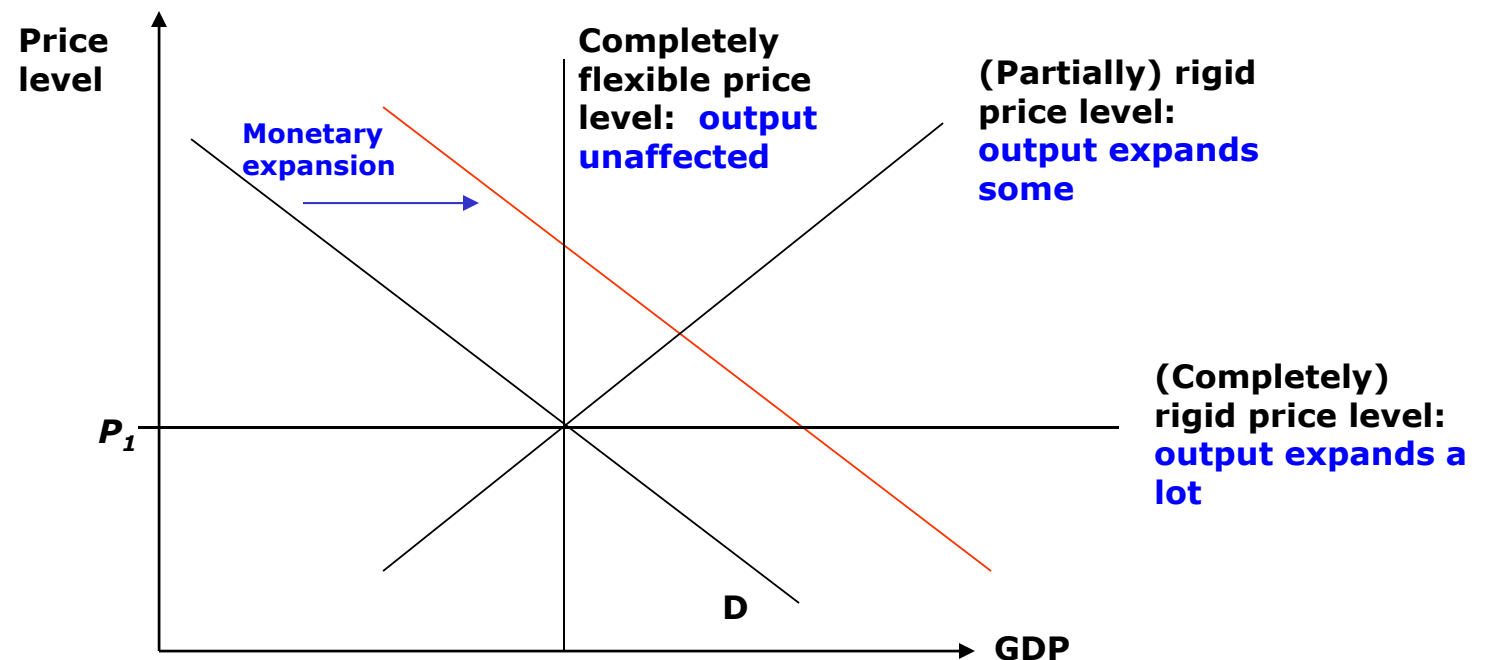

NOMINAL RIGIDITIES IN A DSGE MODEL

FEBRUARY 6, 2017

CYCLICAL IMPLICATIONS OF MONETARY POLICY

- **Conventional Keynesian view: nominal rigidities (in price and/or wage level) cause monetary shifts to have real effects**



https://en.wikipedia.org/wiki/Nominal_rigidity

GENERAL ISSUES

- ❑ **How often do prices change empirically?...**
 - ❑ **Wide heterogeneity across goods/categories of goods**
 - ❑ **Bils and Klenow (2004 *JPE*), Nakamura and Steinsson (2007), Kehoe and Midrigan (2007), Klenow and Krystov (2007), many others...**
 - ❑ **Median ~ 2-3 quarters...**

- ❑ **...and which price changes are most relevant for macro phenomena?**
 - ❑ **Aggregate price level?**
 - ❑ **Individual goods/sectors? Thus require a model of heterogeneity in pricing outcomes?**

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 - ❑ **Aggregate price level?**
 - ❑ **Individual goods/sectors? Thus require a model of heterogeneity in pricing outcomes?**

- ❑ **How to introduce nominal rigidities in basic DSGE model?**
 - ❑ **Time-dependent:** firms (re-)set prices according to some exogenous time interval
 - ❑ **State-dependent:** firms (re-)set prices according to endogenous (potentially firm-specific) state

More tractable to model



CANONICAL DSGE STICKY-PRICE MODEL

- ❑ **Cashless environment**
 - ❑ Woodford (2000): “...effectiveness of monetary policy does not depend on the ability of the central bank to manipulate significant market distortions...”

- ❑ **Nominal prices (hence nominal price level) move sluggishly**
 - ❑ Due to some “costs” or other “timing impediments”

- ❑ **Built on monopolistic foundation**
 - ❑ Infrequent (re-)setting of price requires imperfect competition
 - ❑ Dixit-Stiglitz aggregator readily tractable in DSGE environment
 - ❑ **(Other tractable aggregators soon...)**

