Monopolistic Competition in a DSGE Model: Part II

February 8, 2017

PRODUCT VARIETIES

- **Final Goods Production Function (Dixit-Stiglitz aggregator)**

$$y_t = \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

ε measures elasticity of substitution across any two differentiated varieties

- **Final Goods Production Function ("Benassy" aggregator)**

$$y_t = N_t^{\kappa+1-\frac{\varepsilon}{\varepsilon-1}} \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$\kappa = \varepsilon /(\varepsilon -1) - 1$ recovers Dixit-Stiglitz

κ measures increasing returns to specialization (aka, "variety effect")

INCREASING RETURNS TO SPECIALIZATION

- **Final Goods Production Function**

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- **Returns to Specialization (aka, "variety effect")**

$$v(N_t) \equiv \frac{\left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}}{N_t y_t} \xrightarrow[\text{\& CRTS production fct.}]{\text{symmetric equilibrium } y_{it} = y_t \text{ for all } i} v(N_t) = \frac{\left[\int_0^{N_t} 1^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}}{N_t}$$

Increasing Returns to Specialization

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- **Express returns to specialization in elasticity form**
  - **Analytically convenient**

$$\epsilon(N_t) = \frac{N_t v'(N_t)}{v(N_t)}$$

INCREASING RETURNS TO SPECIALIZATION

- **Returns to Specialization**

$$v(N_t) = \frac{\left[\int_0^{N_t} 1^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}}{N_t} = N_t^{\frac{\varepsilon}{\varepsilon-1}-1}$$

Differentiate →

$$v'(N_t) = \left(\frac{\varepsilon}{\varepsilon-1} - 1\right) \cdot N_t^{\frac{\varepsilon}{\varepsilon-1}-2}$$

Express as elasticity →

$$\epsilon(N_t) = \frac{N_t v'(N_t)}{v(N_t)} = \left(\frac{\varepsilon}{\varepsilon-1} - 1\right) \cdot N_t^{\frac{\varepsilon}{\varepsilon-1}-2} \cdot N_t \cdot N_t^{-\frac{\varepsilon}{\varepsilon-1}+1}$$

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$$\boxed{\epsilon(N_t) = \frac{\varepsilon}{\varepsilon-1} - 1}$$

Increasing returns to specialization

Dixit-Stiglitz

DEPENDS **on monopolistic markup**

aka, "variety effect"

INCREASING RETURNS TO SPECIALIZATION

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Differentiate →

$$v'(N_t) = \kappa \cdot N_t^{\kappa-1}$$

Express as elasticity →

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Increasing returns to specialization

Benassy

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- **Returns to specialization (aka, "variety effect")**

$$\epsilon(N_t) = \frac{\varepsilon}{\varepsilon - 1} - 1$$

Dixit-Stiglitz

$$\epsilon(N_t) = \kappa$$

Benassy

INCREASING RETURNS TO SPECIALIZATION

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- **Returns to Specialization (aka, "variety effect")**
- **Express returns to specialization in elasticity form (Definition #2….)**

$$\epsilon(N_t) = \frac{N_t p'(N_t)}{p(N_t)}$$

instead of

$$\epsilon(N_t) = \frac{N_t v'(N_t)}{v(N_t)}$$

INCREASING RETURNS TO SPECIALIZATION

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$$p(N_t) = N_t^{\frac{1}{\varepsilon-1}}$$

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aka, "variety effect"

Increasing returns to specialization

Dixit-Stiglitz

DEPENDS **on monopolistic markup**

$$\epsilon(N_t) = \frac{1}{\varepsilon-1} \left(= \frac{\varepsilon}{\varepsilon-1} - 1 \right)$$

Exactly the same result

Rationale?….

