Department of Economics The Ohio State University

Economics 8723

Macroeconomic Theory

Midterm Exam – Sketch of Solutions

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The Exam has a total of three (3) problems and pages numbered (following this cover page) one (1) through eleven (11). Each problem's total number of points is shown below. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

Problem 1	**/ 30**
Problem 2	**/ 40**
Problem 3	**/ 30**
TOTAL	**/ 100**

Problem 1: Elasticity of Labor Supply in Neoclassical Labor Market (30 points). Suppose that the labor market is neoclassical (aka, Walrasian). The budget constraint for the household is $c = w \cdot n$, in which c denotes consumption, n denotes labor, and w denotes the real wage. For each utility function provided below:

a) Compute the household's first-order condition with respect to n (and only n).
b) Based on the first-order condition from a), compute the elasticity of labor supply with respect to the real wage w. (**Note:** Hold fixed both c and the multiplier on the budget constraint in the computation of the elasticity.)

Display the elasticity clearly by drawing a box around it, and clearly and carefully provide important algebraic steps/logic that lead to the solution.

a. **(8 points)** $u(c,n) = \ln c - \frac{\psi}{1+1/\iota} n^{1+1/\iota}$.

Solution: Denote by λ the multiplier on the budget constraint. For the given functional form for utility, the first-order condition with respect to n is $-\psi \cdot n^{1/\iota} + \lambda \cdot w = 0$. Restating in terms of the natural log ln(.) gives $\ln \psi + \frac{1}{\iota} \ln n = \ln w + \ln \lambda$, from which it follows that the elasticity of labor supply with respect to the real wage is $\varepsilon_{n,w} \equiv \frac{\partial \ln n}{\partial \ln w} = \iota$.

b. **(10 points)** $u(c,n) = \ln c + \ln(1-n)$.

Solution: Denote by λ the multiplier on the budget constraint. For the given functional form for utility, the first-order condition with respect to n is $-(1-n)^{-1} + \lambda \cdot w = 0$. Computing $\frac{\partial n}{\partial w}\frac{w}{n}$ yields the elasticity of labor supply with respect to the real wage, which is $\varepsilon_{n,w} = \frac{1}{\lambda \cdot w \cdot n}$.

Problem 1 continued

c. **(12 points)** $u(c,n) = \dfrac{\left[c(1-n)^{\psi}\right]^{1-\sigma} - 1}{1-\sigma}$.

Solution: Denote by λ the multiplier on the budget constraint. For the given functional form for utility, the first-order condition with respect to *n* is

$$-\psi \cdot \left[c(1-n)^{\psi}\right]^{-\sigma} \cdot \left[c(1-n)^{\psi-1}\right] + \lambda \cdot w = 0 .$$

After collecting terms, the FOC is $\psi \cdot c^{1-\sigma} \cdot (1-n)^{\psi-1-\psi\sigma} = \lambda \cdot w$. Temporarily defining $L \equiv 1-n$, the FOC can be expressed as

$$\ln\psi + (1-\sigma)\ln c + (\psi - 1 - \psi\sigma)\ln L = \ln\lambda + \ln w ,$$

from which it follows that $\dfrac{\partial \ln L}{\partial \ln w} = \dfrac{1}{\psi - 1 - \psi\sigma}$. To finish computing $\dfrac{\partial \ln n}{\partial \ln w}$, we need to compute the elasticity $\dfrac{\partial \ln n}{\partial \ln L} \approx \dfrac{\partial n}{\partial L}\dfrac{L}{n} = (-1)\cdot\dfrac{1-n}{n}$. Hence, the elasticity of labor supply with respect to the real wage is

$$\frac{\partial \ln n}{\partial \ln w} = \frac{\partial \ln n}{\partial \ln L}\frac{\partial \ln L}{\partial \ln w} = \left(\frac{1-n}{n}\right)\left(\frac{1}{\psi\sigma + 1 - \psi}\right).$$

Problem 2: Vacancy Subsidies, Nash-Bargained Wages, and Proportionally-Bargained Wages (40 points). In an economy that includes proportional subsidies to vacancy posting costs, you will compare and contrast the surplus sharing conditions and the wage outcomes that arise from two different wage-determination mechanisms. Proportional subsidies for vacancy posting costs is the only form of taxation in the economy; vacancy costs in period t are subsidized at the proportional rate $\tau_t^s \ \forall t$. More precisely, the after-subsidy cost to firms for posting any given vacancy is $(1-\tau_t^s)\cdot\gamma$, with $\gamma > 0$ denoting the pre-tax cost of posting a vacancy,

All of the ensuring analysis maintains the parameter setting ρ_x = 1, so there are no "long-lasting employment relationships" at all.

