Economics 8723
Macroeconomic Theory
Problem Set 3
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1. **Taylor Staggered Nominal Price-Setting Model.** There are two groups of monopolistically-competitive firms. One group of firms sets it profit-maximizing nominal price in odd-dated periods (i.e., period 1, period 3, period 5, and so on), while the other group of firms sets it profit-maximizing nominal price in even-dated periods (i.e., period 2, period 4, period 6, and so on). Half of the firms that maximize their nominal prices in odd-dated periods, and the other half of firms maximize their nominal prices in even-dated periods.

   Regardless of group, every monopolistically-competitive firm sets its optimal nominal price for two periods – that is, the profit-maximizing nominal price determined by monopolist *i* in period t (which is labeled P_{it}^*) **is the nominal price for one unit of monopolist *i*'s output in period t AND the nominal price for one unit of monopolist *i*'s output in period t+1,** even if market conditions have changed between periods t and t+1.

   The period-t final goods production function is

   $$y_t = \left[\frac{1}{2}\left(y_t^{ADJ}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2}\left(y_t^{NONADJ}\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

   in which the superscript "*ADJ*" denotes the group of monopolists adjusting nominal prices in period t and the superscript "*NONADJ*" denotes the group of monopolists not adjusting nominal prices in period t. Regardless of group, monopolist *i* faces the downward-sloping demand function in period t

   $$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \cdot y_t,$$

   has zero fixed costs of operation, and operates a constant-returns-to scale production technology, hence its total costs of production in a given time period t are $mc_t \cdot y_{it}$. The price index in period t is denoted P_t.

   In the time period in which monopolist *i* is setting its profit-maximizing nominal price, its profit function is

   $$\max_{P_{it}} \left\{ \left(P_{it} - P_t mc_t\right) y_{it} + E_t \Xi_{t+1|t} \left[\left(P_{it} - P_{t+1} mc_{t+1}\right) y_{it+1} \right] \right\}.$$

a. In period t, half of the monopolistically-competitive firms are permitted to (re-)set their optimal nominal prices. For monopolistically-competitive firm *i* in the group of price adjusters in period t, compute its profit-maximizing nominal price P_{it}.

The final solution should be expressed in the form

$$\frac{P_{it}}{P_t} = ...$$

in which the term in ellipsis **("…")** on the right-hand side is must be determined.

Clearly display the final solution by drawing a box around it, and clearly and carefully provide important algebraic steps/logic that lead to the solution**.** (No need to display each and every step of the algebra – it is left up to you to decide which algebraic steps are the most important ones to display.)

b. For any particular time period t, construct the **aggregate price index** P_t**. Clearly display the final solution by drawing a box around it,** and clearly and carefully provide important algebraic steps/logic that lead to the solution**.** (No need to display each and every step of the algebra – it is left up to you to decide which algebraic steps are the most important ones to display.) **(Note: You should impose symmetric equilibrium within each group but NOT IMPOSE SYMMETRIC EQUILIBRIUM ACROSS GROUPS in constructing the economy's average nominal price.)**

2. Monopolistic Competition and Optimal Fiscal Policy in a Flexible-Price Model. Suppose there is a measure [0, 1] of differentiated monopolistically-competitive firms, all prices are flexible (i.e., there are no nominal price rigidities), and that the representative final goods firm's production function is

$$y_t = \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

with $1 < \varepsilon < \infty$. Each monopolistically-competitive firm operates a linear-in-labor production technology with zero sunk costs, and the labor market is (from the perspective of monopolistically-competitive firms) a perfectly-competitive Walrasian labor market. In a symmetric equilibrium, the real wage is smaller than the marginal product of labor.

The representative household's lifetime utility function is

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t u(c_t, n_t)\right\}$$

with period-t budget constraint

$$c_t + T_t = (1-\tau_t^n)w_t n_t .$$

Period-t labor is denoted by n_t, period-t consumption is denoted by c_t, the period-t real wage is denoted by w_t, the period-t lump-sum tax is denoted by T_t, and the period-t proportional labor income tax rate is denoted by τ_t^n. The properties of the period-t utility are $u_c(c_t, n_t) > 0$, $u_{cc}(c_t, n_t) < 0$, $u_n(c_t, n_t) < 0$, $u_{nn}(c_t, n_t) > 0$, and $u_{cn}(c_t, n_t) = 0$.

Is there a proportional labor income tax rate $\tau_t^n \neq 0$ that equates the representative household's marginal rate of substitution between labor and consumption (mrs_t) with the marginal product of labor mpn_t? If there is, compute it and **clearly display the final solution by drawing a box around it**. If there is not, carefully show why not.

Regardless of whether or not there is a labor income tax rate $\tau_t^n \neq 0$ that equates the representative household's marginal rate of substitution between labor and consumption (mrs_t) with the marginal product of labor mpn_t, clearly and carefully provide important algebraic steps/logic that lead to the solution**.** (No need to display each and every step of the algebra – it is left up to you to decide which algebraic steps are the most important ones to display.)