HOMOTHETIC FUNCTION

- **Dixit-Stiglitz (1977 *AER*) model**
  - **Common specification of imperfect competition in macro models**
  - **Typical building block of sticky price monetary models**
  - **Basic idea: imperfectly-substitutable goods combined yield an aggregate good**

CES function: ε the constant elasticity of substitution between any pair of differentiated goods

$$y_t = \left[\sum_{i=1}^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Discrete number of differentiated goods

$$y_t = \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Continuum of differentiated goods

  - **Important properties of aggregator**
    - **Symmetric in all arguments** ← **Drives efficiency/optimal policy results**
    - **Strictly increasing in all arguments**
    - **Strictly concave in all arguments**
    - **Homogenous of degree one → Homothetic function (monotonic)**

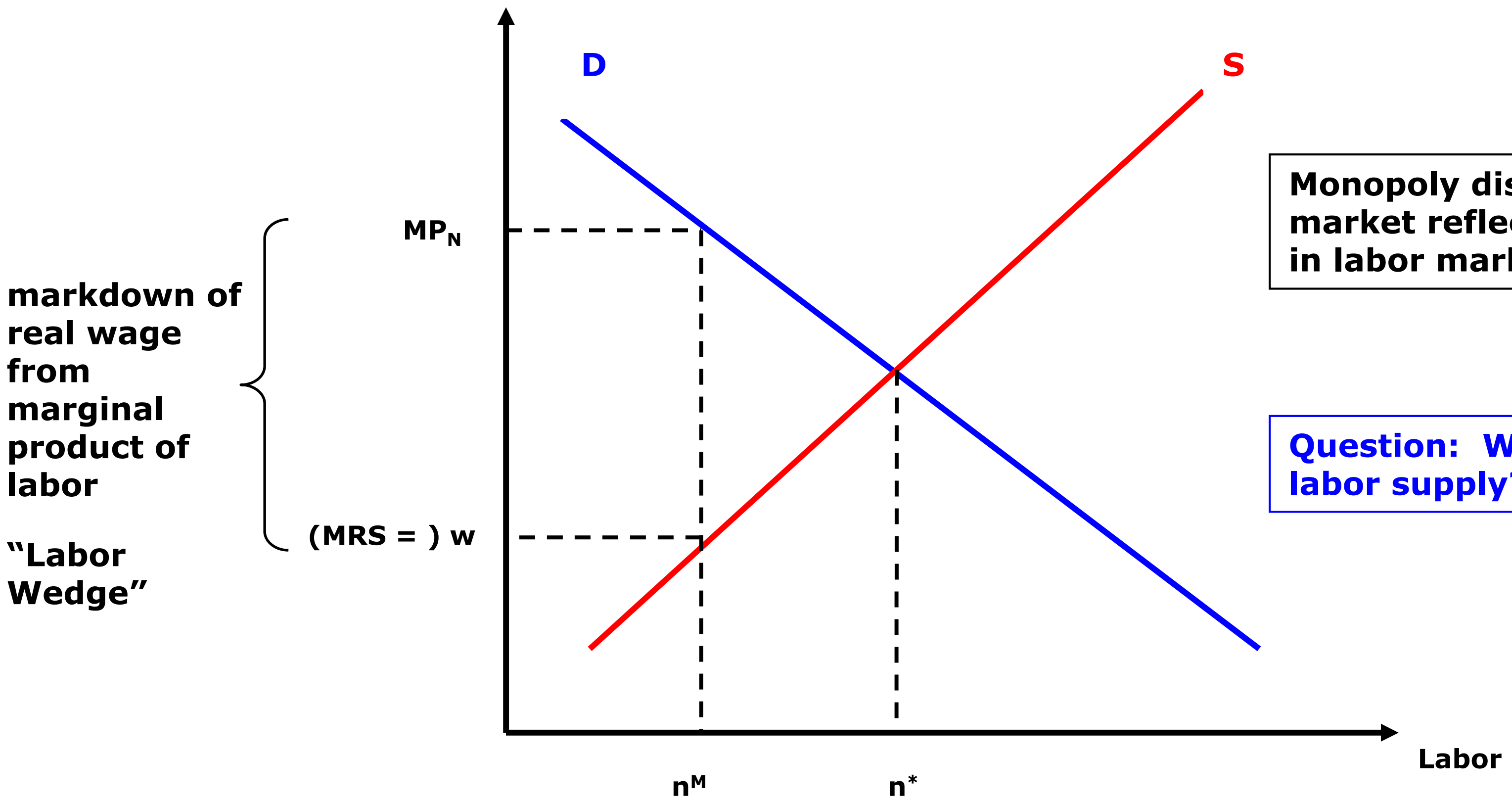
HOMOTHETIC FUNCTION

- **Monotone transformation of a homogenous function**
- **Income expansion paths are rays through origin**
- $\frac{u_i(t \cdot c)}{u_j(t \cdot c)} = \frac{u_i(c)}{u_j(c)}$ **for *t* > 0 (consumer interpretation)**
- **Homogeneity a cardinal property of a function**
- **Homotheticity an ordinal property of a function**

BUSINESS CYCLES

- **Monopoly power (ala D-S & Benassy) creates static distortion in labor market**
  - **Akin to a labor income tax**
  - **Introduces a wedge between u_n/u_c and marginal product of labor**

THE LABOR WEDGE



Monopoly distortion in goods market reflected in distortion in labor markets

Question: What if inelastic labor supply?

BUSINESS CYCLES

- **Monopoly power (ala D-S & Benassy) creates static distortion in labor market**
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- **But a constant wedge – and constant markup – over time**
- **Time-varying endogenous markup?**
- **NK sticky nominal prices…**

BUSINESS CYCLES

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Question: Why do monopolists earn positive economic profits?