Presented below are the payoffs (more precisely, the payoffs out of the total surplus) to each of the two parties relevant for bargaining. The payoff expressions are presented in, at face value, two different forms. The first formulation is

$$\mathbf{W}(w_t) - \mathbf{U}_t = w_t - b \tag{1}$$

and

$$\mathbf{J}(w_t) = z_t - w_t \tag{2}$$

for, respectively, the potential new employee and the potential new employer (note that, due to free entry in posting vacancies, the value of any given vacancy that does not match with a potential new employee, $\mathbf{V}_t$, is driven to $\mathbf{V}_t = 0$).

The period-t payoff expressions can be equivalently (i.e., isomorphically) expressed as

$$\underbrace{\frac{h'(lfp_t) - u'(c_t)b}{k_t^h \cdot u'(c_t)}}_{\equiv \mathbf{W}(w_t) - \mathbf{U}_t} = w_t - b \tag{3}$$

and

$$\underbrace{\frac{(1-\tau_t^s)\gamma}{k_t^f}}_{\equiv \mathbf{J}(w_t)} = z_t - w_t. \tag{4}$$

One conventional view on how wages are determined in the search and matching framework is via **"Nash bargaining."** Another conventional view on how wages are determined in the search and matching framework is via **"Proportional Bargaining."** In both of these wage determination systems, the wage is determined **after** new matches have been formed (i.e., ex-post of the matching process).

In Nash bargaining, the two parties (the potential new employee and the potential new employer) **jointly** decide ("negotiate") the real wage w_t based on maximizing the following expression

$$\text{Nash maximand} = \underbrace{\left(\mathbf{W}(w_t) - \mathbf{U}_t\right)^{\eta}}_{\text{worker's payoff from total surplus}} \underbrace{\left(\mathbf{J}(w_t) - \mathbf{V}_t\right)}_{\text{firm's payoff from total surplus}}^{1-\eta},$$

which (as stated) goes by the terminology "Nash maximand." The (exogenous) bargaining power of the potential new employee is η, the (exogenous) bargaining power of the potential new employer is $1 - \eta$, with $\eta \in (0,1)$, and, due to free entry in vacancy postings, $\mathbf{V}_t = 0$.

In contrast to Nash bargaining, the "proportional bargaining" asserted surplus-splitting condition for period-t wages is

$$\mathbf{W}(w_t) - \mathbf{U}_t = \left(\frac{\eta}{1-\eta}\right) \cdot \mathbf{J}(w_t). \tag{5}$$

a. **(12 points)** Compute the Nash-bargained surplus-splitting condition. **Display the Nash-bargained surplus-splitting condition in an intuitive, economically-informative manner; clearly display your final Nash surplus-splitting condition by drawing a box around it.**

   In the analysis leading to the final Nash surplus-splitting condition, clearly and carefully provide important algebraic steps/logic that lead to the solution**.** (No need to display each and every step of the algebra – it is left up to you to decide which algebraic steps are the most important one to display for, say, a referee that is reviewing your work.)

Solution: The FOC of the Nash maximand with respect to the real wage w is $\eta\left(\mathbf{W}(w_t) - \mathbf{U}_t\right)^{\eta-1} \mathbf{J}(w_t)^{1-\eta} \cdot \mathbf{W}'(w_t) + (1-\eta)\left(\mathbf{W}(w_t) - \mathbf{U}_t\right)^{\eta} \mathbf{J}(w_t)^{-\eta} \cdot \mathbf{J}'(w_t) = 0$.

Problem 2a (more space if needed)

Using the value expressions provided, $\mathbf{W}'(w_t)=1$ and $\mathbf{J}'(w_t)=-1$; substituting these derivatives into the FOC yields, after a few steps of algebraic rearrangement, the Nash surplus-splitting condition is

$$\mathbf{W}(w_t)-\mathbf{U}_t=\left(\frac{\eta}{1-\eta}\right)\mathbf{J}(w_t).$$

Problem 2 continued

b. **(12 points)** Based on the Nash surplus-splitting condition in part a, rewrite it to obtain the period-t real wage equation. **The final solution for the Nash-bargained period-t real wage equation should be expressed in the form** $w_t^{NASH} = ...$ **(**the term in ellipsis **("…")** on the right-hand side is left to you to determine**); clearly display your final Nash-bargained period-t real wage expression by drawing a box around it. (NOTE: The real wage expression may NOT include the terms $\mathbf{W}(w_t)$, $\mathbf{U}_t$, $\mathbf{J}(w_t)$, $\mathbf{V}_t$, $\forall t$.)**

In the analysis leading to the final Nash-bargained period-t real wage expression, clearly and carefully provide important algebraic steps/logic that lead to the solution**.** (No need to display each and every step of the algebra – it is left up to you to decide which algebraic steps are the most important one to display for, say, a referee that is reviewing your work.)

