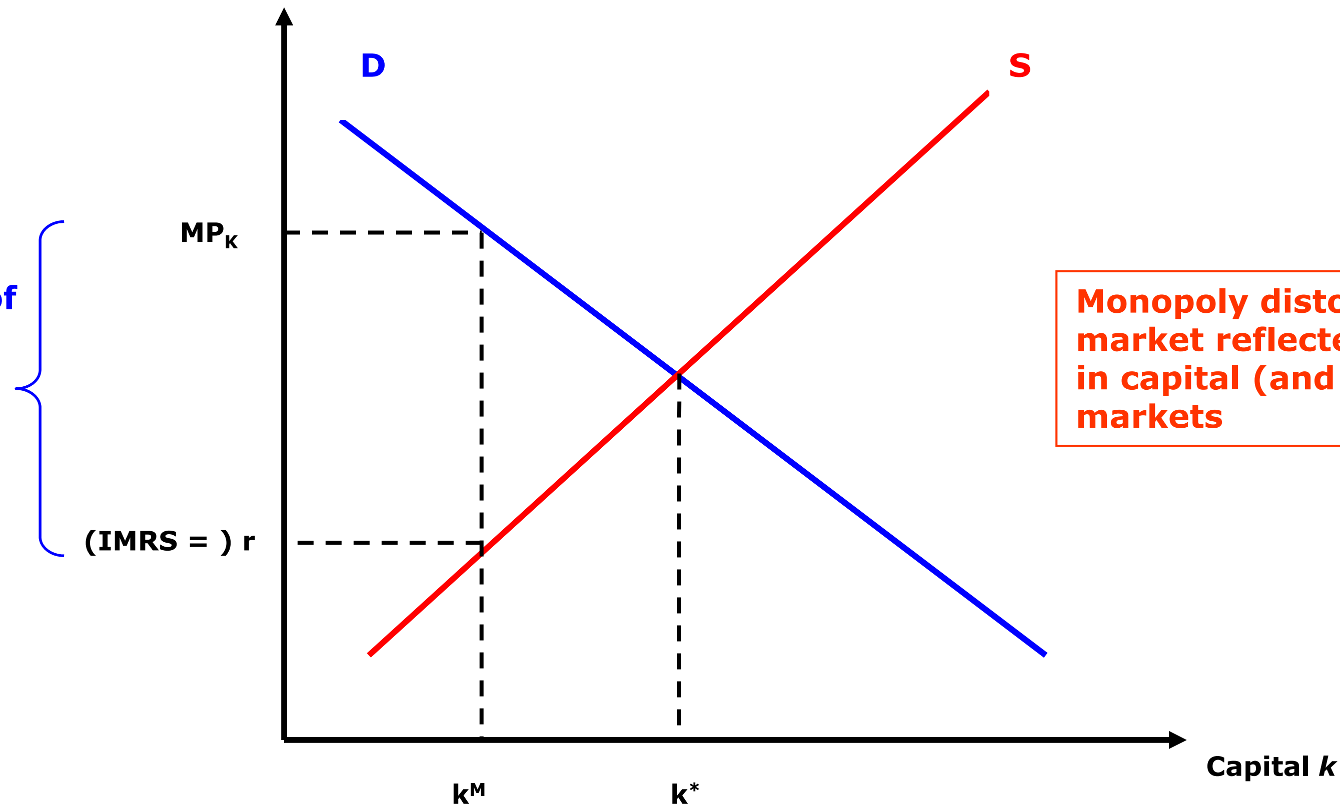


Monopoly distortion in goods market reflected in distortion in labor (and other factor) markets

THE CAPITAL WEDGE

markdown of real *r* from marginal product of capital

"Capital Wedge"



Monopoly distortion in goods market reflected in distortion in capital (and other factor) markets