PRODUCT VARIETIES

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- ***Endogenous* creation of new product varieties and firms**

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- **≈ 10% of U.S. GDP accounted for by new product creation**
- **≈ 9% of lost U.S. GDP accounted for by product destruction**
  - **Based on ≈ 50% and ≈ 45% for U.S. manufacturing sector respectively (Bernard, Redding, and Schott (2010 *AER*))**

- **≈ 10% of consumers' purchases in a year devoted to new goods not previously available**
  - **(Product destruction less cyclical than product creation)**
  - **Broda and Weinstein (2010 *AER*)**

PRODUCT VARIETIES

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Profit Destruction Effect vs. Welfare of Returns to Specialization

Profit Destruction Effect	Welfare of Returns to Specialization
Larger number of monopolistic competitors	**Larger number of monopolistic competitors**
↓	↓
Smaller profits for potential new entrants	**Positive (negative) spillovers in production**

PRODUCT VARIETIES

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Profit Destruction Effect vs. Welfare of Returns to Specialization

Dixit-Stiglitz Production Efficiently Balances Tradeoff

Benassy Production *Inefficiently* Balances Tradeoff

PRODUCT VARIETIES

- **Moreover ....**

$$y_t = \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad y_t = N_t^{\kappa+1-\frac{\varepsilon}{\varepsilon-1}} \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- **...without other mechanisms operative, endogenous entry does NOT generate markup fluctuations for these production functions**

PRODUCT VARIETIES

- **Moreover ….**

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- **…without other mechanisms operative, endogenous entry does NOT generate markup fluctuations for these production functions**

- **Comin and Gertler (2006 *AER*)**
- **Jaimovich and Floetotto (2008 *JME*)**
- **Head and Lampham (1996 *JEDC*)**