Solution: Using the value expressions provided, the Nash surplus-splitting condition can be expressed as

$$w_t - b = \left(\frac{\eta}{1-\eta}\right)\left(z_t - w_t\right).$$

A couple of steps of algebra leads to the Nash-bargained real wage

$$w_t^{NASH} = \eta z_t + (1-\eta)b.$$

Problem 2b (more space if needed)

c. **(8 points)** Using the surplus-splitting condition for proportional bargaining (5), obtain the period-t real wage equation. **The final solution for the proportionally-bargained period-t real wage equation should be expressed in the form** $w_t^{PROP} = ...$ **(**the term in ellipsis **("…")** on the right-hand side is left to you to determine**); clearly display your final proportionally-bargained period-t real wage expression by drawing a box around it.** **(NOTE: The real wage expression may NOT include the terms $\mathbf{W}(w_t)$, $\mathbf{U}_t$, $\mathbf{J}(w_t)$, $\mathbf{V}_t$, $\forall t$.)**

In the analysis leading to the final proportionally-bargained period-t real wage expression, clearly and carefully provide important algebraic steps/logic that lead to the solution**.** (No need to display each and every step of the algebra – it is left up to you to decide which algebraic steps are the most important one to display for, say, a referee that is reviewing your work.)

Solution: Using the value expressions provided, the proportional-bargained surplus-splitting condition can be expressed as $w_t - b = \left(\frac{\eta}{1-\eta}\right)(z_t - w_t)$.

Problem 2c (more space if needed)

A couple of steps of algebra leads to the proportionally-bargained real wage

$$w_t^{PROP} = \eta z_t + (1-\eta)b$$

The following parts of the question ask you to compare and contrast Nash-bargained wages with proportionally-bargained wage based on the results above.

(Note: For the grading process, it will be difficult, if not impossible, to carry errors in earlier parts of the question into part d.)

d. **(8 points)** Is there any numerical value for the vacancy subsidy rate τ_t^s that makes the Nash-bargained real wage in part b **identical** to the proportionally-bargained real wage in part c? **Clearly** show whether or not there is.

Solution: The vacancy subsidy rate τ_t^s affects neither the Nash-bargained real wage nor the proportionally-bargained real wage because it does not impact w in any of the value expressions.

Problem 3: Beveridge Curve (30 points). Suppose the aggregate matching function has the form

$$A^m \cdot m(s,v) = A^m \cdot \left[\gamma \cdot s^{\psi} + (1-\gamma)\cdot v^{\psi}\right]^{\frac{1}{\psi}},$$

in which the two parameters are $\gamma \in (0,1)$ and $\psi \in (-\infty, 1]$ and $A^m > 0$ is an exogenous matching efficiency shock. Unless stated otherwise, suppose that $A^m = 1$.

a. **(6 points)** Supposing that there are prices p_s for active search and p_v for job vacancies, use the given matching function to compute the elasticity of substitution between active job seekers *s* and job vacancies *v*.

Solution: The elasticity of substitution between *s* and *v* is $\dfrac{\partial \ln\left(\dfrac{s^*}{v^*}\right)}{\partial \ln\left(\dfrac{p_v}{p_s}\right)}$. Using the given matching function, the elasticity of substitution is $\dfrac{1}{1-\psi}$.

b. **(6 points)** Using the given matching function, compute the point elasticity of total matches with respect to the number of active job seekers *s*.

Solution: For the sake of brevity of notation let *m* stands for total matches $m(s,v)$. The elasticity of total matches with respect to *s* is

$$\frac{\partial m}{\partial s}\cdot\frac{s}{m} = \frac{1}{\psi}\left[\gamma \cdot s^{\psi} + (1-\gamma)\cdot v^{\psi}\right]^{\frac{1}{\psi}-1}\cdot \psi \cdot \gamma \cdot s^{\psi-1}\cdot\left(\frac{s}{m}\right)$$

(be careful in using both the Chain Rule and the rules of exponents in this step). Cancelling the ψ terms on the right-hand side yields

$$\frac{\partial m}{\partial s}\cdot\frac{s}{m} = \gamma\left[\gamma \cdot s^{\psi} + (1-\gamma)\cdot v^{\psi}\right]^{\frac{1}{\psi}-1}\cdot s^{\psi-1}\cdot\left(\frac{s}{m}\right).$$

Next, including the *m* function in the denominator of the term on far right-hand side, and rearranging a few terms, gives

$$\frac{\partial m}{\partial s}\cdot\frac{s}{m}=\gamma\cdot\left[\gamma\cdot s^{\psi}+(1-\gamma)\cdot v^{\psi}\right]^{-1}\left[\gamma\cdot s^{\psi}+(1-\gamma)\cdot v^{\psi}\right]^{\frac{1}{\psi}}\cdot s^{\psi}\cdot\frac{1}{s}\cdot\left(\frac{s}{\left[\gamma\cdot s^{\psi}+(1-\gamma)\cdot v^{\psi}\right]^{\frac{1}{\psi}}}\right)$$

