OPTIMAL FISCAL POLICY

FEBRUARY 15, 2019

OPTIMAL POLICY

- **Ramsey (1927 *Economic Journal*)**

A CONTRIBUTION TO THE THEORY OF TAXATION

THE problem I propose to tackle is this: a given revenue is to be raised by proportionate taxes on some or all uses of income, the taxes on different uses being possibly at different rates; how should these rates be adjusted in order that the decrement of utility may be a minimum? I propose to neglect altogether

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- **Stiglitz (2015 *Economic Journal*)**
  - **Pros and cons of Ramsey taxation framework**
  - **"In Praise of Frank Ramsey's Contribution to the Theory of Taxation"**

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- **Lump-sum taxation ruled out**
- **Proportional taxation**

Optimal Policy in Macro

- **Lucas and Stokey (1983 *JME*)**
  - **Adapt Ramsey framework to macro**
  - **Jointly optimal fiscal and monetary policy**
  - **Rich theoretical analysis**
- **Chamley (1985 *Econometrica*) and Judd (1985 *J. Public Economics*)**
  - **Zero optimal long-run capital tax**
- **Chari, Christiano, and Kehoe (1991 *JMCB*)**
  - **First computational application of Lucas and Stokey**
- **Chari and Kehoe (1999 *Handbook of Macroeconomics*)**

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(MANY others….)

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- Adopt **primal formulation**
- Assume **commitment**
- Assumption **timeless perspective**

General issues to be aware of/take a stand on for any optimal policy analysis

OPTIMAL POLICY PROBLEMS: GENERAL FORM

- ❒ Set up economic environment
  - ❒ Household problem
  - ❒ Firm problem
  - ❒ Specification of government policy
    - ❒ Policy tools (monetary, fiscal, or both monetary and fiscal)
    - ❒ Government budget constraint(s)

KEY ISSUE:
Lump-sum tax available or not? →

Optimal Policy Problems: General Form

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- **Solve for/define private-sector equilibrium**
  - **For any arbitrary government policy**

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KEY ISSUE:
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- ❒ **Solve for/define private-sector equilibrium**
  - ❒ **For any arbitrary government policy**

- ❒ **Define social welfare criterion**
  - ❒ **Representative-consumer model: expected discounted lifetime utility**
  - ❒ **Heterogeneous-consumer model: not as obvious…how to weight?**

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  - Representative-consumer model: expected discounted lifetime utility
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- Choose government policy rules subject to all equilibrium conditions of economy
  - Basic idea: benevolent policy-maker is a "Social Planner" with the additional restrictions imposed by decentralized equilibrium

OPTIMAL POLICY PROBLEMS: ISSUES

- **Primal Formulation**
  - **Formulate Ramsey optimization problem in terms of only allocations**
    - **By eliminating govt policy variables (and prices) using equilibrium conditions**
    - **Given optimal allocation, construct (implied) policy instruments that support allocation (ala Ramsey (1927))**
      - **Long-standing approach in fiscal policy analysis …**
      - **… but harder to implement in NK monetary policy analysis**

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- Commitment
  - With initial state variable and/or forward-looking equilibrium conditions, policy FOCs for $t = 0$ differ from policy FOCs for $t > 0$
    - Assume government can bind itself to state-contingent policy paths for $t > 0$ (based on policy functions determined in $t = 0$)
    - (Opposite of commitment is discretion)

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  - **With initial state variable and/or forward-looking equilibrium conditions, policy FOCs for $t = 0$ differ from policy FOCs for $t > 0$**
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    - **(Opposite of commitment is discretion)**

- **Timeless Perspective**
  - **Set $t = 0$ state to the steady-state of the $t > 0$ policy FOCs**
  - **Ignoring transition dynamics associated with initially-suboptimal policies**
  - **Interpretation: the optimal policy has *already* been in operation for a long time**

BASICS OF RAMSEY MACRO FISCAL POLICY

- **Household problem**

$$\max_{\{c_t, n_t, k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(n_t) \right] \quad \textbf{s.t.}$$

$$c_t + k_{t+1} + \sum_j \frac{1}{R_t^j} b_{t+1}^j = (1 - \tau_t^n) w_t n_t + \left[1 + (1 - \tau_t^k)\left(r_t - \delta \right) \right] k_t + b_t$$

no lump-sum taxes

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no lump-sum taxes

- **FOCs yield**

$$u'(c_t) = \beta R_t^j u'(c_{t+1}^j) \quad \forall j$$

$$\frac{h'(n_t)}{u'(c_t)} = (1 - \tau_t^n) z_t f_n(k_t, n_t)$$

NOTE: **w/factor-market equilibrium**

$$u'(c_t) = \beta E_t \left\{ u'(c_{t+1}) \left(1 + (1 - \tau_{t+1}^k)(z_{t+1} \cdot f_k(k_{t+1}, n_{t+1}) - \delta) \right) \right\}$$

Basics of Ramsey Macro Fiscal Policy

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f\left(k_t, n_t\right)$$

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Basics of Ramsey Macro Fiscal Policy

- **Ramsey problem (Dual)**

$$\max_{\{c_t, n_t, k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(n_t) \right] \quad \textbf{s.t.}$$

$$c_t + g_t + k_{t+1} - (1-\delta) k_t = z_t f\left(k_t, n_t\right)$$

Sequence of Lagrange multipliers $\beta^t \lambda_t$

$$c_t + k_{t+1} + \sum_j \frac{1}{R_t^j} b_{t+1}^j = (1-\tau_t^n) w_t n_t + \left[1 + (1-\tau_t^k)\left(r_t - \delta\right)\right] k_t + b_t$$

$$u'(c_t) = \beta R_t^j u'(c_{t+1}^j) \quad \forall j$$

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Derivation

Basics of Ramsey Macro Fiscal Policy

- **Ramsey problem (Primal)**

$$\max_{\{c_t, n_t, k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(n_t) \right] \quad \textbf{s.t.}$$

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Sequence of Lagrange multipliers $\beta^t \lambda_t$

Present-value implementability constraint (PVIC): the PV GBC

$$E_0 \sum_{t=0}^{\infty} \beta^t \underbrace{\left[u'(c_t) \cdot c_t - h'(n_t) \cdot n_t \right]}_{\text{Define as } W(c_t,\, n_t)} = A_0$$

Single Lagrange multiplier μ

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Define as $W(c_t, n_t)$ (the bracketed term)

Single Lagrange multiplier μ

- **Ramsey FOCs (for $t > 0$, which sidesteps issue of taxation of $t = 0$ initial capital stock and other assets, of which A_0 is a function)**
  - **Commitment by Ramsey government to its $t > 0$ policies at $t = 0$**
  - **Discretionary Ramsey government does *not* commit to its $t > 0$ policies at $t = 0$**

Basics of Ramsey Macro Fiscal Policy

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$$u'(c_t^{RP}) - \lambda_t^{RP} + \mu \cdot W_c(c_t^{RP}, n_t^{RP}) = 0$$

$$-h'(n_t^{RP}) + \lambda_t^{RP} z_t f_n\left(k_t^{RP}, n_t^{RP}\right) + \mu \cdot W_n(c_t^{RP}, n_t^{RP}) = 0$$

$$-\lambda_t^{RP} + \beta E_t\left\{ \lambda_{t+1}^{RP} \left[z_{t+1} f_k(k_{t+1}^{RP}, n_{t+1}^{RP}) + 1 - \delta \right] \right\} = 0$$

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- **Social Planner FOCs**

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- **Social Planner FOCs**

$$u'(c_t^{SP}) - \lambda_t^{SP} = 0$$

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Evaluate at deterministic steady states

Basics of Ramsey Macro Fiscal Policy

- **Ramsey FOCs (for $t > 0$) at deterministic steady state**

$$u'(c^{RP}) - \lambda^{RP} + \mu \cdot \Omega_c(c^{RP}, n^{RP}) = 0$$

$$-h'(n^{RP}) + \lambda^{RP} z \cdot f_n\left(k^{RP}, n^{RP}\right) + \mu \cdot \Omega_n(c^{RP}, n^{RP}) = 0$$

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- **Social Planner FOCs at deterministic steady state**

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Basics of Ramsey Macro Fiscal Policy