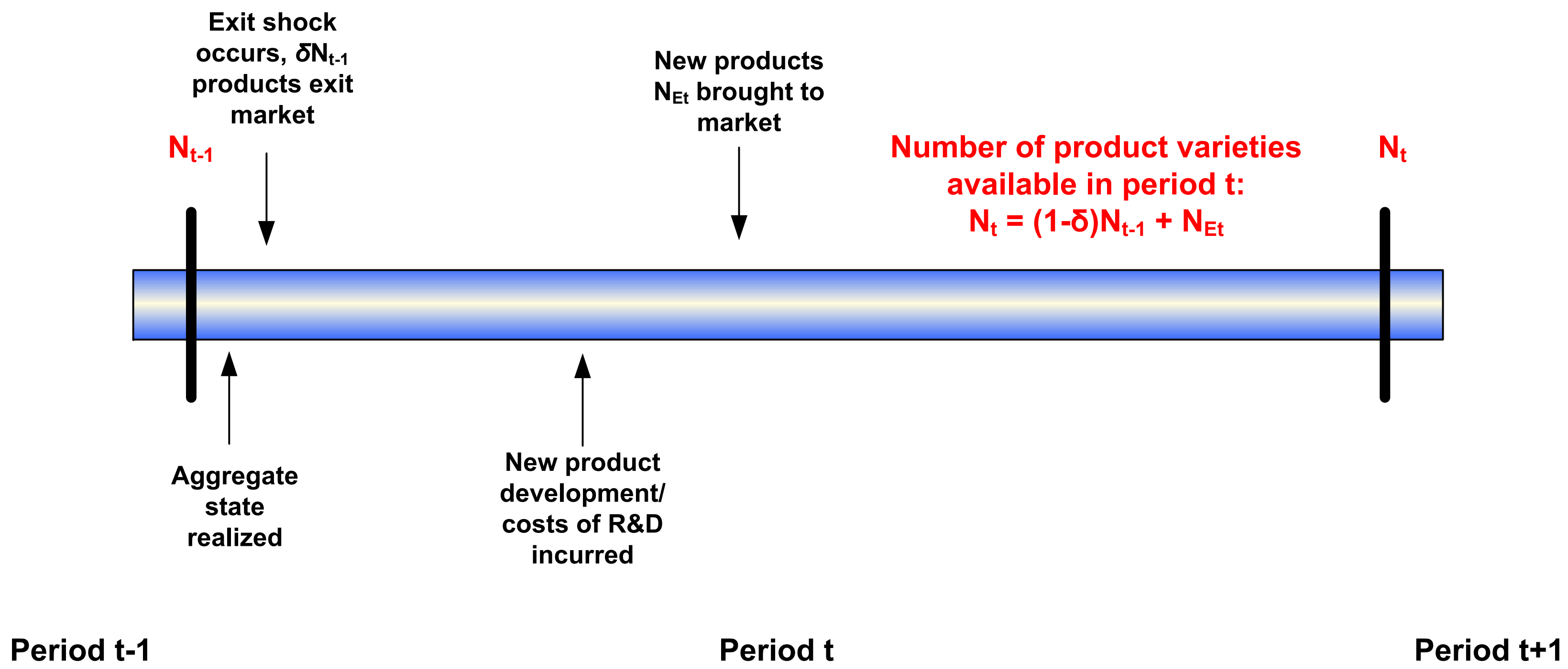
} **DO generate endogenous countercyclical markups…**

(NB: All based on Romer endogenous R&D growth model)

- **Shortcomings**
  - **No *STOCK NATURE* of R&D's newly-developed products**
  - **All new product varieties become obsolete after one period**

Product Varieties

- Bilbiie, Ghironi, and Melitz (2012 *JPE*)
- Sunk R&D costs before appearance of new variety on the market



PRODUCT VARIETIES

- **Bilbiie, Ghironi, and Melitz (2012 *JPE*)**
- **Sunk R&D costs before appearance of new variety on the market**
- **Representative firm with *N* independent "monopolistic" production lines**
  - **Each monopolistic line autonomously develops and prices its own products**

***p* is symmetric relative price of variety**

$$\max_{\{N_t, N_{E,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[(p_t - mc_t) y_t N_t - \Gamma_t \cdot N_{E,t} \right]$$

***Γ* the product development cost, in terms of units of goods**

s.t.

$$N_t = (1-\delta) N_{t-1} + N_{E,t}$$

Law of motion for number of product varieties, which turn over at rate *δ*

PRODUCT VARIETIES

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Law of motion for number of product varieties, which turn over at rate *δ.*