CANONICAL DSGE STICKY-PRICE MODEL

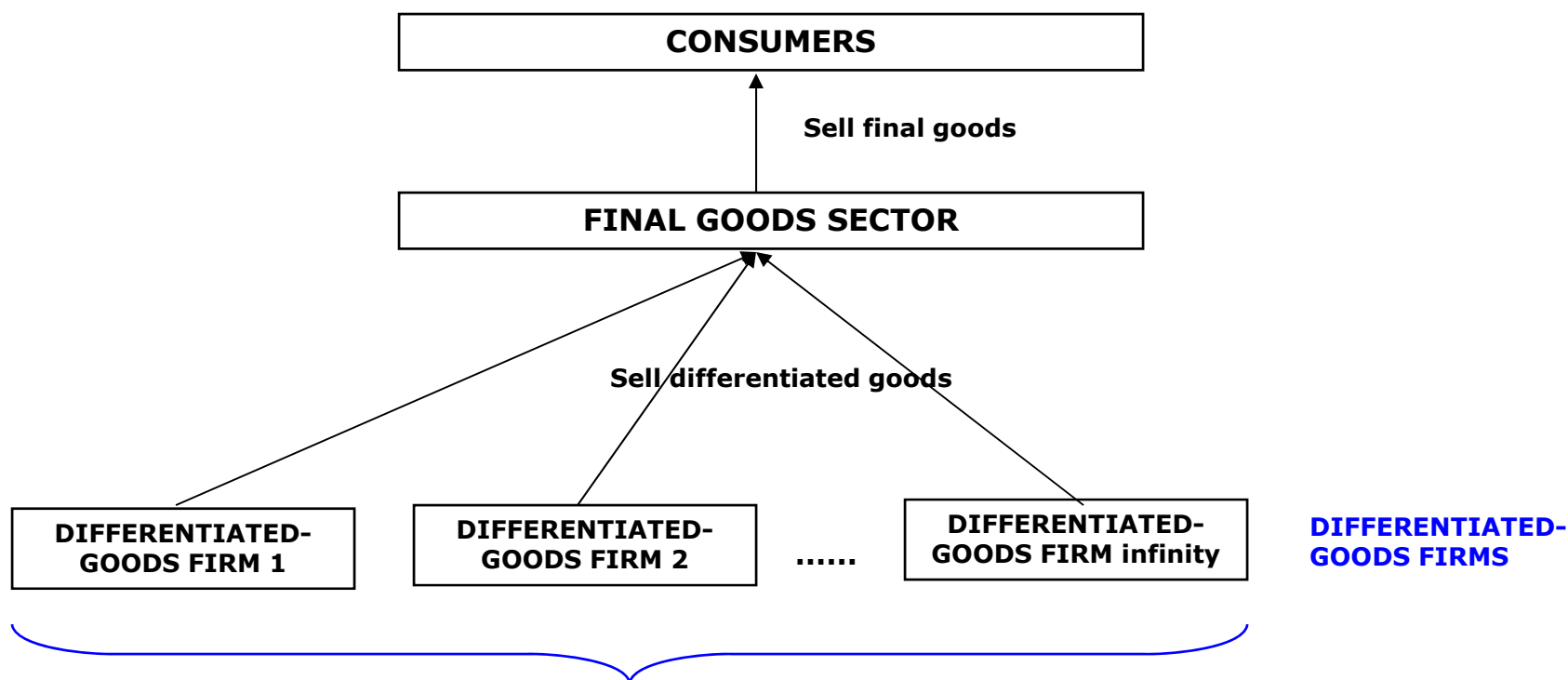
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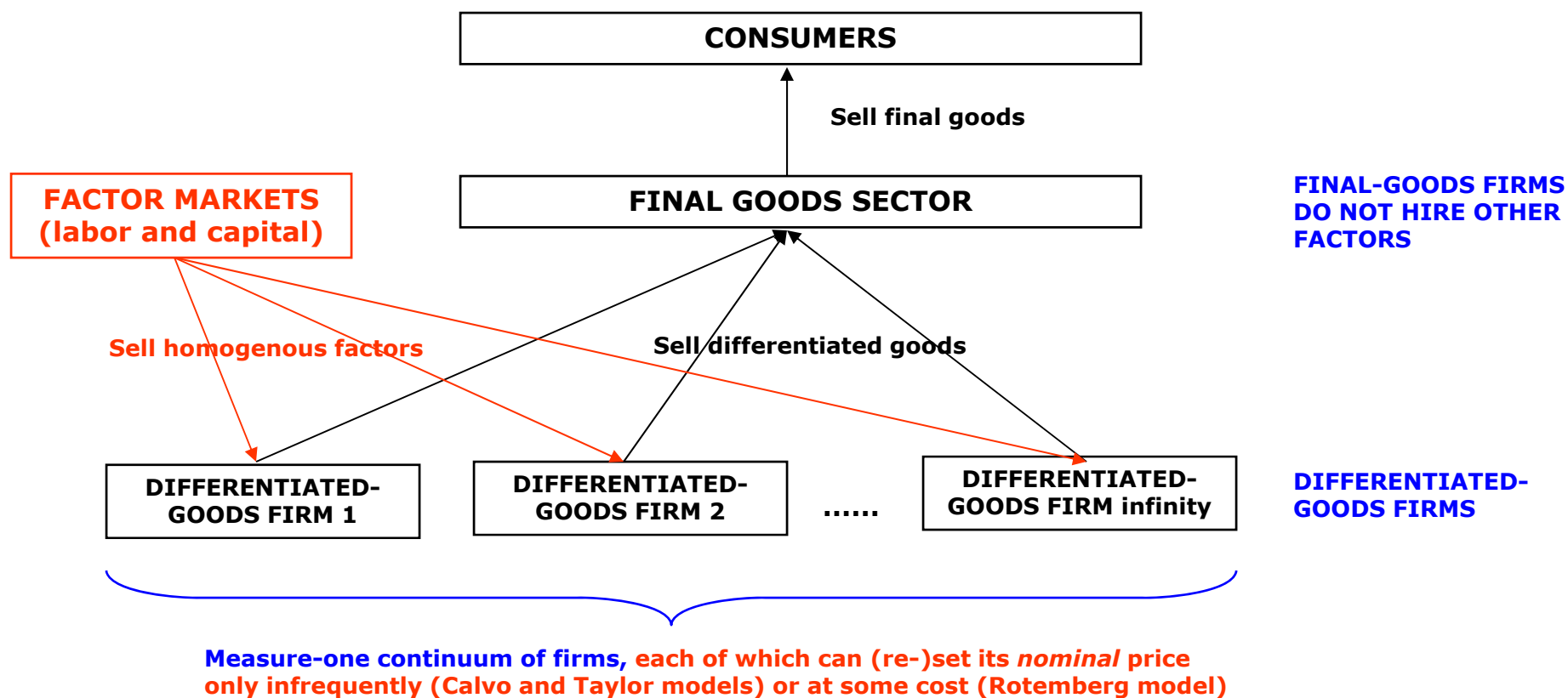
- ❑ **Common DSGE sticky-price mechanisms**
 - ❑ **Rotemberg:** firm can re-optimize price every period, but subject to a quadratic “menu cost”
 - ❑ **Calvo-Yun:** firm receives exogenous “signal” to re-optimize price
 - ❑ **Taylor:** firm can re-optimize price every T periods