- **Ramsey FOCs (for $t > 0$) at deterministic steady state**

$$u'(c^{RP}) - \lambda^{RP} + \mu \cdot W_c(c^{RP}, n^{RP}) = 0 \qquad (1)$$

$$-h'(n^{RP}) + \lambda^{RP} z \cdot f_n\left(k^{RP}, n^{RP}\right) + \mu \cdot W_n(c^{RP}, n^{RP}) = 0 \qquad (2)$$

$$-\cancel{\lambda^{RP}} + \beta \cancel{\lambda^{RP}} \left[z \cdot f_k(k^{RP}, n^{RP}) + 1 - \delta \right] = 0 \qquad (3)$$

- **Social Planner FOCs at deterministic steady state**

$$u'(c^{SP}) - \lambda^{SP} = 0 \qquad (4)$$

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- **(3) and (6) imply Ramsey-optimal k/n ratio = efficient k/n ratio**
  - **(Given maintained assumption of CRS production $f(.)$)**
  - **A crucial result!**
  - **Second-best k/n ratio = first-best k/n ratio**
  - **Chamley (1986 *ECTA*), Judd (1985 *JPub*) seminal references**

ZERO CAPITAL INCOME TAX

- What does this imply for Ramsey-optimal tax rates?
- Recall household optimization
  - With labor income tax and capital income tax (and no lump-sum taxes)

$$\max_{\{c_t, n_t, k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(n_t) \right] \quad \textbf{s.t.} \quad c_t + k_{t+1} = (1-\tau_t^n) w_t n_t + \left[1 + (1-\tau_t^k)(r_t - \delta) \right] k_t$$

  - Steady-state consumption-labor optimality (labor supply condition)

$$\frac{h'(n)}{u'(c)} = (1-\tau^n) z \cdot f_n(k,n)$$

= *w* in equilibrium

  - Steady-state consumption-savings optimality (capital Euler condition)

= *r* in equilibrium

$$u'(c) = \beta u'(c) \left(1 + (1-\tau^k)(z \cdot f_k(k,n) - \delta) \right)$$

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$$u'(c) = \beta u'(c)\left(1 + (1-\tau^k)(z \times f_k(k,n) - \delta)\right)$$

  - **Ramsey-optimal capital income tax rate = 0!**
  - **Don't tax intertemporal margin at all in the long run…**
  - **…even though Ramsey government has to raise revenue through distortionary taxes**

Positive Labor Income Tax

- What does this imply for Ramsey-optimal tax rates?

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- **All revenue must be raised through positive labor income tax**

- Two central macro-Ramsey fiscal policy results

DYNAMICS OF TAX RATES

- **Outside the steady state?**
- **Focus on labor income tax rate (simple to consider)**

  - **Consumption-labor optimality (labor supply condition)**

$$\frac{h'(n_t)}{u'(c_t)} = (1-\tau_t^n) z_t f_n(k_t, n_t)$$

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$$\frac{h'(n_t)}{u'(c_t)} = (1-\tau_t^n) z_t f_n(k_t, n_t)$$

= MRS_t **= MPN_t**

$$\rightarrow MRS_t = (1-\tau_t^n) MPN_t$$

- **Labor income tax is a wedge between labor supply and labor demand**

- **Along the business cycle?**
  - **Consider utility form** $u(c_t) - h(n_t) = \ln c_t - \frac{\kappa}{1+1/\iota} n_t^{1+1/\iota}$

ι is Frisch elasticity of labor supply with respect to real wage

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- **Compute first and second derivatives of *u*(.) and *h*(.)...**
  - **...which are needed to compute W_c(.) and W_n(.)**

- **Do some algebra combining the Ramsey FOCs ...**

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$$\underbrace{\kappa \cdot n_t^{1/\iota} \cdot c_t}_{= MRS_t} = \underbrace{\left[1 + \eta\left(\frac{1+\iota}{\iota}\right)\right]^{-1}}_{\text{= wedge between } MRS_t \text{ and } MPN_t} \cdot \underbrace{z_t f_n(k_t, n_t)}_{= MPN_t}$$

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$$\kappa \cdot n_t^{1/\iota} \cdot c_t = \left[1+\mu\left(\frac{1+\iota}{\iota}\right)\right]^{-1} \cdot z_t f_n(k_t, n_t)$$

- **$= MRS_t$** (left side)
- **= wedge between MRS_t and MPN_t** (bracketed term)
- **$= MPN_t$** (right side)

- **Wedge is a (endogenous...) constant between MRS and MPN in every time period**
  - **μ = 0 (the case of lump-sum taxes) ➔ wedge = 0**
  - **μ > 0 (the Ramsey case) ➔ wedge ≠ 0**

DYNAMICS OF TAX RATES

- **Along the business cycle?**
- **Wedge is a (endogenous…) constant between MRS and MPN in every time period…**

$$k \cdot n_t^{1/i} \cdot c_t = \left[1 + m\left(\frac{1+i}{i} \right) \right]^{-1} \cdot z_t f_n(k_t, n_t)$$

= MRS$_t$ | **= wedge between MRS$_t$ and MPN$_t$** | **= MPN$_t$**

- **…thus labor income tax rate is constant over time (for this utility form)**
  - **Nearly constant if move to slightly different *h*(*n*) function**

Dynamics of Tax Rates

- Along the business cycle?
- Wedge is a (endogenous…) constant between MRS and MPN in every time period…

$$k \cdot n_t^{1/i} \cdot c_t = \left[1 + m\left(\frac{1+i}{i} \right) \right]^{-1} \cdot z_t f_n(k_t, n_t)$$

= MRS_t

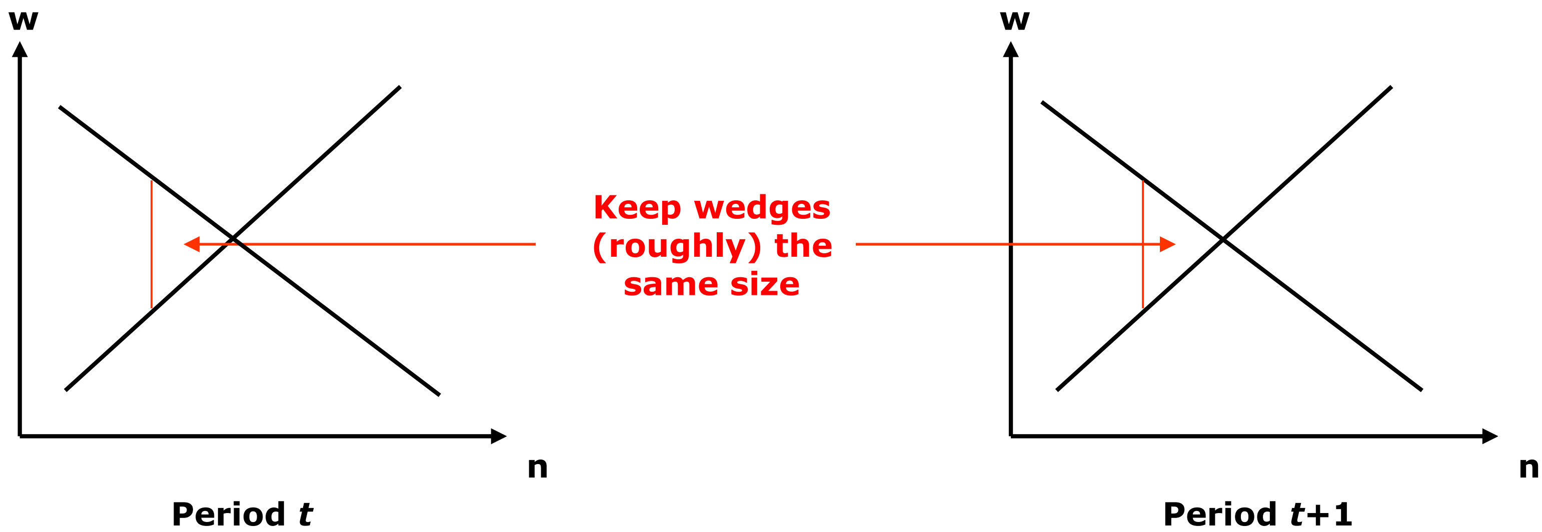
= wedge between MRS_t and MPN_t

= MPN_t

- …thus labor income tax rate is constant over time (for this utility form)
  - Nearly constant if move to slightly different ***h*(*n*)** function

