Department of Economics The Ohio State University

Economics 8723

Macroeconomic Theory

Problem Set 3 – Sketch of Solutions

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1. **Taylor Staggered Nominal Price-Setting Model.** There are two groups of monopolistically-competitive firms. One group of firms sets it profit-maximizing nominal price in odd-dated periods (i.e., period 1, period 3, period 5, and so on), while the other group of firms sets it profit-maximizing nominal price in even-dated periods (i.e., period 2, period 4, period 6, and so on). Half of the firms that maximize their nominal prices in odd-dated periods, and the other half of firms maximize their nominal prices in even-dated periods.

   Regardless of group, every monopolistically-competitive firm sets its optimal nominal price for two periods – that is, the profit-maximizing nominal price determined by monopolist *i* in period t **is the nominal price for one unit of monopolist *i*'s output in period t AND the nominal price for one unit of monopolist *i*'s output in period t+1,** even if market conditions have changed between periods t and t+1.

   The period-t final goods production function is

$$y_t = \left[\frac{1}{2}\left(y_t^{ADJ}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}\left(y_t^{NONADJ}\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

   in which the superscript "*ADJ*" denotes the group of monopolists adjusting nominal prices in period t and the superscript "*NONADJ*" denotes the group of monopolists not adjusting nominal prices in period t. Regardless of group, monopolist *i* faces the downward-sloping demand function in period t

$$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \cdot y_t,$$

   has zero fixed costs of operation, and operates a constant-returns-to scale production technology, hence its total costs of production in a given time period t are $mc_t \cdot y_{it}$. The price index in period t is denoted P_t.

   In the time period in which monopolist *i* is setting its profit-maximizing nominal price, its profit function is

$$\max_{P_{it}} \left\{ \left(P_{it} - P_t mc_t\right) y_{it} + E_t \Xi_{t+1|t} \left[\left(P_{it} - P_{t+1} mc_{t+1}\right) y_{it+1} \right] \right\}.$$

a. In period t, half of the monopolistically-competitive firms are permitted to (re-)set their optimal nominal prices. For monopolistically-competitive firm *i* in the group of price adjusters in period t, compute its profit-maximizing nominal price P_{it}.

The final solution should be expressed in the form

$$\frac{P_{it}}{P_t} = ...$$

in which the term in ellipsis **("…")** on the right-hand side is must be determined.

Clearly display the final solution by drawing a box around it, and clearly and carefully provide important algebraic steps/logic that lead to the solution**.** (No need to display each and every step of the algebra – it is left up to you to decide which algebraic steps are the most important ones to display.)

Solution: Substituting the demand functions into the profit function gives

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{t+1} \Xi_{s|t} \left\{ P_{it} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s - P_s mc_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \right\} \right\}.$$

Rewriting gives

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{t+1} \Xi_{s|t} \left\{ P_{it}^{1-\varepsilon} P_s^{\varepsilon} - P_{it}^{-\varepsilon} P_s^{1+\varepsilon} mc_s \right\} y_s \right\}.$$

The first-order condition with respect to P_{it} is

$$E_t \left\{ \sum_{s=t}^{t+1} \Xi_{s|t} \left[(1-\varepsilon) \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s + \varepsilon \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0 .$$

Next, regrouping terms gives

$$E_t \left\{ \sum_{s=t}^{t+1} \Xi_{s|t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[(1-\varepsilon) + \varepsilon \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0 .$$

Multiplying through by $-1/\varepsilon$ yields

$$E_t\left\{\sum_{s=t}^{t+1}\Xi_{s|t}\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}-\left(\frac{P_{it}}{P_s}\right)^{-1}mc_s\right]\right\}=0.$$

Multiplying by P_s/P_s gives

$$E_t\left\{\sum_{s=t}^{t+1}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}\frac{1}{P_s}-P_{it}^{-1}mc_s\right]\right\}=0$$

and then multiplying by P_{it} gives

$$E_t\left\{\sum_{s=t}^{t+1}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}-mc_s\right]\right\}=0.$$

If the upper limit of the summation were $s=t$, then the pricing function is $\frac{P_{it}}{P_t}=\left(\frac{\varepsilon}{\varepsilon-1}\right)mc_t$, which is a useful check that the analysis up to this point is correct.

