Labor Matching Models: Basic Building Blocks

January 22, 2018

BASIC DSGE ISSUES

- **Labor fluctuations at extensive margin (number of people working) larger than at intensive margin (hours worked per employee)**

- **Labor market structure(s) important to understand/model more deeply**
  - **Theoretical interest: Many results from existing frameworks point to it**
  - **Empirical interest: Labor-market outcomes the most important economic aspect of many (most?) people's lives**
  - **"Labor wedges" – CKM (2007 *EC*), Shimer (2009 *AEJ:Macro*), Karabarbounis (2014 *RED*), *MANY* others**

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- **Explosion of DSGE labor matching models the past ten years**
  - **Sparked in part by Shimer (2005 *AER*) and Hall (2005 *AER*)**
    - **Although their models were not full GE models**
  - **Not yet clear what problems incorporating labor matching has helped solve....**
  - **...but has likely shed insight on some issues (e.g., in cyclical fluctuations and in policy analysis, real wage dynamics matter a lot)**

- **Rogerson and Shimer (2011 *Handbook of Labor Economics*)**

BASIC LABOR MARKET ISSUES

- How can production resources sit idle even when there is "high aggregate demand?"

- Coordination frictions in labor markets
  - Finding a job or an employee takes time and/or resources
  - Not articulated in basic neoclassical/Walrasian framework

- Are labor market transactions "spot" transactions?
  - Or do they occur in the context of ongoing relationships?
  - The answer implies quite different roles for prices (wages)

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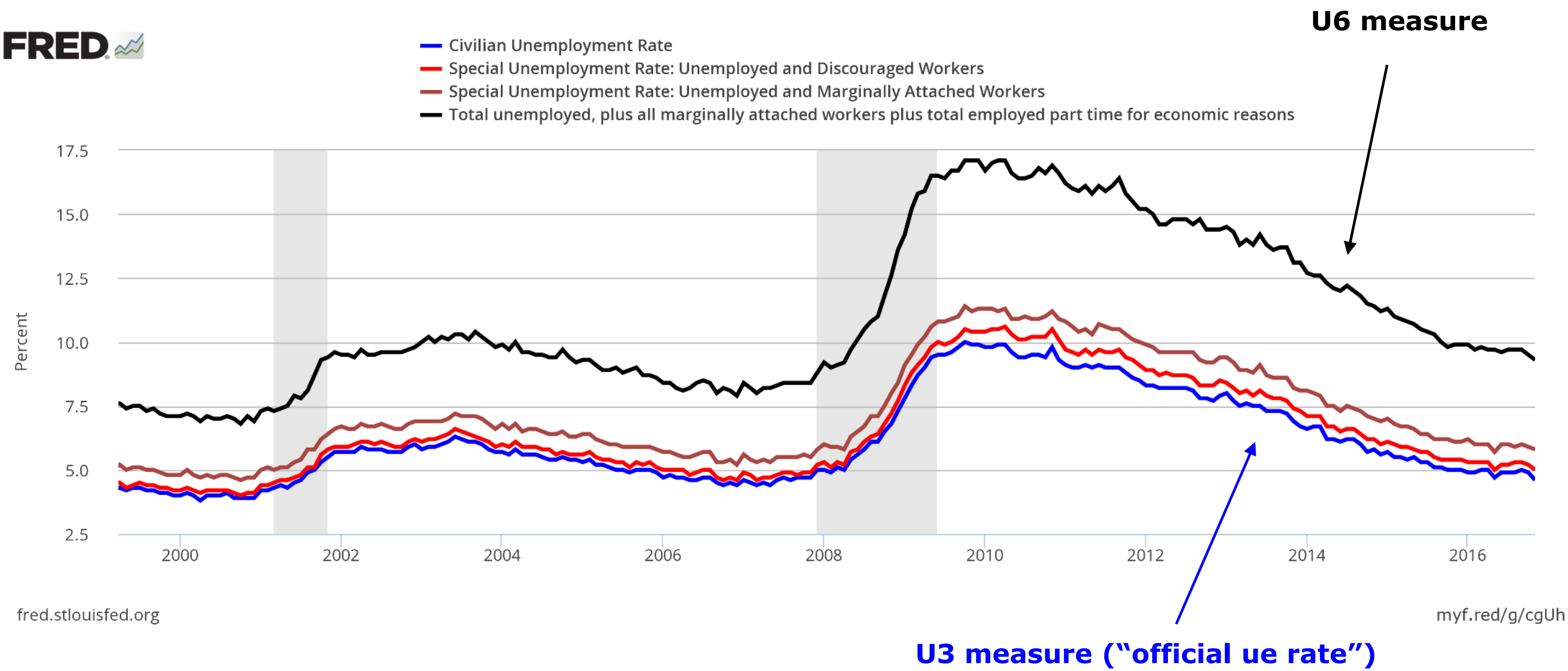
- **“Structural” vs. “frictional” unemployment**
  - **Structural: unemployment induced by fundamental changes in technology, etc – dislocations due to insufficient job training, changing technical/educational needs of workforce, etc.**

  - **Frictional: temporarily unemployed as workers and jobs shuffle from one partner to another**

Basic Labor Market Issues



Basic Labor Market Issues

- **Discouraged workers** (Current Population Survey): Persons not in the labor force who want and are available for a job and **who have looked for work sometime in the past 12 months** (or since the end of their last job if they held one within the past 12 months), but who are **not currently looking because they believe there are no jobs available or there are none for which they would qualify.**

- **Marginally attached workers** (Current Population Survey): Persons not in the labor force who want and are available for work, and who have **looked for a job sometime in the prior 12 months** (or since the end of their last job if they held one within the past 12 months), but **were not counted as unemployed because they had not searched for work in the 4 weeks preceding the survey.** Discouraged workers are a subset of the marginally attached. (See Discouraged workers.)