- Labor income tax smoothing
  - Key Ramsey macro fiscal policy result
  - Keep deadweight losses constant across markets over time
    - aka wedges constant

TAX SMOOTHING VS. WEDGE SMOOTHING



- **Ramsey government smooths wedges across time**

$$MRS_t = \text{WEDGE}_t \cdot MPN_t \; \forall t$$

Wedge (Walrasian labor market)

Labor Search and Matching: General Equilibrium Wedges

February 15, 2019

Matching Efficiency

- **Social Planner**

$$\max_{\{c,n_t,s_t,v_t\}} E_0 \sum_{t=0}^{\infty} b^t \left[u(c_t) - h\left(lfp_t\right) \right]$$

$lfp_t \equiv (1-p_t)s_t + n_t$

s.t.

$$c_t + g_t + \gamma v_t = z_t n_t$$

Resource constraint

$$n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

Aggregate LOM for total employment

MATCHING EFFICIENCY

- **Social Planner**

$$\max_{\{c, n_t, s_t, v_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(lfp_t\right) \right]$$

$lfp_t \equiv (1-p_t)s_t + n_t$

s.t.

$$c_t + g_t + \gamma v_t = z_t n_t$$

Resource constraint

$$n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

Aggregate LOM for total employment

FOCs

(consider deterministic case)

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{\gamma m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

$$= \gamma\theta_t \frac{\xi}{1-\xi}$$

Static Efficiency Condition.

"Efficient Participation Condition"

Can instead derive directly off transformation frontier of model.

Matching Efficiency

- **Social Planner**

$$\max_{\{c,n_t,s_t,v_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(lfp_t\right) \right]$$

$lfp_t \equiv (1-p_t)s_t + n_t$

s.t.

$$c_t + g_t + \gamma v_t = z_t n_t$$

Resource constraint

$$n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

Aggregate LOM for total employment

FOCs

(consider deterministic case)

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{g m_s(s_t,v_t)}{m_v(s_t,v_t)} = g q_t \frac{x}{1-x}$$

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(\frac{\gamma}{m_v(s_{t+1},v_{t+1})}\right)\left(1-m_s(s_{t+1},v_{t+1})\right)}{\frac{\gamma}{m_v(s_t,v_t)} - z_t}$$

Static Efficiency Condition.

"Efficient Participation Condition"

Can instead derive directly off transformation frontier of model.

Intertemporal Efficiency Condition.

"Efficient Vacancies Condition"

Can instead derive directly off transformation frontier of model.

Transformation Frontier

- **Construct model-consistent transformation function**

"The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

$$MRT_{l,k}(y) = \frac{\partial F(y)/\partial y_l}{\partial F(y)/\partial y_k}$$

is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…

…A single-output technology is commonly described by means of a *production function f*(*z*)…Holding the level of output fixed, we can define the *marginal rate of technical substitution (*$MRTS_{l,k}$*)* … Note that $MRTS_{l,k}$ is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."

***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

Transformation Frontier

- **Ceteris paribus…**
- **Does one-unit decrease in $(1 - lfp_t)$ affect c_t?**
  - **If so, how?**
- **Does one-unit decrease in c_t affect c_{t+1}?**
  - **If so, how?**

Transformation Frontier

- Transformation function

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho) n_{t-1} + m(s_t, v_t)$$

TRANSFORMATION FRONTIER

- **Transformation function**

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

- → $v_t = \dfrac{z_t n_t - c_t}{\gamma}$
- **Insert into LOM for n_t to construct** $n_t - (1-\rho)n_{t-1} - m\left(s_t, \dfrac{z_t n_t - c_t}{\gamma}\right) = 0$
- **Use $lfp_t = (1-\rho)n_{t-1} + s_t$ to construct within-period transformation frontier**

$$\Gamma\left(c_t, lfp_t, n_t; \cdot\right) \equiv n_t - (1-\rho)n_{t-1} - m\left(lfp_t - (1-\rho)n_{t-1}, \frac{z_t n_t - c_t}{\gamma}\right) = 0$$

TRANSFORMATION FRONTIER

- **Transformation function**

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

- $\rightarrow v_t = \dfrac{z_t n_t - c_t}{\gamma}$
- **Insert into LOM for n_t to construct** $n_t - (1-\rho)n_{t-1} - m\left(s_t, \dfrac{z_t n_t - c_t}{\gamma}\right) = 0$
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$$\Gamma\left(c_t, lfp_t, n_t; \cdot\right) \equiv n_t - (1-\rho)n_{t-1} - m\left(lfp_t - (1-\rho)n_{t-1}, \frac{z_t n_t - c_t}{\gamma}\right) = 0$$

- **Use IFT to obtain static MRT (participation margin)**

$$MRT_{c_t, lfp_t} = -\frac{\Gamma_{lfp_t}}{\Gamma_{c_t}} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

STATIC **MRT between LFP and Walrasian good**

TRANSFORMATION FRONTIER – INTUITION

- **One-unit decrease in (1 – *lfp*$_t$) …**
- **→ increases *s*$_t$ by one unit …**
- **→ increases *n*$_t$ by** $m_s(s_t, v_t)$ **units …**

Transformation Frontier – Intuition

- **One-unit decrease in (1 – lfp_t) …**
- **→ increases s_t by one unit …**
- **→ increases n_t by $m_s(s_t, v_t)$ units …**
- **→ increases $z_t n_t$ by $z_t m_s(s_t, v_t)$ units …**

TRANSFORMATION FRONTIER – INTUITION

- **One-unit decrease in (1 – lfp_t) …**
- **→ increases s_t by one unit …**
- **→ increases n_t by $m_s(s_t, v_t)$ units …**
- **→ increases $z_t n_t$ by $z_t m_s(s_t, v_t)$ units …**
- **To hold n_t constant, v_t must *decrease* by $m_v(s_t, v_t)$ …**
- **… which decreases $z_t n_t$ by $\frac{z_t m_v(s_t, v_t)}{\gamma}$ units**

$$\Rightarrow MRT_{c_t, lfp_t} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

Transformation Frontier

- **Ceteris paribus…**
- **Does one-unit decrease in (1 – lfp_t) affect c_t?**

$$MRT_{c_t, lfp_t} = -\frac{\Gamma_{lfp_t}}{\Gamma_{c_t}} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

- **Does one-unit decrease in c_t affect c_{t+1}?**
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TRANSFORMATION FRONTIER

- **Transformation function**

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

- → $v_t = \dfrac{z_t n_t - c_t}{\gamma}$ **, then insert into LOM for n_t**
- → $n_t - (1-\rho)n_{t-1} - m\left(s_t, \dfrac{z_t n_t - c_t}{\gamma}\right) = 0$
- **Use $lfp_t = (1-\rho)n_{t-1} + s_t$ to construct within-period transformation frontier**

$$\Gamma\left(c_t, lfp_t, n_t; \cdot\right) \equiv n_t - (1-\rho)n_{t-1} - m\left(lfp_t - (1-\rho)n_{t-1}, \frac{z_t n_t - c_t}{\gamma}\right) = 0$$

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$$MRT_{c_t, lfp_t} = -\frac{\Gamma_{lfp_t}}{\Gamma_{c_t}} = \gamma\frac{m_s(s_t, v_t)}{m_v(s_t, v_t)} \qquad \frac{\partial n_t}{\partial c_t} = -\frac{\Gamma_{c_t}}{\Gamma_{n_t}} = -\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$$

STATIC **MRT between LFP and Walrasian good**

Marginal effect on n_t of a change in c_t …which has *intertemporal* consequences

TRANSFORMATION FRONTIER

- **Transformation function across periods**

$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho)n_t - m\left(lfp_{t+1} - (1-\rho)n_t, \frac{z_{t+1}n_{t+1} - c_{t+1}}{\gamma}\right) = 0$$

- **Use IFT to obtain intertemporal MRT**

$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}}$$

Transformation Frontier

- **Transformation function across periods**

$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho) n(c_t) - m\left(lfp_{t+1} - (1-\rho) n(c_t), \frac{z_{t+1} n_{t+1} - c_{t+1}}{\gamma} \right) = 0$$

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TRANSFORMATION FRONTIER

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- **Use IFT to obtain intertemporal MRT**