CANONICAL DSGE STICKY-PRICE MODEL



Continuum of firms, each of which can (re-)set its *nominal* price only infrequently (Calvo and Taylor models) or at some cost (Rotemberg model)

CANONICAL DSGE STICKY-PRICE MODEL



FINAL-GOODS FIRMS

□ **Aggregator** $y_t = \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$

□ **Nominal profit-maximization problem**

P_t the nominal price of final good; in equilibrium, P_t is nominal price level

$$\max_{y_{it} \geq 0} P_t y_t - \int_0^1 P_{it} y_{it} di \quad P_{it} \text{ the nominal price of differentiated good } i$$

Substitute in CES final-goods aggregator

$$\max_{y_{it} \geq 0} P_t \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{it} y_{it} di$$

FINAL-GOODS FIRMS

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$\max_{y_{it} \geq 0} P_t \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{it} y_{it} di$

Profit-maximization

$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \cdot y_t$

TAKEN AS GIVEN BY DIFFERENTIATED FIRM i

Each differentiated firm i chooses its nominal P_i to maximize profit

Relative price of firm i 's output

Aggregate output a *shifter* of firm i 's demand function

DIFFERENTIATED-GOODS FIRMS

□ Two-stage optimization problem

production y_i pinned down from demand curve →

- Stage 1: Choose optimal nominal P_{it} **subject to pricing impediment**
- (Intermediate “stage”): “choose” to produce the y_{it} corresponding to the implied value of P_{it}/P_t
- Stage 2: Choose factor inputs to produce y_{it} at minimum cost

□ Differentiated producer i production technology

$$y_{it} = \underbrace{z_t f(k_{it}, n_{it})}_{\text{Usual CRS}} - \Phi \quad \text{Assume } = 0 \text{ as before } \rightarrow \text{mc} = \text{ac} \text{ CONSTANT (with respect to quantity)}$$

□ Implementation of two nominal sticky-price models

- **Rotemberg:** firm can re-optimize price every period, but subject to a quadratic “menu cost”
- **Calvo-Yun:** firm receives exogenous “signal” to re-optimize price

DIFFERENTIATED-GOODS FIRMS

□ **Rotemberg Model: Dynamic profit maximization problem**

□ Calvo-Yun Model: Dynamic profit-maximization problem

DIFFERENTIATED-GOODS FIRMS

□ Rotemberg Model: Dynamic profit maximization problem

Note distinction between
 t and s subscripts

$$\max_{P_{is}} E_t \left\{ \sum_{s=t}^{\infty} \beta_{s/t} \left\{ (P_{is} - P_s mc_s) y_{is} - \frac{\psi}{2} \left(\frac{P_{is}}{P_{is-1}} - 1 \right)^2 P_s \right\} \right\}$$

$\beta_{s/t}$ Discount factor between t and s because *dynamic* firm problem;
in equilibrium, = household stochastic discount factor

DIFFERENTIATED-GOODS FIRMS

□ **Rotemberg Model: Dynamic profit maximization problem**

Note distinction between t and s subscripts

$$\max_{P_{is}} E_t \left\{ \sum_{s=t}^{\infty} \Xi_{s|t} \left\{ (P_{is} - P_s mc_s) y_{is} - \frac{\psi}{2} \left(\frac{P_{is}}{P_{is-1}} - 1 \right)^2 P_s \right\} \right\}$$

Discount factor between t and s because *dynamic firm problem*; in equilibrium, = household stochastic discount factor

↓ Rewrite

$$\begin{aligned} \max_{P_{it}} \left\{ (P_{it} - P_t mc_t) y_{it} - \frac{\psi}{2} \left(\frac{P_{it}}{P_{it-1}} - 1 \right)^2 P_t + E_t \Xi_{t+1|t} \left\{ (P_{it+1} - P_{t+1} mc_{t+1}) y_{it+1} - \frac{\psi}{2} \left(\frac{P_{it+1}}{P_{it}} - 1 \right)^2 P_{t+1} \right\} \right\} \\ + \max_{P_{it}} E_t \sum_{s=t+2}^{\infty} \Xi_{s|t+2} \left\{ (P_{is} - P_s mc_s) y_{is} - \frac{\psi}{2} \left(\frac{P_{is}}{P_{is-1}} - 1 \right)^2 P_s \right\} \end{aligned}$$


↓ Substitute demand functions

DIFFERENTIATED-GOODS FIRMS

□ Rotemberg Model: Dynamic profit maximization problem

$$\max_{P_{it}} \left\{ (P_{it} - P_t mc_t) \cdot \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \cdot y_t - \frac{\psi}{2} \left(\frac{P_{it}}{P_{it-1}} - 1 \right)^2 P_t \right\}$$

Periods after t+1 not needed for period-t optimization

$$+ \max_{P_{it}} E_t \mathbb{E}_{t+1|t} \left((P_{it+1} - P_{t+1} mc_{t+1}) \cdot \left(\frac{P_{it+1}}{P_{t+1}} \right)^{-\varepsilon} \cdot y_{t+1} - \frac{\psi}{2} \left(\frac{P_{it+1}}{P_{it}} - 1 \right)^2 P_{t+1} \right) + \dots$$


DIFFERENTIATED-GOODS FIRMS

□ Rotemberg Model: Dynamic profit maximization problem

$$\max_{P_{it}} \left\{ (P_{it} - P_t mc_t) \cdot \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \cdot y_t - \frac{\psi}{2} \left(\frac{P_{it}}{P_{it-1}} - 1 \right)^2 P_t \right\}$$