- **Part-time workers** (Current Population Survey and American Time Use Survey): Persons who work less than 35 hours per week.

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- Aggregate matching function
- Law of motion for employment
- Vacancy posting costs
- Some wage determination mechanism (Nash or many others…)
- Intensive (aka “hours”) margin?
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- Some wage determination mechanism (Nash or many others…)
- Intensive (aka "hours") margin?
  - Often absent…
- Endogenous consumption/labor "supply" decision?
  - Typically absent…
  - Can consider it implicitly in the background (might depend on the wage determination mechanism…)
  - …or consider it explicitly by introducing a third activity for individuals ("outside the labor force")

Basic Building Blocks

- **Aggregate matching function**

$$m(u_t, v_t)$$

Typically assumed to be Cobb-Douglas (see Petrongolo and Pissarides 2001 *JEL*)

  - **Brings together individuals looking for work (*u*) and employers looking for workers (*v*)**
  - **A technology from the perspective of the economy (just like aggregate production function)**
  - **Black box that describes all the possible coordination, matching, informational, temporal, geographic, etc. frictions in finding workers and jobs**

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- **Employment is a state variable (one specific timing; try others)**

$$n_{t+1} = (1-\rho_x)n_t + m(u_t, v_t)$$

Aggregate law of motion of employment

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Churning of jobs; a job is not an absorbing state

$$n_{t+1} = \underbrace{(1-\rho_x)n_t}_{\text{Number of existing jobs that end: } \rho_x \text{ exogenous separation rate, but can also endogenize}} + \underbrace{m(u_t, v_t)}_{\text{Number of new jobs (matches) that form in } t \text{ and will become active in } t+1}$$

Aggregate law of motion of employment

Number of existing jobs that end: ρ_x exogenous separation rate, but can also endogenize

Number of new jobs (matches) that form in *t* and will become active in *t*+1

ANALOGY: $k_{t+1} = (1-\delta)k_t + i_t$

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  - **A primitive cost**

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- **→ marginal cost of vacancy posting = …?…**
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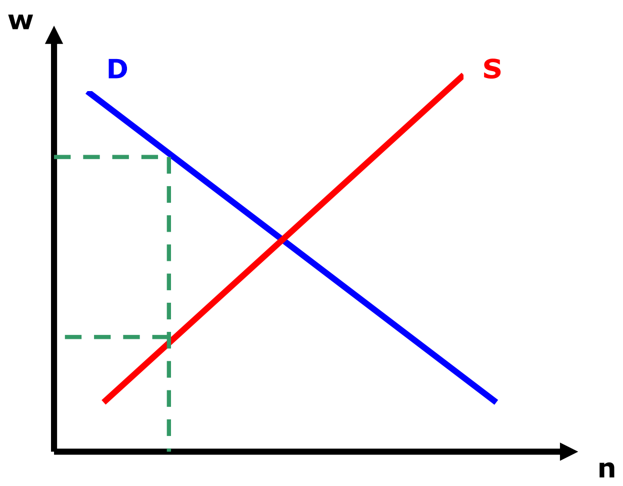
- **Realistic for recruiting departments?**

- **If not, suppose convex (concave?) costs of posting vacancies**
- **Total vacancy posting costs = $\gamma g(v_t)$**
- **Does marginal cost = average cost ?….**

BASIC BUILDING BLOCKS

- **Wage determination**
  - **Labor transactions not neoclassical(-based), so no simple supply-and-demand based pricing**

Walrasian "wedge" between *$MRS_{C,L}$* and *MP_N*



Basic Building Blocks

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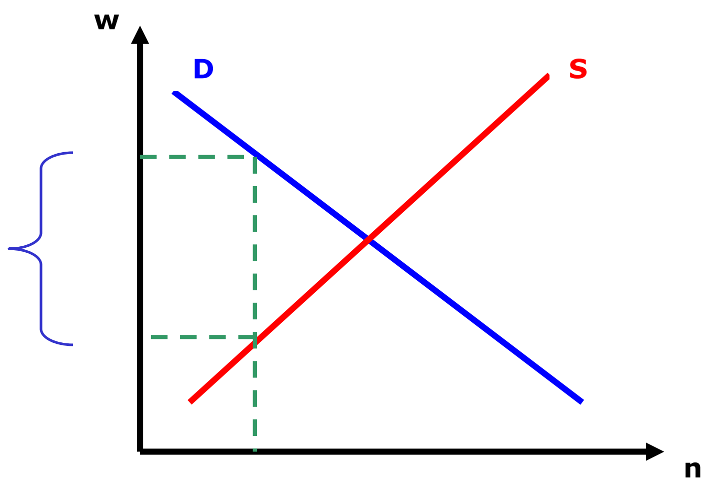
  - **Labor transactions not neoclassical(-based), so no simple supply-and-demand based pricing**
  - **Local (bilateral, *not* market-based) monopolies (local rents) exist between each worker-employer pair**
    - **Exist due to the matching friction and ex-ante costs of hiring**
    - **Allows a wide range (too wide?) of wage-determination schemes – one of the points of Hall (2005 *AER*)**