$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}} \qquad G_{c_{t+1}} = \frac{m_v(s_{t+1}, v_{t+1})}{\gamma}$$

Transformation Frontier

- **Transformation function across periods**

$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho) n(c_t) - m\left(lfp_{t+1} - (1-\rho) n(c_t), \frac{z_{t+1} n_{t+1} - c_{t+1}}{\gamma} \right) = 0$$

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$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}} \qquad G_{c_{t+1}} = \frac{m_v(s_{t+1}, v_{t+1})}{\gamma} \qquad G_{c_t} = -(1-\rho)\frac{\partial n_t}{\partial c_t} + (1-\rho) m_s(s_{t+1}, v_{t+1}) \frac{\partial n_t}{\partial c_t}$$

TRANSFORMATION FRONTIER

- **Transformation function across periods**

$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho)n(c_t) - m\left(lfp_{t+1} - (1-\rho)n(c_t), \frac{z_{t+1}n_{t+1} - c_{t+1}}{\gamma} \right) = 0$$

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$$\frac{\partial n_t}{\partial c_t} = -\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$$

And convert back into consumption units …

$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}} = \frac{(1-\rho)\left(\frac{\gamma}{m_v(s_{t+1}, v_{t+1})}\right)\left(1 - m_s(s_{t+1}, v_{t+1})\right)}{\frac{\gamma}{m_v(s_t, v_t)} - z_t}$$

Transformation Frontier – Intuition

- **One unit reduction in c_t …**
- **→ increases v_t by $1/\gamma$ units**
- **→ increases n_t by $\dfrac{m_v(s_t, v_t)}{\gamma}$ units**

Transformation Frontier – Intuition

- **One unit reduction in c_t …**
- **→ increases v_t by $1/\gamma$ units**
- **→ increases n_t by $\frac{m_v(s_t, v_t)}{\gamma}$ units**
- **→ *increases* c_t by $\frac{z_t m_v(s_t, v_t)}{\gamma}$ units**

Transformation Frontier – Intuition

- One unit reduction in c_t …
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- → increases n_t by $\dfrac{m_v(s_t, v_t)}{\gamma}$ units
- → ***increases*** c_t by $\dfrac{z_t m_v(s_t, v_t)}{\gamma}$ units
- … so resulting change in c_t is …

Must be netted out…

…in order to hold period-*t* output constant

$$\frac{\gamma - z_t m_v(s_t, v_t)}{\gamma} (< 1)$$

TRANSFORMATION FRONTIER – INTUITION

- One unit reduction in c_t …
- → increases v_t by $1/\gamma$ units
- → increases n_t by $\dfrac{m_v(s_t, v_t)}{\gamma}$ units
- → *increases* c_t by $\dfrac{z_t m_v(s_t, v_t)}{\gamma}$ units
- … so resulting change in c_t is …

Must be netted out…

…in order to hold period-*t* output constant

$$\frac{\gamma - z_t m_v(s_t, v_t)}{\gamma} (< 1)$$

- → increase in v_t by $\dfrac{1}{\gamma - z_t m_v(s_t, v_t)}$ units for ***ONE-UNIT*** DECREASE IN c_t

Transformation Frontier – Intuition

- **Increase in v_t by** $\dfrac{1}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_t, v_t)$ by** $\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**

TRANSFORMATION FRONTIER – INTUITION

- ❑ **Increase in v_t by** $\frac{1}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- ❑ **→ increase in $m(s_t, v_t)$ by** $\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- ❑ **→ increase in $m(s_{t+1}, v_{t+1})$ by** $(1-\rho)\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**

TRANSFORMATION FRONTIER – INTUITION

- **Increase in v_t by** $\dfrac{1}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_t, v_t)$ by** $\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_{t+1}, v_{t+1})$ by** $(1-\rho)\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **To hold n_{t+1} constant, s_{t+1} must *decrease* by $m_s(s_{t+1}, v_{t+1})$ …**

TRANSFORMATION FRONTIER – INTUITION

- **Increase in v_t by** $\dfrac{1}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_t,v_t)$ by** $\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_{t+1},v_{t+1})$ by** $(1-\rho)\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **To hold n_{t+1} constant, s_{t+1} must *decrease* by $m_s(s_{t+1},v_{t+1})$ …**
- **→ increase in v_{t+1} by** $(1-\rho)\left(\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}\right)\left(1 - m_s(s_{t+1}, v_{t+1})\right)$ **units …**

TRANSFORMATION FRONTIER – INTUITION

- **Increase in v_{t+1} by** $(1-\rho)\left(\frac{m_v(s_t,v_t)}{\gamma - z_t m_v(s_t,v_t)}\right)\left(1-m_s(s_{t+1},v_{t+1})\right)$ **units …**

- **→ increases by c_{t+1}** $\frac{(1-\rho)\left(\frac{\gamma}{m_v(s_t,v_t)}\right)\left(1-m_s(s_{t+1},v_{t+1})\right)}{\frac{\gamma}{m_v(s_t,v_t)} - z_t}$ **units**

Transformation Frontier

- **Ceteris paribus…**
- **Does one-unit decrease in (1 – lfp_t) affect c_t?**

$$MRT_{c_t, lfp_t} = -\frac{\Gamma_{lfp_t}}{\Gamma_{c_t}} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

- **Does one-unit decrease in c_t affect c_{t+1}?**

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Matching Efficiency

- Efficiency characterized by

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{\gamma m_s(s_t, v_t)}{m_v(s_t, v_t)} = \gamma \theta_t \frac{\xi}{1-\xi}$$

= **Static MRT$_t$**

Static Efficiency Condition.

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(\frac{\gamma}{m_v(s_{t+1}, v_{t+1})}\right)(1 - m_s(s_{t+1}, v_{t+1}))}{\frac{\gamma}{m_v(s_t, v_t)} - z_t}$$

= **Intertemporal MRT$_t$**

Intertemporal Efficiency Condition.

MATCHING EFFICIENCY

- **Efficiency characterized by**

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{g m_s(s_t, v_t)}{m_v(s_t, v_t)} = g q_t \frac{x}{1-x}$$

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Static Efficiency Condition.

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= Intertemporal MRT$_t$

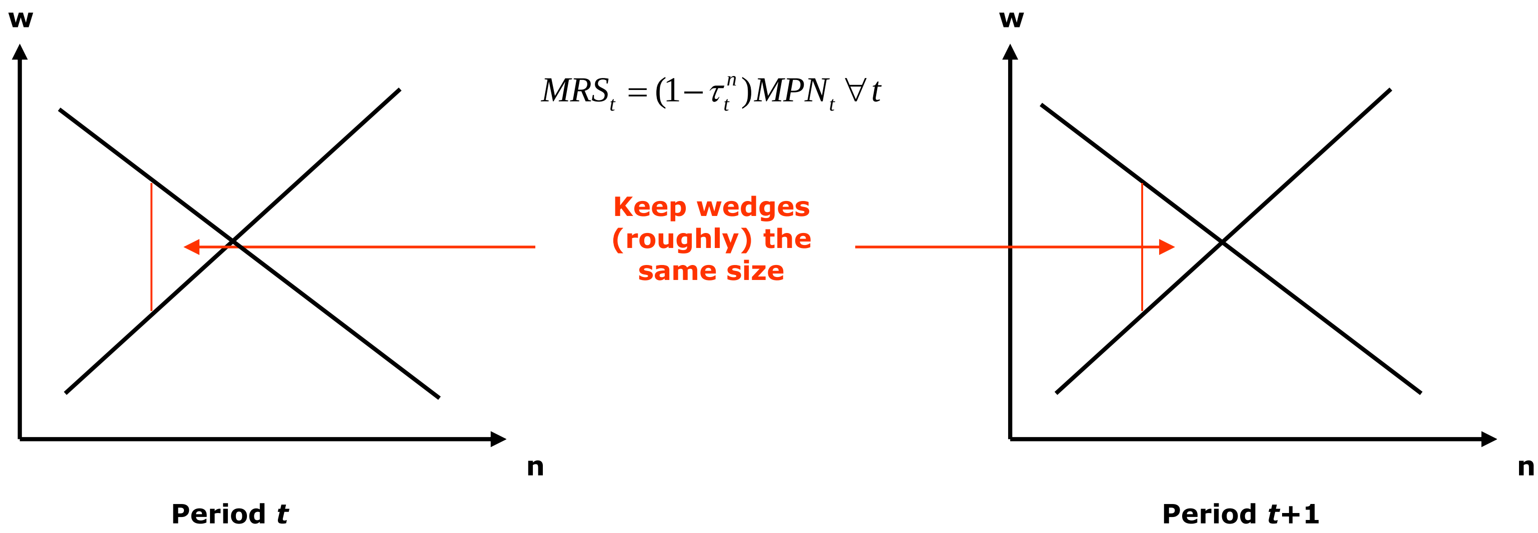
Intertemporal Efficiency Condition.