Periods after t+1 not needed for period-t optimization

$$+ \max_{P_{it}} E_t \Xi_{t+1|t} \left((P_{it+1} - P_{t+1} mc_{t+1}) \cdot \left(\frac{P_{it+1}}{P_{t+1}} \right)^{-\varepsilon} \cdot y_{t+1} - \frac{\psi}{2} \left(\frac{P_{it+1}}{P_{it}} - 1 \right)^2 P_{t+1} \right) + \dots$$

↓

□ FOC wrt P_{it}

$$\left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} y_t - \left(\frac{P_{it} - P_t mc_t}{P_{it}} \right) \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \varepsilon y_t - \psi \left(\frac{P_{it}}{P_{it-1}} - 1 \right) \frac{P_t}{P_{it-1}} + \psi E_t \left\{ \Xi_{t+1} \left(\frac{P_{it+1}}{P_{it}} - 1 \right) \frac{P_{t+1} P_{it+1}}{P_{it} \cdot P_{it}} \right\} = 0$$

□ If $\psi = 0$

$$\frac{P_{it}}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) mc_t$$

Flexible-price Dixit-Stiglitz pricing rule

DIFFERENTIATED-GOODS FIRMS

□ Rotemberg Model: Optimal-pricing condition

$$\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t - \left(\frac{P_{it} - P_t mc_t}{P_{it}}\right) \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \varepsilon y_t - \psi \left(\frac{P_{it}}{P_{it-1}} - 1\right) \frac{P_t}{P_{it-1}} + \psi E_t \left\{ \Xi_{t+1} \left(\frac{P_{it+1}}{P_{it}} - 1\right) \frac{P_{t+1} P_{it+1}}{P_{it} \cdot P_{it}} \right\} = 0$$



In symmetric equilibrium

$$\left(\frac{P_t}{P_t}\right)^{-\varepsilon} y_t - \left(\frac{P_t - P_t mc_t}{P_t}\right) \left(\frac{P_t}{P_t}\right)^{-\varepsilon} \varepsilon y_t - \psi \left(\frac{P_t}{P_{t-1}} - 1\right) \frac{P_t}{P_{t-1}} + \psi E_t \left\{ \Xi_{t+1|t} \left(\frac{P_{t+1}}{P_t} - 1\right) \cdot \frac{P_{t+1} \cdot P_{t+1}}{P_t \cdot P_t} \right\} = 0$$

DIFFERENTIATED-GOODS FIRMS

□ **Rotemberg Model: Optimal-pricing condition**

$$\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t - \left(\frac{P_{it} - P_t mc_t}{P_{it}}\right) \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \varepsilon y_t - \psi \left(\frac{P_{it}}{P_{it-1}} - 1\right) \frac{P_t}{P_{it-1}} + \psi E_t \left\{ \Xi_{t+1} \left(\frac{P_{it+1}}{P_{it}} - 1\right) \frac{P_{t+1} P_{it+1}}{P_{it} \cdot P_{it}} \right\} = 0$$

↓ **In symmetric equilibrium**

$$\left(\frac{P_t}{P_t}\right)^{-\varepsilon} y_t - \left(\frac{P_t - P_t mc_t}{P_t}\right) \left(\frac{P_t}{P_t}\right)^{-\varepsilon} \varepsilon y_t - \psi \left(\frac{P_t}{P_{t-1}} - 1\right) \frac{P_t}{P_{t-1}} + \psi E_t \left\{ \Xi_{t+1|t} \left(\frac{P_{t+1}}{P_t} - 1\right) \cdot \frac{P_{t+1} \cdot P_{t+1}}{P_t \cdot P_t} \right\} = 0$$

↓ **Cancel terms**
 ↓ **Definition of inflation: $1 + \pi_{t+1} = P_{t+1}/P_t$**
 ↓ **Combine terms**
 ↓ **Algebra**

$$[1 - \varepsilon + \varepsilon mc_t] y_t - \psi \pi_t (1 + \pi_t) + \psi E_t \left\{ \Xi_{t+1} \cdot \pi_{t+1} \cdot \pi_{t+1} \cdot (1 + \pi_{t+1}) \right\} = 0$$

**New Keynesian
Phillips Curve**

DIFFERENTIATED-GOODS FIRMS

□ Rotemberg Model: Dynamic profit maximization problem

□ **Calvo-Yun Model: Dynamic profit-maximization problem**

Note distinction between t and s subscripts

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} \left\{ P_{it} y_{is} - P_s mc_s y_{is} \right\} \right\} \xrightarrow{\text{Substitute in demand function}}$$

Exogenous probability of not being able to (re-)set price – the “Calvo fairy”

Discount factor between t and s because dynamic firm problem; in equilibrium, = household stochastic discount factor

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Dynamic profit maximization problem

Price change opportunities arrive according to Poisson process

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} \left\{ P_{it} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s - P_s mc_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \right\} \right\}$$

Rewrite

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} \left\{ P_{it}^{1-\varepsilon} P_s^\varepsilon - P_{it}^{-\varepsilon} P_s^{1+\varepsilon} mc_s \right\} y_s \right\}$$

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Dynamic profit maximization problem

Price change opportunities arrive according to Poisson process

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} \left\{ P_{it} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s - P_s mc_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \right\} \right\}$$

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□ FOC wrt P_{it}

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} \left[(1-\varepsilon) \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s + \varepsilon \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0$$

Continue manipulating

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Optimal-pricing condition

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[(1-\varepsilon) + \varepsilon \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0$$

↓ Rewrite: multiply by $-1/\varepsilon$

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\frac{\varepsilon-1}{\varepsilon} - \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0$$

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Optimal-pricing condition

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[(1-\varepsilon) + \varepsilon \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0$$

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Rewrite: multiply each term by P_s/P_s and multiply entire expression by P_{it}

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\frac{\varepsilon-1}{\varepsilon} \frac{P_{it}}{P_s} - mc_s \right] \right\} = 0$$

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Optimal-pricing condition

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[(1-\varepsilon) + \varepsilon \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0$$