neoclassical-based equilibrium quarantines this range of wages



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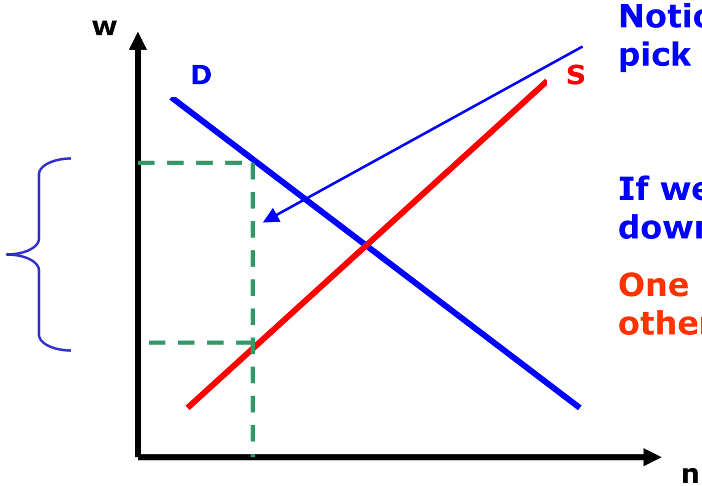

Notion of *matching equilibrium* can pick out these *w*'s…

If we have a systematic way of pinning down a particular *w*

One method: Nash bargaining (many others…)

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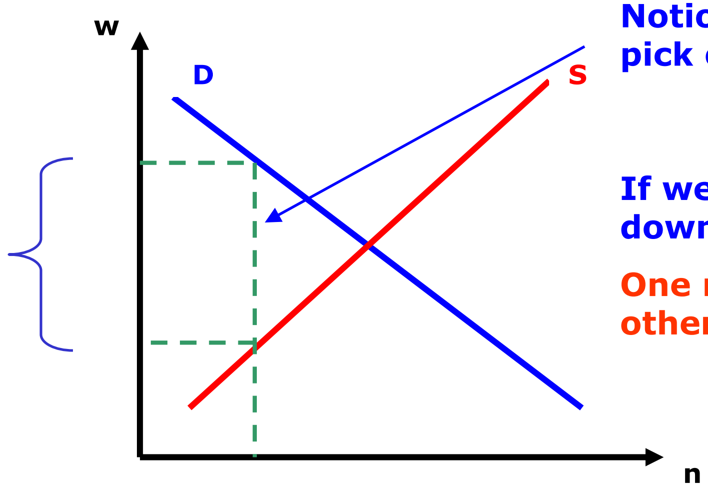

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If we have a systematic way of pinning down a particular *w*

One method: Nash bargaining (many others…)

<u>IMPORTANT:</u> wage plays a very different role than in neoclassical(-based) labor market – *not* purely allocative, now also plays a distributive role

Basic Building Blocks

- **(Generalized) Nash Bargaining**

Bargaining powers η and 1-η measure "strength" of each party in negotiations

$$\max_{w_t}\underbrace{\left(\mathbf{W}(w_t)-\mathbf{U}(w_t)\right)}_{\substack{\text{Net payoff to an individual of agreeing}\\ \text{to wage } w \text{ and beginning production}}}^{\eta}\underbrace{\left(\mathbf{J}(w_t)-\mathbf{V}(w_t)\right)}_{\substack{\text{Net payoff to a firm of agreeing to}\\ \text{wage } w \text{ and beginning production}}}^{1-\eta}$$

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Original Nash 1950 was η = 0.5

- **The unique problem whose solution satisfies three axioms (Nash 1950)**
  - **Pareto optimality**
  - **Scale invariance**
  - **Independence of irrelevant alternatives**

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- **Given an extensive-form foundation by Binmore (1980) and Binmore, Rubinstein, Wolinksy (1986)**
  - Nash solution the limiting solution of a Rubinstein alternating-offers game (as time interval between successive offers → zero)
  - In which (*η*, 1- *η*) measure discount factors of each party between successive offers

Analysis of Model

- Study firm vacancy posting decision
  - Representative firm chooses desired number of workers to hire
    - Typical setup in DSGE labor matching models…
    - …in contrast to partial equilibrium labor matching models (one firm/one job) – but equivalent if sufficient linearity

ANALYSIS OF MODEL

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- **Study household/worker decision(s)**
  - **No labor-force participation decision in baseline model**
  - **Full consumption insurance the norm in DSGE matching models**
    - **All individuals live in a "large" (infinite) household, so full risk-sharing (Problem Set 1)**

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- How do matching markets clear?

- How are wages determined?