- **Ramsey theory: stabilizing THESE wedges is optimal**
  - **MRTs in DSGE search and matching model: Arseneau and Chugh (2012 *JPE*)**
- **Contribution to understanding efficiency in DGE models with "entry" margins**
  - **MRTs in new monetarist models: Aruoba and Chugh (2010 *JET*)**
  - **MRTs in customer market models: Arseneau, Chahrour, Chugh, and Finkelstein Shapiro (2015 *JMCB*)**
  - **MRTs in endogenous product variety framework: Chugh and Ghironi (2018)**

Tax Smoothing in Frictional Labor Markets

February 15, 2019

TAX SMOOTHING



- **Ramsey government smooths wedges across time**
- **Result and intuition depend on neoclassical view of labor markets**
  - **Labor tax is the only wedge → tax-smoothing is wedge-smoothing**

- **Question: Is tax smoothing optimal in search and matching labor markets?**

Overview of Model

- Infinitely-lived representative household, measure one of members
  - Employed members
  - Unemployed members
  - Members outside the labor force ("leisure")

Full consumption insurance – standard in DSGE labor search models

Incompleteness of government debt markets NOT driving our results (Aiyagari et al (2002 *JPE*))

- Exogenous stochastic government spending
  - Financed via labor income taxation and one-period real state-contingent debt
  - Government provides unemployment benefits
  - Government provides vacancy subsidies
    - For completeness of tax instruments (Ramsey issue)

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- Infinitely-lived representative household, measure one of members
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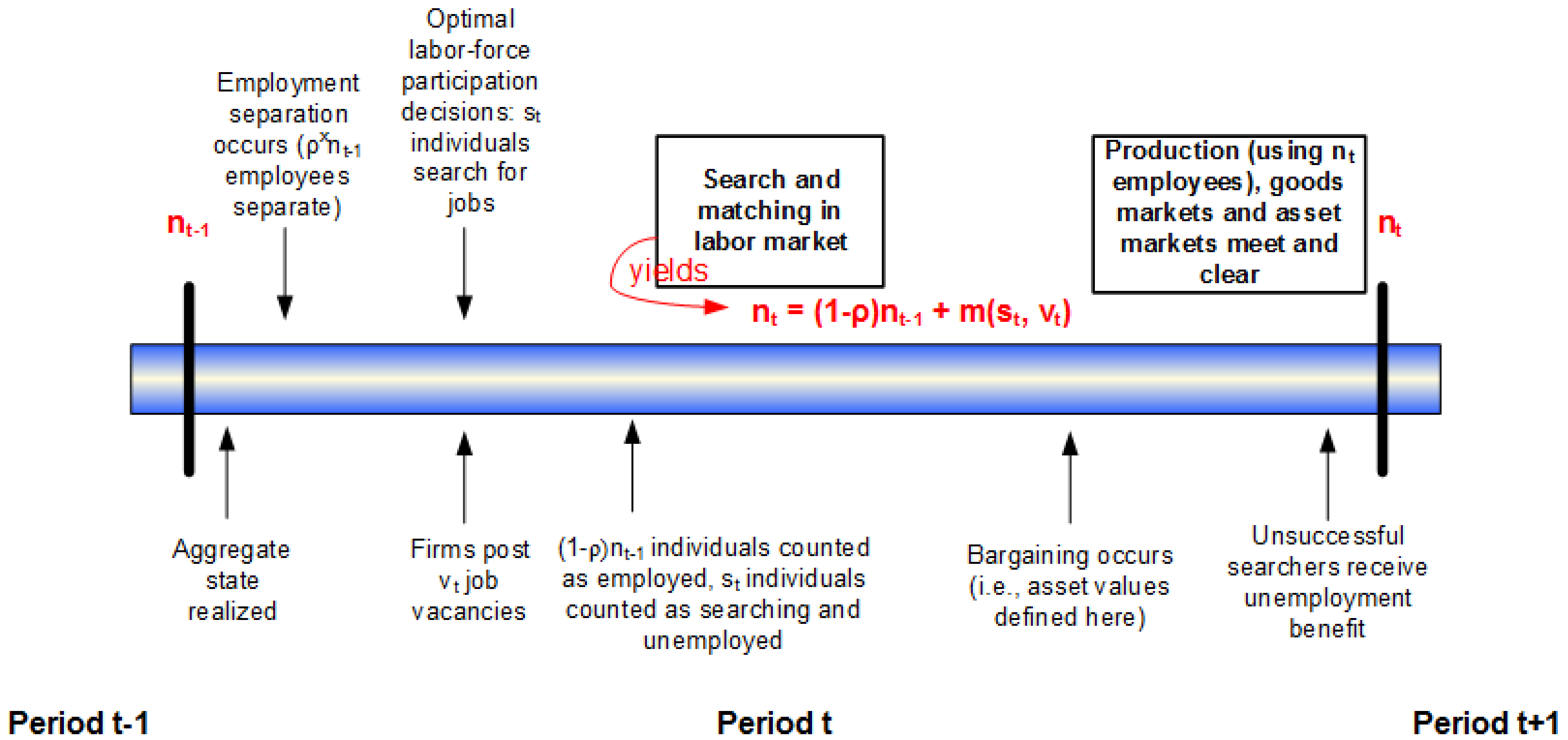
  - Financed via labor income taxation and one-period real state-contingent debt
  - Government provides unemployment benefits
  - Government provides vacancy subsidies
    - For completeness of tax instruments (Ramsey issue)

- Labor market with matching frictions and wage-setting frictions

- Only an extensive labor margin, no intensive labor margin

- Timing: "instantaneous production"

Overview of Model



- **Unemployed are the unsuccessful searchers: $ue_t = (1-p_t)s_t$**
  - **p_t = probability an individual finds a job and begins working immediately**

HOUSEHOLD OPTIMIZATION

- **Maximize expected lifetime utility**

$$\max_{\{c,n_t,s_t,b_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\big((1-p_t)s_t + n_t\big) \right]$$

disutility of employment + unsuccessful search

Household Optimization

- **Maximize expected lifetime utility**

$$\max_{\{c, n_t, s_t, b_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\big((1-p_t)s_t + n_t\big) \right]$$

disutility of employment + unsuccessful search

s.t.

$$c_t + b_t = n_t(1-\tau_t^n)w_t + (1-p_t)s_t\chi + R_t b_{t-1} + (1-\tau^d)d_t$$

Flow budget constraint

measure *n* earn after-tax wage income

measure *ue* = (1-*p*)*s* receive ue benefit χ (government financed)

Baseline analysis: set $\tau^d = 1$ → no *profit*-taxation issues driving results

HOUSEHOLD OPTIMIZATION

- Maximize expected lifetime utility

$$\max_{\{c,n_t,s_t,b_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left((1-p_t)s_t + n_t\right) \right]$$

disutility of employment + unsuccessful search

s.t.

$$c_t + b_t = n_t(1-\tau_t^n)w_t + (1-p_t)s_t\chi + R_t b_{t-1} + (1-\tau^d)d_t$$

Flow budget constraint

measure n earn after-tax wage income

measure $ue = (1-p)s$ receive ue benefit χ (government financed)

Baseline analysis: set $\tau^d = 1$ → no *profit*-taxation issues driving results

$$n_t = (1-\rho)n_{t-1} + s_t p_t$$

Perceived LOM for employment ("instantaneous production")

(exogenous) measure of pre-existing employment relationships terminate

flow of new employment relationships = measure of searchers s_t x probability a searcher successfully lands a job

↓ **FOCs with respect c_t, n_t, s_t, b_t**

HOUSEHOLDS

- **Household LFP condition (the labor supply condition)**

$$\frac{h'(lfp_t)}{u'(c_t)} = p_t\left[(1-\tau_t^n)w_t + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{1-p_{t+1}}{p_{t+1}}\right)\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})}-\chi\right)\right\}\right] + (1-p_t)\chi$$

