Department of Economics The Ohio State University

Economics 8723
**Macroeconomic Theory**
**Problem Set 1**
Professor Sanjay Chugh
Spring 2017

**Perfect Risk-Sharing in GE Search and Matching.** Both Hansen (1985 *JME*) and Rogerson (1988 *JME*) prove that the general equilibrium that emerges from an economy in which individuals have identical utility functions, have non-convex choice sets, and in which **labor markets are Walrasian** are **identical** to the general equilibrium that emerges from a representative-household economy in which the household utility function is quasi-linear in labor.

Following a similar approach as Hansen and Rogerson, **prove** that the general equilibrium that emerges from an economy in which individuals have identical utility functions, have non-convex choice sets, and in which **labor markets are based on search and matching** are **identical** to the general equilibrium that emerges from a representative-household economy in which the household utility function is quasi-linear in labor.

For the sake of simplicity in your proof:

1. Omit physical capital $k$.
2. The intensive margin is $h = 1$ for those who are employed.
3. Omit endogenous LFP (i.e., the only two pools of individuals are those that have a job, $n$, and those that are unemployed and actively searching for a job, $u$, with $n+u=1$).
4. The aggregate law of motion for employment is

$$n_{t+1} = (1-\rho_x)n_t + m(u_t, v_t) .$$

Other than the "usual" variables, define clearly any new variables introduced in the proof.

The following two pages provide a sketch of the representative household in which a continuum [0, 1] of individual family members live. A measure $n$ of these individuals are employed, and a measure $u$ of these individuals are unemployed and are actively seeking a job opportunity. Regardless of whether an individual is employed or unemployment, the "large-family" household that pools all individuals' earnings ends up giving each member the same quantity of consumption (i.e., "risk sharing" occurs).

# HOUSEHOLD PROBLEM

- **Dynamic household utility-maximization problem**
  - **A continuum [0, 1] of households (standard assumption)**
  - **A continuum [0, 1] of atomistic individuals live in each household**
  - **Representative household has continuum of "family members"**

$$\max_{c_t, n_t, a_t} \left[ E_0 \sum_{t=0}^{\infty} \beta^t \left( u(c_t) - A \cdot n_t \right) \right]$$

$A > 0$

$$\text{s.t. } c_t + a_t = n_t w_t h_t + (1 - n_t) b + R_t a_{t-1}$$

**An (arbitrary) asset to make pricing interest rates explicit**

**Wage (-setting process) taken as given by household**

**Measure $n_t$ of family members earn labor income (because they work) (and recall we've normalized $h$ = 1)**

**Measure 1-$n_t$ of family members receive unemployment benefits and/or engaged in home production**

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- **Consumption-savings optimality condition:** $1 = R_t E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}$

**Stochastic discount factor**

- **No LFP margin in starter model**
  - **Each family member either works or is looking for work**