Labor Matching Models: Basic DSGE Implementation

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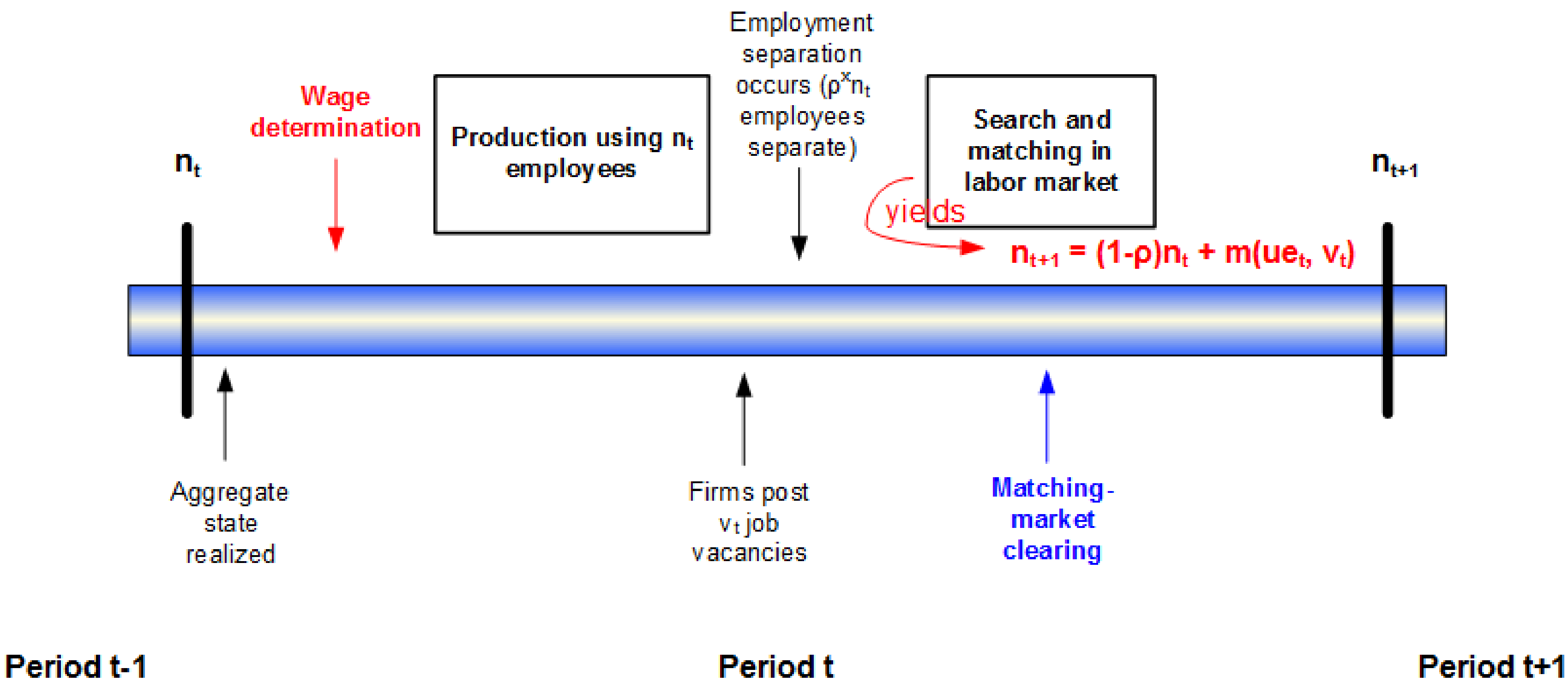
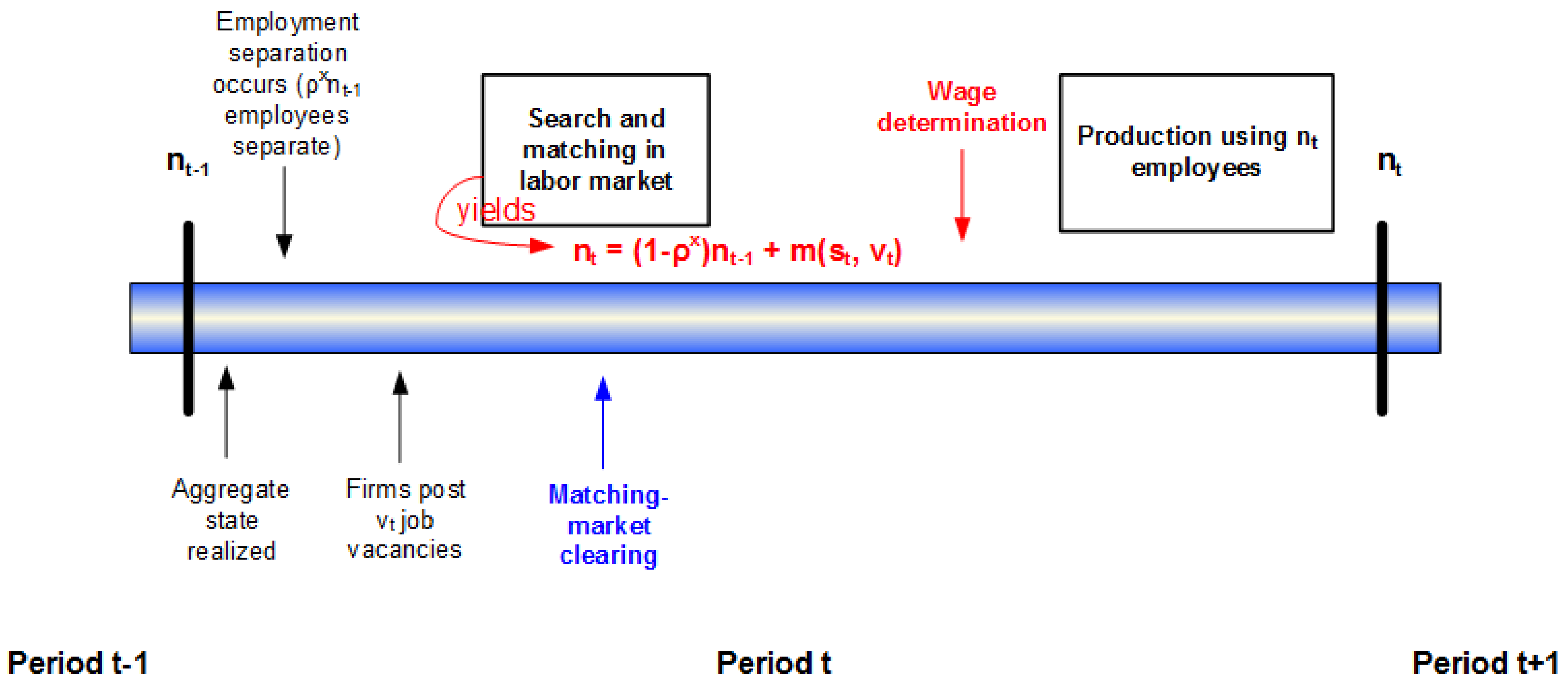
TIMELINE