Expanding the summation, dividing the summation by P_t, and rearranging terms gives

$$\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon}+E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it}}{P_{t+1}}\right)^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}$$
$$=\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon}y_tmc_t+E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it}}{P_{t+1}}\right)^{-\varepsilon}y_{t+1}mc_{t+1}\right\}$$

which states that monopolist i's present-value marginal revenues until the next price adjustment equates with the present-value marginal costs until the next price adjustment.

Next, use the definition of the gross inflation rate $\pi_{t+1}\equiv P_{t+1}/P_t$ and the relationship $\frac{P_{it}}{P_{t+1}}=\frac{P_{it}}{P_t}\frac{P_t}{P_{t+1}}=\frac{P_{it}}{P_t}\frac{1}{\pi_{t+1}}$ to re-write the previous expression as

$$\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it}}{P_t}\frac{P_t}{P_{t+1}}\right)^{1-\varepsilon} y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}$$

$$= \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t mc_t + E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it}}{P_t}\frac{P_t}{P_{t+1}}\right)^{-\varepsilon} y_{t+1} mc_{t+1}\right\}$$

and then, collecting terms, as

$$\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} \frac{\varepsilon-1}{\varepsilon} E_t\left\{\Xi_{t+1|t}\pi_{t+1}^{\varepsilon} y_{t+1}\right\}$$

$$= \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t mc_t + \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} E_t\left\{\Xi_{t+1|t}\pi_{t+1}^{1+\varepsilon} y_{t+1} mc_{t+1}\right\}$$

Isolating the $\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}$ on one side gives

$$\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t mc_t + \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} E_t\left\{\Xi_{t+1|t}\pi_{t+1}^{1+\varepsilon} y_{t+1} mc_{t+1}\right\}}{y_t + E_t\left\{\Xi_{t+1|t}\pi_{t+1}^{\varepsilon} y_{t+1}\right\}}.$$

One final rearrangement yields the solution

$$\frac{P_{it}}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{y_t mc_t + E_t\left\{\Xi_{t+1|t}\pi_{t+1}^{1+\varepsilon} y_{t+1} mc_{t+1}\right\}}{y_t + E_t\left\{\Xi_{t+1|t}\pi_{t+1}^{\varepsilon} y_{t+1}\right\}}\right),$$

in which **the numerator is the present-value of real marginal costs, and the denominator is the present-value of real marginal revenues.**

b. For any particular time period t, construct the **aggregate price index** P_t**. Clearly display the final solution by drawing a box around it,** and clearly and carefully provide important algebraic steps/logic that lead to the solution**.** (No need to display each and every step of the algebra – it is left up to you to decide which algebraic steps are the most important ones to display.) **(Note: You should impose symmetric equilibrium within each group but NOT IMPOSE SYMMETRIC EQUILIBRIUM ACROSS GROUPS in constructing the economy's average nominal price.)**

Solution: Start with the final goods' producers profit function

$$P_t\left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{it}y_{it}di = 0,$$

in which the right-hand side follows from perfect competition in the final goods market (i.e., zero equilibrium economic profits for final goods' producers).[1] Substituting the demand functions $y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon}\cdot y_t$ leads to

$$P_t\left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}} = \int_0^1 P_{it}\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon}\cdot y_t\cdot di.$$

Given the production function of final goods $y_t = \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}$, cancelling terms in the previous expression yields

$$P_t = \int_0^1 P_{it}^{1-\varepsilon}\cdot P_t^{\varepsilon}\cdot di,$$

which is easy to rewrite as

$$P_t^{1-\varepsilon} = \int_0^1 P_{it}^{1-\varepsilon}\cdot di.$$

Next, we know that in period t, half of the firms optimally (re-)set their nominal price and the other half does not. Labeling these prices as P_t^{ADJ} and P_t^{NONADJ} and substituting them into them into the previous expression gives

$$P_t^{1-\varepsilon} = \frac{1}{2}\left(P_t^{ADJ}\right)^{1-\varepsilon} + \frac{1}{2}\left(P_t^{NONADJ}\right)^{1-\varepsilon},$$

which characterizes the aggregate price index.