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Flexible-price Dixit-Stiglitz pricing rule

If prices are completely flexible (i.e., if $\alpha = 0$)

$$\frac{P_{it}}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1} \right) mc_t$$

DIFFERENTIATED-GOODS FIRMS

- Calvo-Yun Model: Optimal-pricing condition

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\underbrace{\frac{\varepsilon-1}{\varepsilon} \frac{P_{it}}{P_s}}_{\text{Real marginal revenue}} - mc_s \right] \right\} = 0$$

- With price impediment, optimal P_i balances current and future marginal revenues against current and future marginal costs until the next (expected) price re-optimization

DIFFERENTIATED-GOODS FIRMS

- Calvo-Yun Model: Optimal-pricing condition

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \mathbb{E}_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\underbrace{\frac{\varepsilon-1}{\varepsilon} \frac{P_{it}}{P_s}}_{\text{Real marginal revenue}} - mc_s \right] \right\} = 0$$

- With price impediment, optimal P_i balances current and future marginal revenues against current and future marginal costs until the next (expected) price re-optimization
- Differentiated firm i 's (and hence the aggregate) markup will be time-varying
 - As "initial marginal revenues" > "initial marginal costs" to balance against "later marginal revenues" < "later marginal costs"
 - Inflation erodes *relative* price of firm i
- Conduct full non-linear analysis (around distorted steady state)
 - "Textbook" New Keynesian analysis is around efficient steady state

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Optimal-pricing condition

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\frac{\varepsilon-1}{\varepsilon} \frac{P_{it}}{P_s} - mc_s \right] \right\} = 0$$

↓
 Divide entire expression by P_t
 PDV of marginal revenues on LHS, PDV of marginal costs on RHS

$$\begin{aligned} & \frac{P_t}{P_t} \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} y_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} y_s \right\} \\ &= \frac{P_t}{P_t} \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} y_t mc_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s mc_s \right\} \end{aligned}$$

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Optimal-pricing condition

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\frac{\varepsilon-1}{\varepsilon} \frac{P_{it}}{P_s} - mc_s \right] \right\} = 0$$

↓ Divide entire expression by P_t

↓ PDV of marginal revenues on LHS, PDV of marginal costs on RHS

$$\begin{aligned} & \frac{P_t}{P_t} \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} y_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} y_s \right\} \\ &= \frac{P_t}{P_t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_t mc_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s mc_s \right\} \end{aligned}$$

↓ Rewrite

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Optimal-pricing condition

$$\begin{aligned}
 \longrightarrow \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} y_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_t} \frac{P_t}{P_s} \right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} y_s \right\} \\
 = \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_t mc_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_t} \frac{P_t}{P_s} \right)^{-\varepsilon} y_s mc_s \right\}
 \end{aligned}$$

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Optimal-pricing condition

$$\begin{aligned}
 \longrightarrow \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} y_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_t} \frac{P_t}{P_s} \right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} y_s \right\} \\
 = \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_t mc_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_t} \frac{P_t}{P_s} \right)^{-\varepsilon} y_s mc_s \right\} \\
 \downarrow \text{Rewrite} \\
 \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} y_t + \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \left(\frac{P_s}{P_t} \right)^{\varepsilon} y_s \right\} \\
 = \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_t mc_t + \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \left(\frac{P_s}{P_t} \right)^{1+\varepsilon} y_s mc_s \right\} \\
 \downarrow \text{Rewrite again}
 \end{aligned}$$

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Optimal-pricing condition

$$\begin{aligned} \left(\frac{P_{it}}{P_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} y_t + \left(\frac{P_{it}}{P_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \left(\frac{P_s}{P_t}\right)^{\varepsilon} y_s \right\} \\ = y_t mc_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \left(\frac{P_s}{P_t}\right)^{1+\varepsilon} y_s mc_s \right\} \end{aligned}$$



Rewrite again

$$\frac{P_{it}}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \left(\frac{y_t mc_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \left(\frac{P_s}{P_t}\right)^{1+\varepsilon} y_s mc_s \right\}}{y_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \left(\frac{P_s}{P_t}\right)^{\varepsilon} y_s \right\}} \right)$$

**Calvo Model:
optimal nominal
price = PDV mc
divided by PDV mr**

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Optimal-pricing condition

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\frac{\varepsilon-1}{\varepsilon} \frac{P_{it}}{P_s} - mc_s \right] \right\} = 0$$

DIFFERENTIATED-GOODS FIRMS

□ Calvo-Yun Model: Optimal-pricing condition

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\frac{\varepsilon-1}{\varepsilon} \frac{P_{it}}{P_s} - mc_s \right] \right\} = 0$$

□ Define

$$P_t x_t^1 = E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \frac{P_{it}}{P_s} \right\} \quad \text{PDV of nominal marginal revenues until next price change}$$

$$P_t x_t^2 = E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s mc_s \right\} \quad \text{PDV of nominal marginal costs until next price change}$$

□ Optimal-pricing condition: $x_t^1 = x_t^2$

- Emphasizes that optimal P_i balances current and future mr against current and future mc

□ Write x_t^1, x_t^2 recursively

OPTIMAL-PRICING CONDITION

□ Some notation and definitions

$$P_{it}$$

Nominal price of good i in period t

$$p_{it} \equiv \frac{P_{it}}{P_t}$$

Relative price of good i in period t

$$P_{it+1} = P_{it}$$

Evolution of nominal price if no price change



$$p_{it+1} = \frac{P_{it+1}}{P_{t+1}} = \frac{P_{it}}{P_{t+1}}$$

As long as no nominal price change, a firm's relative price erodes at the rate of inflation

$$= \frac{P_{it}}{P_t} \frac{P_t}{P_{t+1}}$$

($\pi_{t+1} = P_{t+1}/P_t$)

$$= \frac{p_{it}}{\pi_{t+1}}$$

OPTIMAL-PRICING CONDITION

□ Write x_t^1 recursively

$$P_t x_t^1 = E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \frac{P_{it}}{P_s} \right\}$$