("Lagged production" timing – use for now…)

Timeline



("Instantaneous production" timing…)

FIRM VACANCY-POSTING PROBLEM

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_{t+1}^f} \left[E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f f(h_t) - \underbrace{w_t n_t^f h_t}_{} - \underbrace{\gamma g(v_t)}_{} \right) \right]$$

Discount factor between time 0 and *t* because ***dynamic*** firm problem; in equilibrium, = household stochastic discount factor

Number of vacancies to post (how many job advertisements)

Desired target ***future*** firm employment

Total output – sold in perfectly-competitive goods market

Total wage bill depends on both extensive and intensive employment

Total cost of posting ***v*** vacancies

- **Subject to (perceived) law of motion for firm's employment stock**

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- For starters
  - Shut down intensive margin: $h_t = 1$
  - Linear posting costs: $g(v) = v$
  - Firm production function: $y_t = z_t n_t$

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Perceived law of motion for evolution of employment stock

Number of existing jobs that remain intact: ρ_x exogenous separation rate, but can also endogenize

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- **Market-determined probability k^f taken as given**

Firm Vacancy-Posting Problem

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Perceived law of motion for evolution of employment stock

FOCs with respect to v_t, n_{t+1}

$$-\gamma + \mu_t k^f(\theta_t) = 0$$

$$-\mu_t + E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + (1-\rho_x)\mu_{t+1} \right) \right\} = 0$$

Combine

FIRM VACANCY-POSTING PROBLEM

- **Vacancy posting condition (aka job creation condition)**

$$\underbrace{\gamma}_{\text{Cost of posting a vacancy}} = \underbrace{k^f(\theta_t)E_t\left\{\Xi_{t+1|t}\left(z_{t+1} - w_{t+1} + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})}\right)\right\}}_{\text{Expected benefit of posting a vacancy}}$$

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of attracting a worker) x (expected future benefit of an additional worker)

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γ / k^f is capital value of an existing employee – because one *less* worker firm has to find in the future

EMPLOYEES ARE ASSETS

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= marginal output – wage payment + expected asset value of an additional worker

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- **Vacancy-posting is a type of investment decision**
  - **Intertemporal dimension makes discount factor potentially important**
    - **Makes general equilibrium effects potentially important**
- **Two prices affect posting decision (aside from intertemporal price)**
  - **Wage**
  - **Matching probability k^f (which depends on the market variable θ)**

Household Problem

- Dynamic household utility-maximization problem
  - A continuum [0, 1] of households (a standard assumption)
  - A continuum [0, 1] of atomistic individuals live in each household
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$$\max_{c_t, n_t, a_t} \left[E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - A \cdot n_t \right) \right]$$