[1] Or, equivalently, $y_t = \left[\frac{1}{2}\left(y_t^{ADJ}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}\left(y_t^{NONADJ}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$ instead of $y_t = \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ -- the final solution does not depend on which formulation is used.

2. **Monopolistic Competition and Optimal Fiscal Policy in a Flexible-Price Model.** Suppose there is a measure [0, 1] of differentiated monopolistically-competitive firms, all prices are flexible (i.e., there are no nominal price rigidities), and that the representative final goods firm's production function is

$$y_t = \left[\int_0^1 y_{it}^{\frac{\varepsilon - 1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon - 1}},$$

with $1 < \varepsilon < \infty$. Each monopolistically-competitive firm operates a linear-in-labor production technology with zero sunk costs, and the labor market is (from the perspective of monopolistically-competitive firms) a perfectly-competitive Walrasian labor market. In a symmetric equilibrium, the real wage is smaller than the marginal product of labor.

The representative household's lifetime utility function is

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \right\}$$

with period-t budget constraint

$$c_t + T_t = (1 - \tau_t^n) w_t n_t .$$

Period-t labor is denoted by n_t, period-t consumption is denoted by c_t, the period-t real wage is denoted by w_t, the period-t lump-sum tax is denoted by T_t, and the period-t proportional labor income tax rate is denoted by τ_t^n. The properties of the period-t utility are $u_c(c_t, n_t) > 0$, $u_{cc}(c_t, n_t) < 0$, $u_n(c_t, n_t) < 0$, $u_{nn}(c_t, n_t) > 0$, and $u_{cn}(c_t, n_t) = 0$.

Is there a proportional labor income tax rate $\tau_t^n \neq 0$ that equates the representative household's marginal rate of substitution between labor and consumption (mrs_t) with the marginal product of labor mpn_t? If there is, compute it and **clearly display the final solution by drawing a box around it**. If there is not, carefully show why not.

Regardless of whether or not there is a labor income tax rate $\tau_t^n \neq 0$ that equates the real wage w_t with the marginal product of labor mpn_t clearly and carefully provide important algebraic steps/logic that lead to the solution**.** (No need to display each and every step of the algebra – it is left up to you to decide which algebraic steps are the most important ones to display.)

Solution: Due to the aggregate symmetric equilibrium price of the final good $p_t = 1$ (which can be computed using the measure [0, 1] of monopolistic firms, but was not required), the real wage, which is equivalent to the marginal cost of production, is smaller than the marginal product of labor. More precisely, the real wage is $w_t = \left(\frac{\varepsilon - 1}{\varepsilon}\right) mpn_t < mpn_t$.

To understand the details behind this, consider that (from household optimization) the period-t consumption-labor optimality condition is

$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = (1 - \tau_t^n) \cdot w_t \,,$$

which in turn implies that labor-market equilibrium is characterized by

$$\begin{aligned} -\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} &= (1 - \tau_t^n) \cdot w_t \\ &= (1 - \tau_t^n) \cdot \left(\frac{\varepsilon - 1}{\varepsilon}\right) \cdot mpn_t \end{aligned}$$

With a zero income tax, $mrs_t < mpn_t$ because $\frac{\varepsilon - 1}{\varepsilon} < 1$ given $1 < \varepsilon < \infty$. Thus, the tax rate that equates mrs_t with mpn_t is a **negative** tax rate (i.e., a subsidy), $\tau_t^n = \frac{\varepsilon}{\varepsilon - 1} - 1$. (E.g., if $\varepsilon = 11$, then $\tau_t^n = -0.10$.)