↓ Divide by P_t ; write out first two terms

$$x_t^1 = \frac{P_t}{P_t} \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \frac{P_{t+1}}{P_t} \left(\frac{P_{it}}{P_{t+1}} \right)^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

OPTIMAL-PRICING CONDITION

□ Write x_t^1 recursively

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↓ $P_{it+1} = P_{it}$ while no price change opportunity

$$x_t^1 = \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \frac{P_{t+1}}{P_t} \left(\frac{P_{it+1}}{P_{t+1}} \right)^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

↓ Use definitions

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1} p_{it+1}^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

↓ Use $p_{it+1} = p_{it} / \pi_{t+1}$

OPTIMAL-PRICING CONDITION

□ Write x_t^1 recursively

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1} \left(\frac{p_{it}}{\pi_{t+1}} \right)^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

↓ Rearrange

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} p_{it}^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

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↓ Multiply and divide by $p_{it+1}^{1-\varepsilon}$

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Have generated a recursive term

OPTIMAL-PRICING CONDITION

□ Write x_t^1 recursively

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1} \left(\frac{p_{it}}{\pi_{t+1}} \right)^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

↓ Rearrange

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Have generated a recursive term

↓ Express recursively

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^\varepsilon \left(\frac{p_{it}}{p_{it+1}} \right)^{1-\varepsilon} x_{t+1}^1 \right\} \quad \mathbf{x^1 \text{ expressed recursively}}$$

OPTIMAL-PRICING CONDITION

- Both x_t^1, x_t^2 recursively

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon - 1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^\varepsilon \left(\frac{p_{it}}{p_{it+1}} \right)^{1-\varepsilon} x_{t+1}^1 \right\} \quad \mathbf{x^1 \text{ expressed recursively}}$$

$$x_t^2 = p_{it}^{-\varepsilon} y_t mc_t + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{1+\varepsilon} \left(\frac{p_{it}}{p_{it+1}} \right)^{-\varepsilon} x_{t+1}^2 \right\} \quad \mathbf{x^2 \text{ expressed recursively}}$$

- Optimal-pricing condition expressed compactly

$$x_t^1 = x_t^2$$

- Aggregate price index follows from Dixit-Stiglitz aggregation

AGGREGATE PRICE LEVEL

- Aggregate price index follows from Dixit-Stiglitz aggregation

$$P_t^{1-\varepsilon} = \int_0^1 P_{it}^{1-\varepsilon} di = \int_0^{\alpha} P_{it-1}^{1-\varepsilon} di + \int_{\alpha}^1 P_{it}^{*1-\varepsilon} di$$

non-adjusters
adjusters

↓
↓

Emphasize *optimal new price*
 Obtained by substituting demand functions into D-S aggregator

AGGREGATE PRICE LEVEL

- Aggregate price index follows from Dixit-Stiglitz aggregation

TRACTABILITY DUE TO THE CALVO RANDOM-ADJUSTMENT ASSUMPTION

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non-adjusters
adjusters

Emphasize *optimal* new price

Obtained by substituting demand functions into D-S aggregator

$$= \alpha P_{t-1}^{1-\varepsilon} + (1-\alpha) P_t^{*1-\varepsilon}$$

↑
↑

KEY: Because adjusters were *randomly* selected, average (aggregate) price of non-adjusters is identical to previous period's average (aggregate) price

Fraction $1 - \alpha$ re-set price optimally (and symmetrically)

AGGREGATE PRICE LEVEL

□ Aggregate price index follows from Dixit-Stiglitz aggregation

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non-adjusters
adjusters

Emphasize *optimal* new price

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↑
↑

KEY: Because adjusters were *randomly* selected, average (aggregate) price of non-adjusters is identical to previous period's average (aggregate) price
Fraction $1 - \alpha$ re-set price optimally (and symmetrically)

Together form the "aggregate supply" block of New Keynesian sticky-price model

$$1 = \alpha \pi_t^{\varepsilon-1} + (1-\alpha) p_t^{*1-\varepsilon}$$

$x_t^1 = x_t^2$

EQUILIBRIUM EVOLUTION OF AGGREGATE INFLATION – depends on relative price set by firms currently adjusting nominal price

Optimal pricing condition

PRICE DISPERSION

- ❑ **Calvo model implies dispersion of relative prices**
 - ❑ As does Taylor model ...
 - ❑ ...but not Rotemberg model (quadratic cost of nominal price adjustment)

- ❑ **Dispersion often ignored until recently...**
 - ❑ ...due to linearization around a zero-inflation steady state (typical simple New Keynesian model soon...)
 - ❑ With better numerical tools, easier to take account of dispersion

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- ❑ **Price dispersion the basic source of welfare losses of non-zero inflation**
 - ❑ **Because it implies quantity dispersion across intermediate producers...**
 - ❑ **...which is inefficient because Dixit-Stiglitz aggregator is symmetric and concave in every good i**

The basic driving force of optimal policy in any NK model

PRICE DISPERSION

□ **For firm i ,** $y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t = z_t f(k_{it}, n_{it})$

□ **Integrating over i**

$$y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = z_t \int_0^1 f(k_{it}, n_{it}) di$$

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Symmetric choices of k/n ratio across all firms i ...

$$y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = z_t f\left(\frac{k_t}{n_t}, 1\right) \int_0^1 n_{it} di$$

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$$y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = z_t f\left(\frac{k_t}{n_t}, 1\right) \int_0^1 n_{it} di$$


$\underbrace{\hspace{10em}}_{\equiv S_t}$

A measure of dispersion: relative price dispersion leads to dispersion of factor usage across differentiated firms, hence dispersion of quantity across differentiated firms

↓ Express s_t recursively

PRICE DISPERSION

$$s_t = \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} di = \int_{\alpha}^1 \left(\frac{P_{it}^*}{P_t} \right)^{-\varepsilon} di + \int_0^{\alpha} \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} di$$


Re-setters all choose same price

$$= (1 - \alpha) \left(\frac{P_t^*}{P_t} \right)^{-\varepsilon} + \int_0^{\alpha} \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} di$$