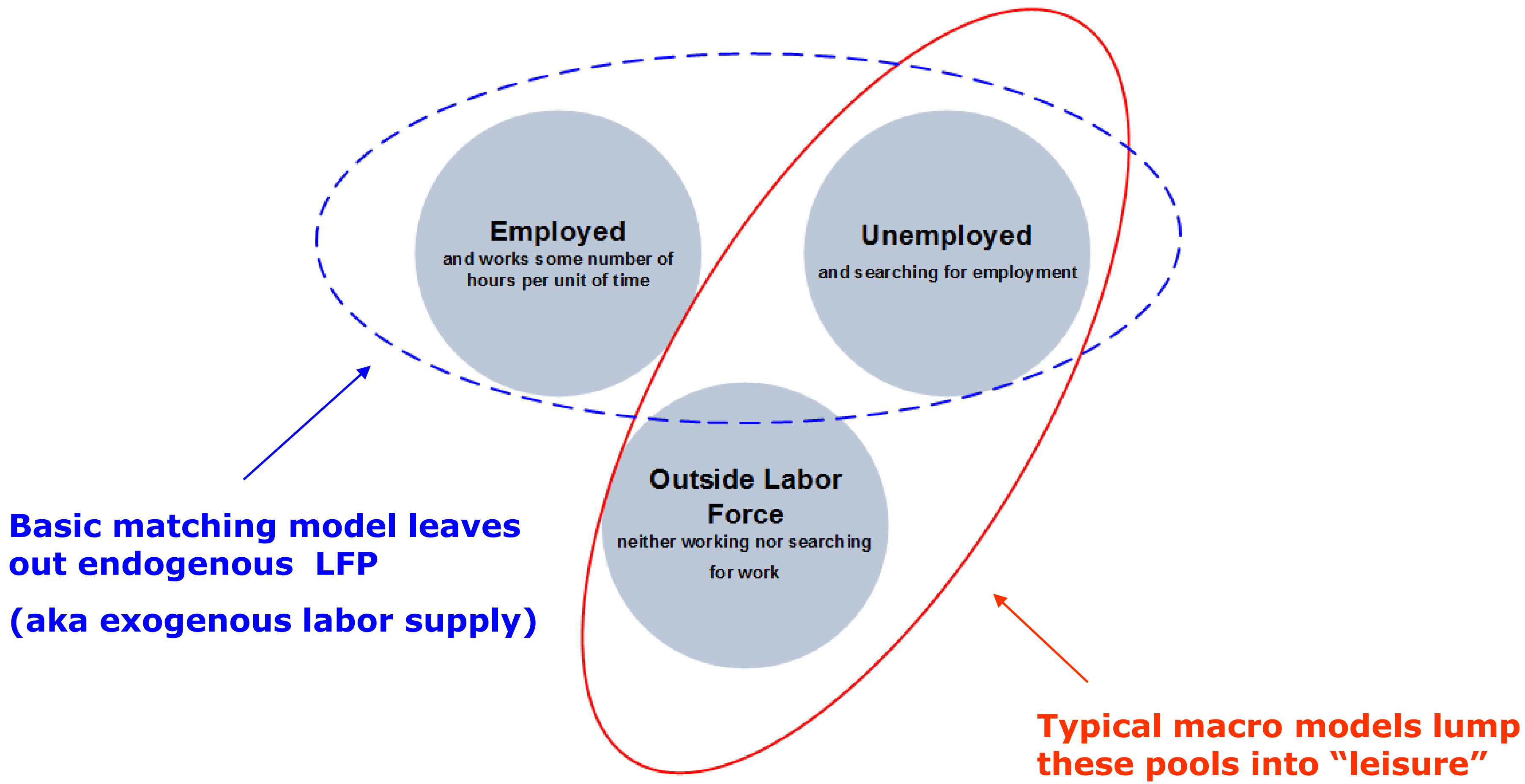
$$\text{s.t.} \quad c_t + a_t = \underbrace{n_t w_t h_t}_{\text{Measure } n_t \text{ of family members earn labor income (because they work) (and recall we've normalized } h = 1\text{)}} + \underbrace{(1-n_t) b}_{\text{Measure } 1-n_t \text{ of family members receive unemployment benefits and/or engaged in home production}} + R_t a_{t-1}$$

Measure n_t of family members earn labor income (because they work) (and recall we've normalized h = 1)

Measure 1-n_t of family members receive unemployment benefits and/or engaged in home production

An (arbitrary) asset to make pricing interest rates explicit

Wage (-setting process) taken as given by household


Employed
and works some number of hours per unit of time
Unemployed
and searching for employment
Outside Labor Force
neither working nor searching for work
Basic matching model leaves out endogenous LFP
(aka exogenous labor supply)
Typical macro models lump these pools into “leisure”

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KEY: **Assuming infinite *family* structure delivers *full consumption insurance* – i.e., all employed and unemployed individuals have equal consumption**

Individual family members are risk-neutral with respect to their labor-market realization

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KEY: Assuming infinite *family* structure delivers ***full consumption insurance*** – i.e., all employed and unemployed individuals have equal consumption

Individual family members are **risk-neutral** with respect to their labor-market realization

$$\max_{c_t, n_t, a_t} \left[E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - A \cdot n_t \right) \right]$$

$$\text{s.t.} \quad c_t + a_t = \underbrace{n_t w_t h_t}_{} + \underbrace{(1-n_t)b}_{} + R_t a_{t-1}$$

An (arbitrary) asset to make pricing interest rates explicit

Wage (-setting process) taken as given by household

Measure n_t of family members earn labor income (because they work) (and recall we've normalized h = 1)

Measure 1-n_t of family members receive unemployment benefits and/or engaged in home production

- Consumption-savings optimality condition: $1 = R_t E_t \underbrace{\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}}_{}$

Stochastic discount factor

- No LFP margin in starter model
  - Each family member either works or is looking for work

WAGE BARGAINING

- (Generalized) Nash Bargaining

$$\max_{w_t} \underbrace{\left(\mathbf{W}(w_t) - \mathbf{U}(w_t)\right)^{h}}_{\text{Net payoff to an individual/household of agreeing to wage } w \text{ and beginning production}} \underbrace{\left(\mathbf{J}(w_t) - \mathbf{V}(w_t)\right)^{1-h}}_{\text{Net payoff to a firm of agreeing to wage } w \text{ and beginning production}}$$

Bargaining over how to divide the surplus

WAGE BARGAINING

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Bargaining over how to divide the surplus

Net payoff to an individual/household of agreeing to wage *w* and beginning production

Net payoff to a firm of agreeing to wage *w* and beginning production

- **Value equations**
  - ***W*: value to (representative) household of having one additional member employed**
  - ***U*: value to (representative) household of having one additional member unemployed and searching for work**
  - ***J*: value to (representative) firm of having one additional employee**
  - ***V*: value to (representative) firm of having a vacancy that goes unfilled**
    - **Free entry in vacancy-posting → *V* = 0**

- **Define *W* and *U* in terms of household problem**
  - **i.e., based on envelope conditions of household value function**