- **FOCs**

***N_t* :** $$-\mu_t^N + (p_t - mc_t) y_t + (1-\delta) E_t \left\{ \Xi_{t+1|t} \mu_{t+1}^N \right\} = 0$$

***$N_{E,t}$* :** $$-\Gamma_t + \mu_t^N = 0$$

PRODUCT VARIETIES

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$N_{E,t}$: $$-\Gamma_t + \mu_t^N = 0$$

- **Product creation condition**
  - **Characterizes optimal investment in R&D/product development**
  - **Free-entry condition for product development**

$$\Gamma_t = (p_t - mc_t) y_t + (1-\delta) E_t \left\{ \Xi_{t+1|t} \Gamma_{t+1} \right\}$$

PRODUCT VARIETIES

- **Final goods production (Dixit-Stiglitz aggregator)**

$$y_t = \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

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$\kappa = \varepsilon/(\varepsilon - 1) - 1$ recovers Dixit-Stiglitz

κ measures increasing returns to specialization

MARKUPS, RELATIVE PRICE, & LOVE OF VARIETY

❒ **Symmetric equilibrium across differentiated product varieties**

	Dixit-Stiglitz	Benassy	
Markup $\mu(N_t)$	$\mu = \varepsilon/(\varepsilon-1)$	$\mu = \varepsilon/(\varepsilon-1)$	
Relative price $p(N_t)$	$= N^{\mu-1}$	$= N^{\kappa}$	
Returns to specialization $\epsilon(N_t)$	$= \varepsilon/(\varepsilon-1) - 1$	$= \kappa$	

PRODUCT VARIETIES

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- **Translog aggregator**
  - **No closed-form primal exists**
  - **Primitive is expenditure function**
  - **(Feenstra, 2003 *Economics Letters*, for further details)**

PRODUCT VARIETIES

- **Translog unit expenditure function on differentiated intermediate varieties**

$$\ln P_t = \frac{1}{2\sigma}\cdot\left(\frac{1}{N_t}-\frac{1}{\tilde{N}}\right)+\frac{1}{N_t}\int_{\omega\in\Omega_t}\ln p_{\omega t}\,d\omega+\frac{\sigma}{2N_t}\int_{\omega\in\Omega_t}\int_{\omega'\in\Omega_t}\ln p_{\omega t}\left(\ln p_{\omega t}-\ln p_{\omega' t}\right)d\omega d\omega'$$

- **Notation**

- $\tilde{N}$ **mass of potential set of varieties** Ω **that could ever exist**
- σ **price elasticity of spending share on an individual variety**

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- $\mu(N_t)=1+\dfrac{1}{\sigma N_t}$ **translog markup**
- **Depends on number of product varieties**
  - **vs. D-S or Benassy markup** $\mu(N_t)=\dfrac{\varepsilon}{\varepsilon-1}$

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- **Depends on number of product varieties**
  - **vs. D-S or Benassy markup** $\mu(N_t) = \frac{\varepsilon}{\varepsilon - 1}$
- **Ret. to specialization** $\epsilon(N_t) = \frac{1}{2\sigma N_t} = \frac{1}{2}\left(\mu(N_t) - 1 \right)$

= μ – 1 for Dixit-Stiglitz
= κ for Benassy

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- **"Profit-destruction externality" inherent in translog aggregator**
  - **Causes *OVER*-production of varieties…**

- **Scope for corrective policy**

MARKUPS, RELATIVE PRICE, & LOVE OF VARIETY

- **Symmetric equilibrium across differentiated product varieties**

	Dixit-Stiglitz	Benassy	Translog
Markup $\mu(N_t)$	$\mu = \varepsilon/(\varepsilon-1)$	$\mu = \varepsilon/(\varepsilon-1)$	$\mu = 1 + 1/(\sigma N)$
Relative price $p(N_t)$	$= N^{\mu-1}$	$= N^{\kappa}$	$= \exp\left(\frac{1}{2}\frac{\tilde{N}-N}{\sigma\tilde{N}N}\right)$
Returns to specialization $\epsilon(N_t)$	$= \varepsilon/(\varepsilon-1) - 1$	$= \kappa$	$= 1/(2\sigma N)$ $= ½\,(\mu(N) - 1)$