PRICE DISPERSION

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↓ **$P_{it} = P_{it-1}$ for firms that cannot re-set price**

$$= (1-\alpha) \left(\frac{P_t^*}{P_t} \right)^{-\varepsilon} + \int_0^{\alpha} \left(\frac{P_{it-1}}{P_t} \right)^{-\varepsilon} di$$

↓ **Multiply by $(P_{t-1}/P_t)^{-\varepsilon}$**

$$= (1-\alpha) \left(\frac{P_t^*}{P_t} \right)^{-\varepsilon} + \left(\frac{P_{t-1}}{P_t} \right)^{-\varepsilon} \int_0^{\alpha} \left(\frac{P_{it-1}}{P_{t-1}} \right)^{-\varepsilon} di$$

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$$s_t = \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} di = \frac{1}{\alpha} \int_0^{\alpha} \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} di$$

Multiply by $(P_{t-1}/P_t)^{-\varepsilon}$

$$= (1 - \alpha) \left(\frac{P_t^*}{P_t} \right)^{-\varepsilon} + \underbrace{\left(\frac{P_{t-1}}{P_t} \right)^{-\varepsilon}}_{\equiv \pi_t^\varepsilon} \int_0^{\alpha} \underbrace{\left(\frac{P_{it-1}}{P_{t-1}} \right)^{-\varepsilon}}_{\equiv \alpha s_{t-1}} di$$

Because of Calvo random adjustment and all adjusters choose same price

NOTE:

$\alpha = 0$: $s_t = 1$ (no dispersion)

$\alpha > 0$: $s_t > 1$ (dispersion)

RESOURCE CONSTRAINT

□ Summarized by *three* conditions

And using factor market clearing conditions here

$$k_t = \int_0^1 k_{it} di, \quad n_t = \int_0^1 n_{it} di$$

$$y_t = c_t + k_{t+1} - (1 - \delta)k_t + g_t$$

“Usual” resource constraint

$$y_t = \frac{z_t f(k_t, n_t)}{s_t}$$

Some output is a pure deadweight loss
(note $s_t < 1$ cannot occur)

$$s_t = (1 - \alpha)p_t^{*-\varepsilon} + \alpha\pi_t^\varepsilon s_{t-1}$$

Law of motion for deadweight loss

RESOURCE CONSTRAINT

□ Summarized by *three conditions*

$$y_t = c_t + k_{t+1} - (1 - \delta)k_t + g_t$$

= 0 in Yun model

"Usual" resource constraint

$$y_t = \frac{z_t f(k_t, n_t)}{s_t}$$

Some output is a pure deadweight loss
(note $s_t < 1$ cannot occur)

$$s_t = (1 - \alpha)p_t^{*-\varepsilon} + \alpha\pi_t^\varepsilon s_{t-1}$$

Law of motion for deadweight loss

□ Law of motion for s_t represented using laws of motion for both P_{t-1} and P_{t-1}^*

□ See equations (25) and (26)

OTHER MODEL DETAILS

- ❑ **Cash/credit to motivate money demand**
 - ❑ i.e., Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1991)
- ❑ **(Habit persistence (i.e., time-non-separability) in leisure)**
 - ❑ As in Kydland and Prescott (1982); unimportant for main results...
- ❑ **Exogenous AR(1) money growth process**
 - ❑ Also “endogenous” money supply process, but not as interesting
- ❑ **Exogenous AR(1) TFP process**

Could have been
cashless...

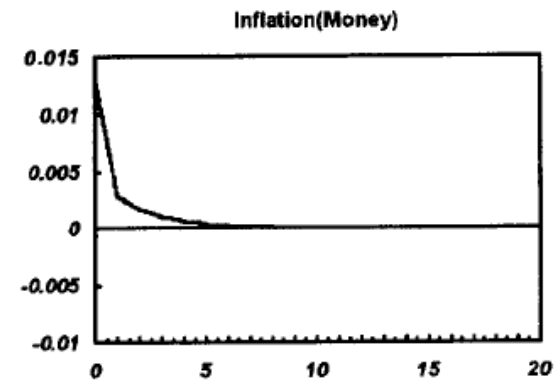
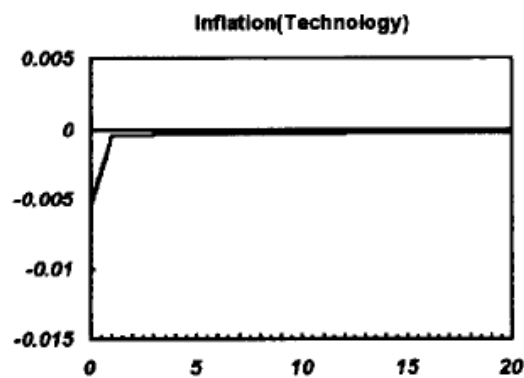
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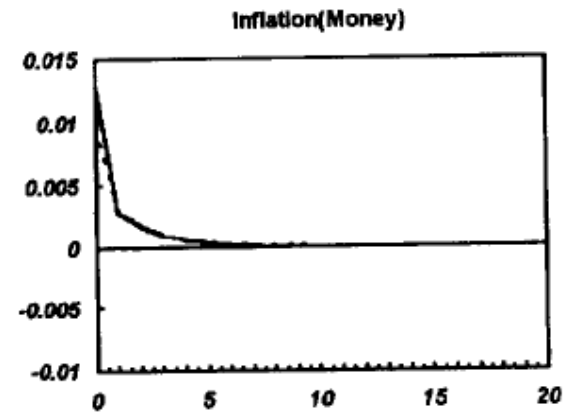
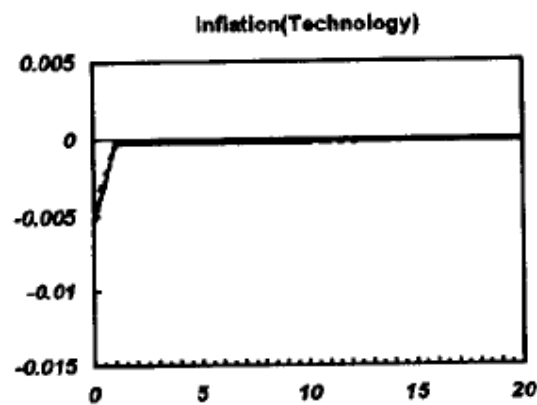
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- Exogenous AR(1) money growth process**
 - Also “endogenous” money supply process, but not as interesting
- Exogenous AR(1) TFP process**
- Indexation of prices to average (i.e., steady-state) inflation**
 - For firms not re-setting price, $P_{it} = \pi P_{it-1}$
- Approximated and simulated using linear methods**
 - Using King, Plosser, Rebelo (1988) linear approximation method