WAGE BARGAINING

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Bargaining over how to divide the surplus

- **Nash surplus-sharing rule**

$$h\left(\mathbf{W}'(w_t) - \mathrm{U}'(w_t)\right)\mathbf{J}(w_t) = (1-h)(-\mathbf{J}'(w_t))\left(\mathbf{W}(w_t) - \mathrm{U}(w_t)\right)$$

(FOC with respect to w_t)

- **Must specify value equations *W*(.), *U*(.), *J*(.)**

Value Equations

- Individual/household value equations (constructed from household problem)

$$\mathbf{W}(w_t) = \underbrace{w_t}_{\text{Contemporaneous return is wage}} + \underbrace{E_t\left\{\Xi_{t+1|t}\left[(1-\rho_x)\mathbf{W}(w_{t+1}) + \rho_x \mathbf{U}(w_{t+1})\right]\right\}}_{\text{Expected future return takes into account transition probabilities}}$$

Value to household of having the marginal individual employed

Value Equations

- ❒ **Individual/household value equations (constructed from household problem)**

Each searching individual has probability $k^h(\theta)$ of finding a job opening: depends on a *market* variable, θ, so taken as given

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Value to household of having the marginal individual employed

$$\mathbf{U}(w_t) = \underbrace{b}_{\text{Contemporaneous return is unemployment benefit/home production}} + E_t\left\{\underbrace{\mathrm{X}_{t+1|t}\left[k^h(q_t)\mathbf{W}(w_{t+1}) + (1- k^h(q_t))\mathbf{U}(w_{t+1})\right]}_{\text{Expected future return takes into account transition probabilities}}\right\}$$

Value to household of having the marginal individual unemployed and searching

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Value to household of having the marginal individual unemployed and searching

- **Firm value equation**

$$\mathbf{J}(w_t) = \underbrace{z_t - w_t}_{\text{Contemporaneous return is marginal output net of wage payment}} + \underbrace{E_t \left\{ \Xi_{t+1|t} (1-\rho_x) \mathbf{J}(w_{t+1}) \right\}}_{\text{Expected future return takes into account transition probabilities}}$$

Value to firm of the marginal employee

WAGE BARGAINING

❒ **The Nash surplus-sharing rule**

$$\eta\left(\mathbf{W}'(w_t) - \mathbf{U}'(w_t)\right)\mathbf{J}(w_t) = (1-\eta)(-\mathbf{J}'(w_t))\left(\mathbf{W}(w_t) - \mathbf{U}(w_t)\right)$$

(FOC with respect to w_t)

Insert marginal values

$$\eta\mathbf{J}(w_t) = (1-\eta)\left(\mathbf{W}(w_t) - \mathbf{U}(w_t)\right)$$

Firm's surplus *J* a constant fraction of household's surplus *W – U*

NOTE: NOT a general property of Nash bargaining; here due to the linearity of *W, U,* and *J* with respect to wage

Wage Bargaining

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Using definitions of *W, U,* and *J,* the job-creation condition, and some algebra

$$\boxed{w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b}$$

Bargained wage a convex combination of gains from consummating the match and the gains from walking away from the match

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Contemporaneous marginal output…

…plus term that captures savings on future posting costs if match continues

Bargained wage a convex combination of gains from consummating the match and the gains from walking away from the match

NOTE: With C-D matching function,

$\theta = k^h(\theta)/k^f(\theta)$

LABOR MARKET MATCHING

- **Aggregate matching function displays CRS**

$$m(u_t, v_t)$$

u_t = 1 – n_t is measure of individuals searching for work

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- For any given individual vacancy or individual (partial equilibrium), matching probabilities depend only on ***v/u***

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Market tightness: **measures relative number of traders on opposite sides of market**

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$$\frac{m(u_t, v_t)}{v_t} = m\left(\frac{u_t}{v_t}, 1\right) = m\left(\theta_t^{-1}, 1\right) \equiv k^f(\theta_t)$$

Probability a given vacancy/job posting attracts a worker

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Probability a given individual finds a job opening

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Labor Market Matching

- **Aggregate matching function displays CRS**

$$m(u_t, v_t)$$

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- **For any given individual vacancy or individual (partial equilibrium), matching probabilities depend only on *v/u***

NOTE: With C-D matching function,

$\theta = k^h(\theta)/k^f(\theta)$

In matching models, θ is key driving force of efficiency and thus optimal policy prescriptions (Mortensen 1982 *AER* and Hosios 1990 *ReStud* key references)

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Probability a given individual finds a job opening

$$\theta_t \equiv \frac{v_t}{u_t}$$

<u>Market tightness:</u> measures relative number of traders on opposite sides of market

- **Market tightness an allocational signal**
  - **Because matching probabilities depend on it**
  - **e.g., the higher (lower) is *v/u*, the easier (harder) it is for a given individual to find a job opening**

Labor Market Equilibrium

- **Aggregate law of motion of employment**

$$n_{t+1} = (1-\rho_x)n_t + m(u_t, v_t)$$

- **Matching-market equilibrium**

$$m(u_t, v_t) = u_t \cdot k^h(\theta_t) = v_t \cdot k^f(\theta_t)$$

- **Vacancy-posting (aka job-creation) condition**

$$\gamma = k^f(\theta_t) E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

- **Wage determination (Nash bargaining)**

$$w_t = \eta \left[z_t + \gamma \theta_t \right] + (1-\eta) b$$

LABOR MARKET EQUILIBRIUM

- **Aggregate law of motion of employment**

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$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