Cancelling the *s* terms on the right-hand side and cancelling the *m* terms on the right-hand side yields the final solution for the elasticity of substitution

$$\frac{\partial m}{\partial s}\cdot\frac{s}{m}=\frac{\gamma\cdot s^{\psi}}{\gamma\cdot s^{\psi}+(1-\gamma)\cdot v^{\psi}}.$$

One verification (which was not required in your solution) that this elasticity is correct is to suppose that $\psi=0$. If $\psi=0$, the elasticity of substitution simplifies to

$$\begin{aligned}\frac{\partial m}{\partial s}\cdot\frac{s}{m}&=\gamma\cdot\left(\frac{1}{\gamma\cdot s^{0}+(1-\gamma)\cdot v^{0}}\right)\cdot s^{0}\\&=\gamma\cdot\left(\frac{1}{\gamma+1-\gamma}\right)\\&=\gamma\end{aligned}$$

which is the elasticity of total matches with respect to the number of active job seekers *s* for the Cobb-Douglas matching function $m(s,v)=s^{\gamma}v^{1-\gamma}$.

c. **(5 points)** Using the given matching function, compute the slope $\frac{\partial v}{\partial s}$.

Solution: Using the implicit function theorem, the slope is

$$\begin{aligned}\frac{\partial v}{\partial s}&=-\frac{\partial m/\partial s}{\partial m/\partial v}\\&=-\left(\frac{\gamma}{1-\gamma}\right)\cdot\left(\frac{s}{v}\right)^{\psi-1}.\end{aligned}$$

d. **(7 points)** The diagram below contains the number of job seekers *s* on the horizontal axis and the number of job vacancies on the vertical axis.

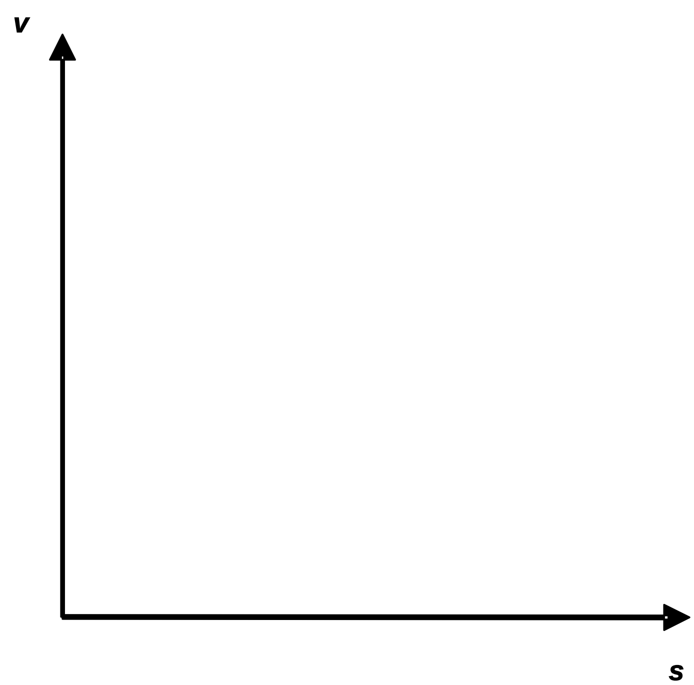
Holding constant the aggregate number of matches, qualitatively sketch the relationship between *v* and *s*. If there is any curvature in the relationship, it should be clearly shown. Provide **brief economic interpretation.** (Note: "economic interpretation" does **not** mean verbally restating the mathematics.)

Solution: For the CES matching function, as long as the parameter ψ is neither $\psi = 1$ nor limiting towards $\psi \to \infty$, the relationship between v and s is shown below.

This inverse relationship between v and s is known as the **Beveridge Curve**. As stated in the accompanying Wikipedia site,

https://en.wikipedia.org/wiki/Beveridge_curve

even though the British economist William Beveridge investigated issues regarding unemployment in the 1940's and 1950's, he himself never drew this inverse relationship. It was instead first developed by two economists in the late 1950's. At some point circa 1980, the inverse relationship was labeled as the "Beveridge Curve," but the origin of the label remains obscure.



e. **(6 points)** Suppose that matching efficiency A^m rises above one. Does the relationship between v and s in the diagram in part d shift if A^m rises from $A^m = 1$ to $A^m > 1$? If it does shift, in which direction does it shift? If it does not shift, explain why not.

Solution: At the value $A^m = 1$, there are some number of matches created at any particular (v,s) point. If matching efficiency rises above $A^m = 1$, then, holding fixed the number of matches created, for any value of v, there would be fewer units of s needed to create the same fixed number of matches. Hence the Beveridge Curve shifts inwards.