  - **MRS between lfp_t and c_t = expected payoff of searching**
    - **Unemployment benefit (with probability $1 - p_t$)**
    - **After-tax wage + continuation value (with probability p_t)**

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  - **MRS between *lfp_t* and *c_t* = expected payoff of searching**
    - **Unemployment benefit (with probability 1 – *p_t*)**
    - **After-tax wage + continuation value (with probability *p_t*)**

To recover standard labor supply function (e.g., RBC)

1. **ρ = 1 (all employment relationships terminate at end of every period)**
2. **p = 1 (probability a searcher finds a job)**
3. **χ = 0 (no ue benefit because no notion of "ue")**

$$\frac{h'(lfp_t)}{u'(c_t)} = (1-\tau_t^n)w_t$$

FIRMS

- **Production**
  - **Requires a matched job-worker pair: posting cost γ per vacancy**
  - **Individual job i produces $y_{it} = z_t$**
  - **Aggregate output $y_t = n_t z_t$ (symmetry across jobs)**

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- **Dynamic profit-maximization problem**

$$\max_{\{n_t, v_t\}} \sum_{t=0}^{\infty} \Xi_{t|0} \left[z_t n_t - w_t n_t - (1 - \tau_t^s)\gamma v_t \right]$$

Ensures completeness of tax instruments

$$n_t = (1-\rho)n_{t-1} + v_t q_t$$

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(exogenous) measure of pre-existing employment relationships terminate

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Firm's perceived LOM for total employment ("instantaneous hiring")

(exogenous) measure of pre-existing employment relationships terminate

flow of new employment relationships = # job-openings x probability an opening attracts a searching individual

- **Vacancy-creation condition**

$$\frac{\gamma(1-\tau_t^s)}{q_t} = z_t - w_t + (1-\rho)E_t\left[\Xi_{t+1|t} \frac{\gamma(1-\tau_{t+1}^s)}{q_{t+1}} \right]$$

cost of posting vacancy (inclusive of subsidy or tax)

benefit of posting vacancy

LABOR MARKET

- Labor-market tightness $\theta_t = v_t / u_t$
  - Important aggregate variable in matching-based models
  - Matching probabilities p and q depend only on θ given CRTS matching
  - Key statistic for matching efficiency

Labor Market

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  - **Matching probabilities p and q depend only on θ given CRTS matching**
  - **Key statistic for matching efficiency**

- **Matching function** $m(s_t, v_t) = \psi s_t^{\xi} v_t^{1-\xi}$
- **LOM for aggregate employment** $n_t = (1-\rho) n_{t-1} + m(s_t, v_t)$

- **Nash bargaining over wage payment solves**

$$\max_{w_t} \left(\mathbf{W}_t - \mathbf{U}_t\right)^{h} \mathbf{J}_t^{1-h} \longrightarrow \frac{\mathbf{W}_t - \mathbf{U}_t}{1-t_t^n} = \frac{h}{1-h}\mathbf{J}_t$$

Gain to household of successfully forming another employment relationship

Value to firm of hiring another worker

$$\longrightarrow w_t = hz_t + (1-h)\frac{c}{1-t_t^n} + h(1-r)E_t\left\{\mathrm{X}_{t+1|t}\left[1-(1-p_{t+1})\frac{1-t_{t+1}^n}{1-t_t^n}\right]\frac{g(1-t_{t+1}^s)}{q_{t+1}}\right\}$$

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Although main results also hold if we discard Nash bargaining and assume ad-hoc real wage rigidity:

w_t = *w*bar in every period *t*

$$\max_{w_t} \left(\mathbf{W}_t - \mathbf{U}_t\right)^{h} \mathbf{J}_t^{1-h} \longrightarrow \frac{\mathbf{W}_t - \mathbf{U}_t}{1-t_t^n} = \frac{h}{1-h}\mathbf{J}_t$$

Gain to household of successfully forming another employment relationship ($\mathbf{W}_t - \mathbf{U}_t$)

Value to firm of hiring another worker ($\mathbf{J}_t$)

$$\longrightarrow w_t = h z_t + (1-h)\frac{c}{1-t_t^n} + h(1-r)E_t\left\{X_{t+1|t}\left[1-(1-p_{t+1})\frac{1-t_{t+1}^n}{1-t_t^n}\right]\frac{g(1-t_{t+1}^s)}{q_{t+1}}\right\}$$

GOVERNMENT AND RESOURCE FRONTIER

- **Exogenous government spending financed via**
  - **Labor income tax**
  - **One-period state contingent real debt**

$$t_t^n w_t n_t + b_t + t^d d_t = g_t + R_t b_{t-1} + (1- p_t) s_t c + t_t^s g v_t$$

  - **Government provides unemployment benefits**
    - **Rather than assuming χ is "home production"**

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- **Resource constraint**

$$c_t + g_t + \gamma v_t = z_t n_t$$

  - **= govt budget constraint + hh budget constraint**
  - **Assuming χ is govt-financed allows it to drop out of resource constraint**
    - **Makes model more comparable to existing Ramsey models**

Government and Resource Frontier

- Exogenous government spending financed via
  - Labor income tax
  - One-period state contingent real debt

$$\tau_t^n w_t n_t + b_t + \tau^d d_t = g_t + R_t b_{t-1} + (1-p_t)s_t \chi + \tau_t^s \gamma v_t$$

  - Government provides unemployment benefits
    - Rather than assuming χ is "home production"

- Resource constraint

$$c_t + g_t + \gamma v_t = z_t n_t$$

  - = govt budget constraint + hh budget constraint
  - Assuming χ is govt-financed allows it to drop out of resource constraint
    - Makes model more comparable to existing Ramsey models

- Precise nature of χ (ue benefit? home production? value of leisure?) not typically specified in DSGE matching models
  - Model articulates both ue benefit and value of leisure

PRIVATE-SECTOR EQUILIBRIUM

- **Stochastic processes** $\{c_t, n_t, s_t, w_t, \theta_t, R_t, b_t\}_{t=0}^{\infty}$ **that satisfy**
  - **Household's bond Euler equation**
  - **Vacancy-creation condition**
  - **Labor force participation condition**
  - **Nash wage outcome**
  - **Law of motion for employment** $n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$
  - **Present-value government budget constraint (key condition in Ramsey models)**
  - **Resource constraint** $c_t + g_t + \gamma v_t = z_t n_t$
  - **Given processes** $\{g_t, z_t, \tau_t^n, \tau_t^s\}_{t=0}^{\infty}$

Standard conditions in basic Ramsey models

Ramsey Problem

- **Ramsey problem – "Hybrid" Approach**

$$\max_{\{c_t, n_t, s_t, v_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(lfp_t) \right]$$

$lfp_t \equiv (1-p_t)s_t + n_t$

s.t.

$$c_t + g_t + \gamma \cdot v_t = z_t n_t$$

Lagrange multiplier for each *t*

PVIC

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u'(c_t) \cdot c_t - h'(lfp_t) \cdot lfp_t - u'(c_t) \cdot (1-\tau^d) \cdot d_t \right] = A_0$$

Single Lagrange multiplier μ

Ramsey Problem

- ❑ **Ramsey problem – "Hybrid" Approach**

$$\max_{\{c_t, n_t, s_t, v_t, w_t, \tau_t^n, \tau_t^s\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(lfp_t) \right]$$

$lfp_t \equiv (1\text{-}p_t)s_t + n_t$

s.t.

$$c_t + g_t + \gamma \cdot v_t = z_t n_t$$

Lagrange multiplier for each t

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Single Lagrange multiplier μ

$$n_t = (1-\rho) n_{t-1} + m(s_t, v_t)$$

$$\frac{h'(lfp_t)}{u'(c_t)} = p_t \left[(1- t_t^n) w_t + (1- r) E_t \left\{ X_{t+1|t} \left(\frac{1- p_{t+1}}{p_{t+1}} \right) \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - c \right) \right\} \right] + (1- p_t) c$$

$$\frac{g(1- t_t^s)}{q_t} = z_t - w_t + (1- r) E_t \left[X_{t+1|t} \frac{g(1- t_{t+1}^s)}{q_{t+1}} \right]$$

$$w_t = h z_t + (1- h) \frac{c}{1- t_t^n} + h(1- r) E_t \left\{ X_{t+1|t} \left[1- (1- p_{t+1}) \frac{1- t_{t+1}^n}{1- t_t^n} \right] \frac{g(1- t_{t+1}^s)}{q_{t+1}} \right\}$$