- **Basic labor-theory literature: impose ss, comparative static exercises, etc. (exogenous real interest rate)**
  - **Pissarides Chapter 1, RSW 2005 *JEL***

General Equilibrium

- ❒ **Aggregate law of motion for employment**
- ❒ **Vacancy-posting (aka job-creation) condition**
- ❒ **Wage determination**

The labor market equilibrium (*partial* equilibrium)

- ❒ **Consumption-savings optimality condition (endogenizes real interest rate)**

$$1 = R_t E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}$$

- ❒ **Aggregate resource constraint**

Often interpreted as the output of a home production sector – only the unemployed produce in the home sector

$$c_t + g_t + \gamma v_t = z_t n_t h_t + (1 - n_t) b$$

Vacancy posting costs and "outside option" are real uses of resources

- ❒ **Exogenous LOMs for any driving processes (TFP, etc)**

STEADY STATE OF LABOR MARKET

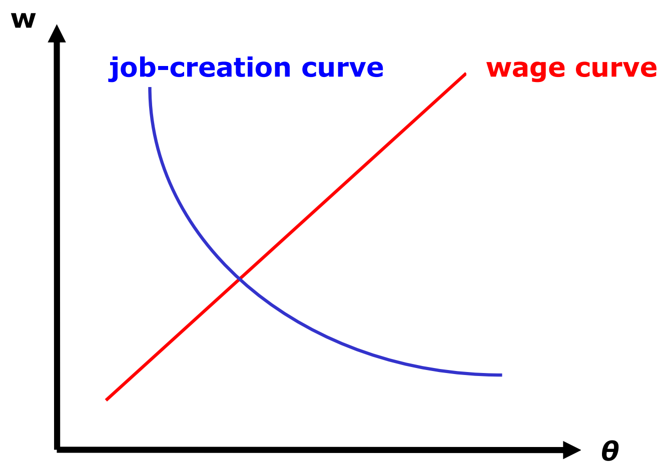
- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1) $$1-u=(1-\rho_x)(1-u)+m(u,v)$$ **(using *n* = 1 – *u*)**

(2) $$\gamma=\beta k^f(\theta)\left(z-w+\frac{(1-\rho_x)\gamma}{k^f(\theta)}\right)$$ ***w* negatively and nonlinearly related to *θ* (given CRS matching function)**

(3) $$w=\eta\left[z+\gamma\theta\right]+(1-\eta)b$$ ***w* positively and linearly related to *θ***

Pissarides 2000, Figure 1.1



NOTE: wage function entirely due to ASSUMPTION of Nash-negotiated wages

STEADY STATE OF LABOR MARKET

- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1)
$$u = \frac{\rho_x - m(u,v)}{\rho_x}$$

For a given (w,θ), v and u negatively related (given CRS matching function)

(2)
$$\gamma = \beta k^f\left(\frac{v}{u}\right)\left(z - w + \frac{(1-\rho_x)\gamma}{k^f\left(\frac{v}{u}\right)}\right)$$

For a given (w,θ), v and u positively related (given CRS matching function)

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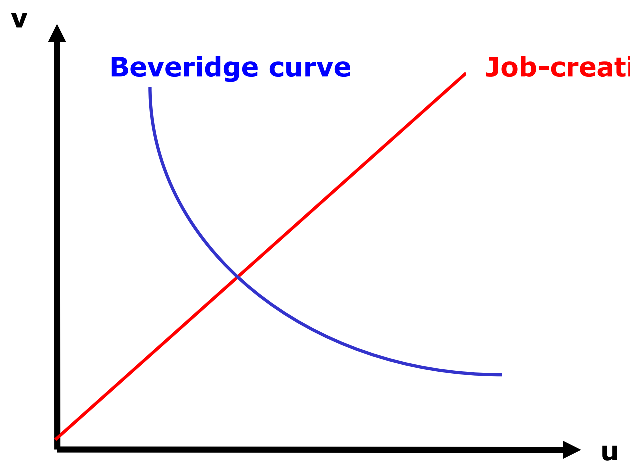

Pissarides 2000, Figure 1.2

<u>BEVERIDGE CURVE</u>: Empirical relationship in both long run and short run (i.e., cyclical)

STEADY STATE OF LABOR MARKET

- **Labor-market equilibrium is (*w*, *u*, *θ*) satisfying (1), (2), (3)**

STEADY STATE OF LABOR MARKET

- **Labor-market equilibrium is (w, u, θ) satisfying (1), (2), (3)**

- **Comparative statics**
  - **A rise in b…**
    - **…raises w**
    - **…lowers θ**
    - **…lowers v and raises u**

Higher value (outside option) of unemployment requires a higher wage to induce individuals to work, which reduces firm incentives to create jobs

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  - **A fall in β (or a rise in ρ_x)…**
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    - **…raises u**
    - **…ambiguous effect on v**

    Higher real rate and/or faster job separations (i.e., "faster depreciation of employment stock") makes posting vacancies (FOR FIXED u) less attractive for firms (both erode firm profits)

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- **See Pissarides Chapter 1 and RSW (2005 *JEL*) for more**

- **Dynamic stochastic partial equilibrium (Shimer 2005 *AER*, Hall 2005 *AER*, Hagedorn and Manovskii 2008 *AER*)**