NOMINAL EFFECTS OF STICKY PRICES

- Effects on inflation not very different compared to flex-price case



Flexible Prices

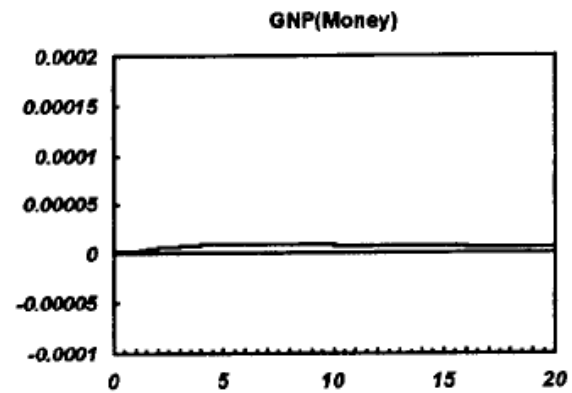
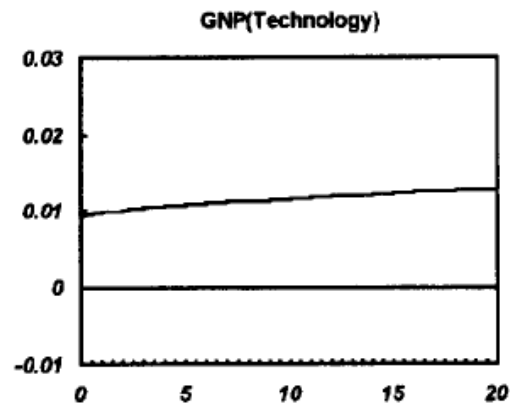


Sticky Prices

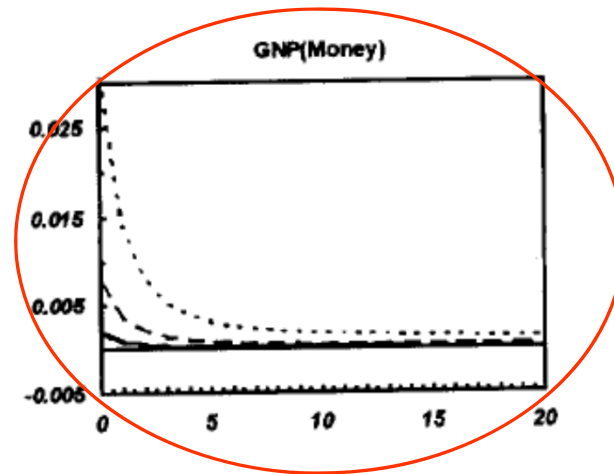
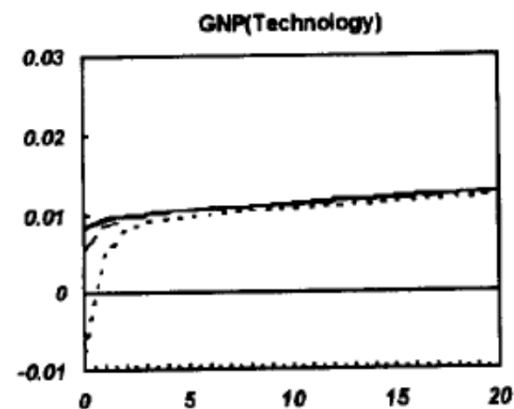
Lack of inflation persistence in basic Calvo-Yun model (now) well-known –

REAL EFFECTS OF STICKY PRICES

- Effects on GDP much bigger the stickier are prices



Flexible Prices



Sticky Prices

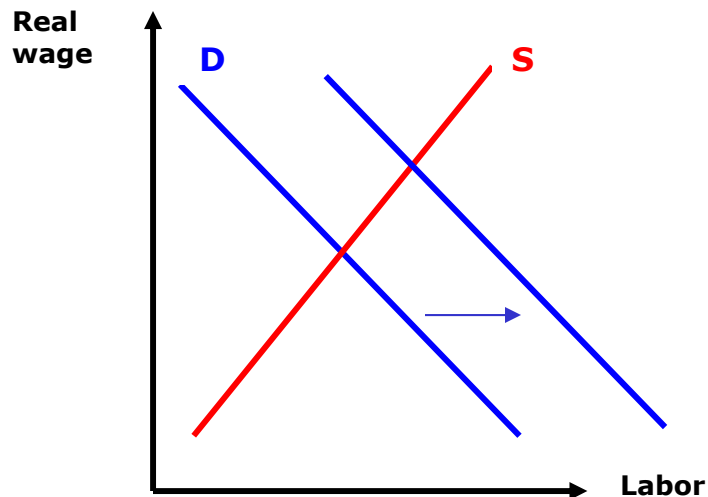
MARKUP DYNAMICS

- Money (i.e., **nontechnology**) shock → output expands

- With z_t, k_t fixed, output expansion due to increased (equilibrium) employment
 - **Downward-sloping product demand curves → individual (differentiated) firms must expand their output (partial equilibrium)**
 - **Recall aggregate output a shifter of firm i demand function**
 - **Can only be achieved in short-run if a given firm i hires more labor at any real wage**

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$$\frac{w_t}{z_t f_n(k_t, n_t)} = \frac{1}{\mu_t} = mc_t$$

Sticky-price model delivers
endogenously-countercyclical
 markup...
 ...and **endogenous variations** in
 "labor wedge"