NOT captured in PVIC

Lagrange multiplier for each t

CALIBRATION

- **Baseline calibration**
  - **So that exogenous policy (non-Ramsey) equilibrium broadly matches U.S. labor market fluctuations**

  - **Preferences and key parameters**

$$u(c_t) - h(lfp_t) = \ln c_t - \frac{k}{1+1/\iota} lfp_t^{1+1/\iota}$$

  - **Participation (labor supply) elasticity (ι = 0.18)**
  - **Low worker bargaining power (η = 0.05)**
  - **High unemployment benefit (98% of real wage)**

The two key parameters of HM calibration

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  - **Low worker bargaining power (η = 0.05)**
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The two key parameters of HM calibration

- **Rest of parameters, matching-related and otherwise, standard**
  - **β = 0.99**
  - **ρ = 0.10**
  - **ξ = 0.40**
  - **AR(1) parameters for LOMs for TFP and government spending**
  - **Etc.**

DYNAMICS

Ramsey | Exogenous Policy Benchmark | Data ← Gertler and Trigari (2009 *JPE*)

		Calibration		Calibration		
		<u>HM</u>	<u>0% and Hosios</u>	<u>HM</u>		
Labor Tax Rate	Mean					22%
	Rel SD					1.4
Market tightness (θ)	Rel SD					11.3
Vacancies	Rel SD					6.3
Unemployment	Rel SD					5.2
LFP	Rel SD					0.20
Real wage	Rel SD					0.52
Static wedge	SD (%)					
Intertemporal wedge	SD (%)					

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		Calibration		Calibration		
		<u>HM</u>	<u>0% and Hosios</u>	<u>HM</u>		
Labor Tax Rate	Mean			22%		22%
	Rel SD			1.4		1.4
Market tightness (θ)	Rel SD			10.9		11.3
Vacancies	Rel SD			6.9		6.3
Unemployment	Rel SD			5.4		5.2
LFP	Rel SD			0.20		0.20
Real wage	Rel SD			0.28		0.52
Static wedge	SD (%)					
Intertemporal wedge	SD (%)					

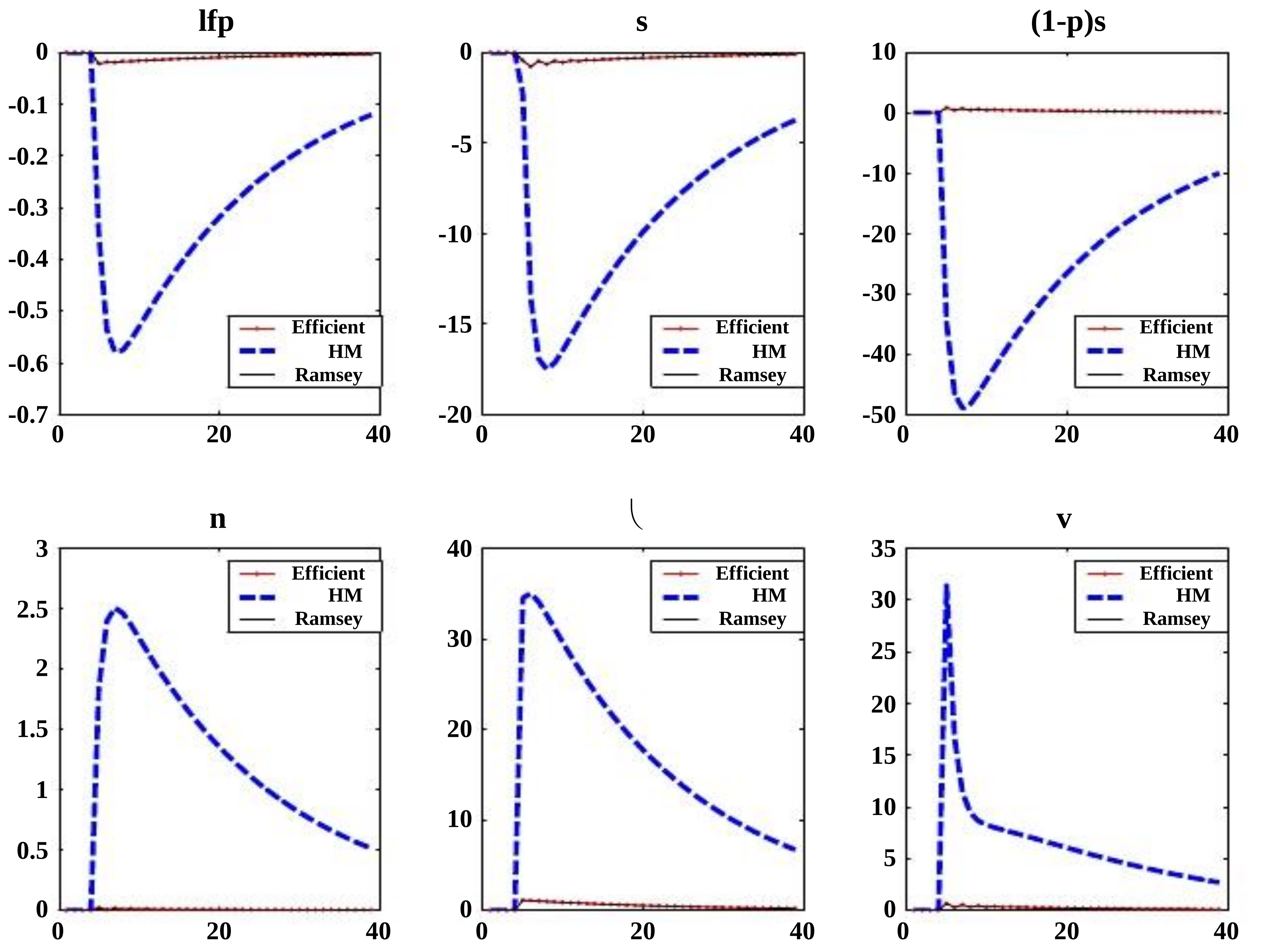
DYNAMICS

Ramsey | Exogenous Policy Benchmark | Data ← Gertler and Trigari (2009 *JPE*)

		Calibration		Calibration		
		HM	0% and Hosios	HM		
Labor Tax Rate	Mean	11%		22%		22%
	Rel SD	5.6		1.4		1.4
Market tightness (θ)	Rel SD	1.1		10.9		11.3
Vacancies	Rel SD	1.3		6.9		6.3
Unemployment	Rel SD	1.4		5.4		5.2
LFP	Rel SD	0.13		0.20		0.20
Real wage	Rel SD	0.50		0.28		0.52
Static wedge	SD (%)					
Intertemporal wedge	SD (%)					

DYNAMICS

- **Ramsey fluctuations IDENTICAL to efficient fluctuations for ANY (η, χ) pair**
  - **In terms of fluctuations around a given steady state**
  - **Steady-state levels of (τ^n, τ^s) depend on (η, χ) pair**


lfp
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
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20
40
Efficient
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s
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-10
-15
-20
0
20
40
Efficient
HM
Ramsey
(1-p)s
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  - **By appropriately adjusting (τ^n, τ^s) over the business cycle**

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Gertler and Trigari (2009 *JPE*)

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- **Wedge dynamics?**
  - **Ramsey smooths both static wedge....**
  - **...and intertemporal wedge**

DYNAMICS

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LFP	Rel SD	0.13	0.13	0.20		0.20
Real wage	Rel SD	0.50	1.1	0.28		0.52
Static wedge	SD (%)	0.08	0	22.9	0.66	
Intertemporal wedge	SD (%)	0	0	12.3	0.63	

STATIC AND INTERTEMPORAL CONDITIONS

- Efficiency characterized by

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{g m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

$$= g q_t \frac{x}{1-x}$$

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(\frac{\gamma}{m_v(s_{t+1}, v_{t+1})}\right)\left(1 - m_s(s_{t+1}, v_{t+1})\right)}{\frac{\gamma}{m_v(s_t, v_t)} - z_t}$$

- Decentralized equilibrium conditions characterized by

$$\frac{h'(lfp_t)}{u'(c_t)} = \left[\frac{\chi(1-\xi)}{\gamma \cdot \xi \cdot \theta_t} + (1-\tau_t^n)(1-\tau_t^s)\frac{\eta(1-\xi)}{\xi(1-\eta)}\right]\gamma\theta_t \frac{\xi}{1-\xi}$$

= wedge between static MRS$_t$ and static MRT$_t$

To obtain zero static wedge in every period, need $\tau^n = \tau^s = 0$ in every period, $\eta = \xi$, $\chi = 0$

STATIC AND INTERTEMPORAL CONDITIONS

- **Efficiency characterized by**

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{g m_s(s_t, v_t)}{m_v(s_t, v_t)} = g q_t \frac{x}{1-x}$$

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= wedge between static MRS_t and static MRT_t

(See eqn. (29) for intertemporal wedge)

To obtain zero static wedge in every period, need $\tau^n = \tau^s = 0$ in every period, $\eta = \xi$, $\chi = 0$

To obtain zero intertemporal wedge in every period, need $\tau^n = \tau^s = 0$ in every period, $\eta = \xi$, $\chi = 0$

CONCLUSIONS

- **Labor tax smoothing not optimal in DSGE search and matching model**
  - **Calibrated to match key labor market dynamics under exogenous tax policy**
  - **Rigid real wage (delivered through Nash-Hosios bargaining as benchmark) the important feature of the model**
  - **Result conditional on Cobb-Douglas m(.) and Nash bargaining**

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- **But wedge smoothing IS optimal**
  - **Basic Ramsey theory**
  - **Irrespective of wage model and particular matching process**
- **Ramsey fluctuations in allocations efficient regardless of calibration**

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  - Result conditional on Cobb-Douglas m(.) and Nash bargaining
- But wedge smoothing IS optimal
  - Basic Ramsey theory
  - Irrespective of wage model and particular matching process
- Ramsey fluctuations in allocations efficient regardless of calibration
- Welfare-relevant notions of wedges
  - Matching-model concepts of efficiency and MRTs for use in virtually any matching application
- Could think of "labor wedge" as featuring both static and intertemporal dimensions
  - Use as framework to empirically measure labor wedges

APPENDIX:
GENERAL EQUILIBRIUM WEDGES

FEBRUARY 15, 2019

Transformation Function

- **Construct model-consistent transformation function**

TRANSFORMATION FUNCTION

- **Construct model-consistent transformation function**

"The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

$$MRT_{l,k}(y) = \frac{\partial F(y)/\partial y_l}{\partial F(y)/\partial y_k}$$

is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…

***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

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…A single-output technology is commonly described by means of a *production function f*(*z*)…Holding the level of output fixed, we can define the *marginal rate of technical substitution* (*MRTS*$_{l,k}$) … Note that *MRTS*$_{l,k}$ is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."

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***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

- **RBC model**

- **Does one-unit decrease in 1-n_t affect c_t?**
  - **If so, how?**

- **Does one-unit decrease in c_t affect c_{t+1}?**
  - **If so, how?**

TRANSFORMATION FUNCTION

- **Transformation function of RBC model**

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f\left(k_t, n_t\right)$$

Goods resource constraint

- **One-unit decrease in 1-n_t → one-unit increase in n_t**

- **One-unit increase in n_t → output increases by $z_t f_n(k_t, n_t)$ units**

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- **One-unit decrease in 1-n_t → one-unit increase in n_t**
- **One-unit increase in n_t → output increases by $z_t f_n(k_t, n_t)$ units**
- **Increase of output by $z_t f_n(k_t, n_t)$ units → c_t increases by $z_t f_n(k_t, n_t)$ units**

$$MRT_{c_t, n_t} \equiv z_t f_n(k_t, n_t)$$

TRANSFORMATION FUNCTION

- **Construct model-consistent transformation function**

"The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

$$MRT_{l,k}(y) = \frac{\partial F(y)/\partial y_l}{\partial F(y)/\partial y_k}$$

is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…

…A single-output technology is commonly described by means of a *production function f*(*z*)…Holding the level of output fixed, we can define the *marginal rate of technical substitution (MRTS$_{l,k}$)* … Note that *MRTS$_{l,k}$* is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."

***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

- **RBC model**

- **Does one-unit decrease in 1-n_t affect c_t?**
  - $MRT_{c_t, n_t} \equiv z_t f_n(k_t, n_t)$

- **Does one-unit decrease in c_t affect c_{t+1}?**
  - **If so, how?**

Transformation Function

- **Transformation function of RBC model (between t and $t+1$)**

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t) \qquad c_{t+1} + g_{t+1} + k_{t+2} - (1-\delta)k_{t+1} = z_{t+1} f(k_{t+1}, n_{t+1})$$

- **One-unit decrease in c_t → one-unit increase in k_{t+1}**
- **One-unit increase in k_{t+1} → output increases by $z_{t+1}f_k(k_{t+1},n_{t+1})$ units**
- **Increase of output by $z_{t+1}f_k(k_{t+1},n_{t+1})$ units**

TRANSFORMATION FUNCTION

- **Transformation function of RBC model (between *t* and *t* + 1)**

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t) \qquad c_{t+1} + g_{t+1} + k_{t+2} - (1-\delta)k_{t+1} = z_{t+1} f(k_{t+1}, n_{t+1})$$

- **One-unit decrease in c_t → one-unit increase in k_{t+1}**
- **One-unit increase in k_{t+1} → output increases by $z_{t+1}f_k(k_{t+1},n_{t+1})$ units**
- **Increase of output by $z_{t+1}f_k(k_{t+1},n_{t+1})$ units**
- **→ c_{t+1} increases by**

$$MRT_{c_t, c_{t+1}} \equiv 1 + z_{t+1} f_k(k_{t+1}, n_{t+1}) - \delta$$

TRANSFORMATION FUNCTION

- **Construct model-consistent transformation function**

"The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

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***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

- **RBC model**

- **Does one-unit decrease in 1-n_t affect c_t?**
  - $MRT_{c_t,n_t} \equiv z_t f_n(k_t, n_t)$

- **Does one-unit decrease in c_t affect c_{t+1}?**
  - $MRT_{c_t,c_{t+1}} \equiv 1 + z_{t+1} f_k(k_{t+1}, n_{t+1}) - \delta$

APPENDIX:
RAMSEY PRESENT VALUE IMPLEMENTABILITY CONSTRAINT

FEBRUARY 15, 2019

PVIC

$$c_t + \sum_j \frac{1}{R_t^j} b_{t+1}^j + k_{t+1} = (1-\tau_t^n) w_t n_t + \left[1 + (1-\tau_t^k)(r_t - \delta)\right] k_t + b_t$$

$$u'(c_t) = \beta R_t^j u'(c_{t+1}^j) \quad \forall j$$

$$\frac{h'(n_t)}{u'(c_t)} = (1-\tau_t^n) w_t$$

$$u'(c_t) = \beta E_t \left\{ u'(c_{t+1}) \left(1 + (1-\tau_{t+1}^k)(r_{t+1} - \delta) \right) \right\}$$

PVIC

$$\sum_{t=0}^{\infty}\beta^t\lambda_t\cdot c_t+\sum_{t=0}^{\infty}\beta^t\sum_j\lambda_t\cdot\frac{1}{R_t^j}b_{t+1}^j+\sum_{t=0}^{\infty}\beta^t\lambda_t k_{t+1}$$

$$=\sum_{t=0}^{\infty}\beta^t\lambda_t\cdot(1-\tau_t^n)w_t n_t+\sum_{t=0}^{\infty}\beta^t\lambda_t\left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t+\sum_{t=0}^{\infty}\beta^t\lambda_t\cdot b_t$$

$$u'(c_t)=\beta R_t^j u'(c_{t+1}^j)\quad\forall j$$

$$\frac{h'(n_t)}{u'(c_t)}=(1-\tau_t^n)w_t$$

$$u'(c_t)=\beta E_t\left\{u'(c_{t+1})\left(1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right)\right\}$$

PVIC

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot c_t + \sum_{t=0}^{\infty}\beta^t \sum_j u'(c_t)\cdot\frac{1}{R_t^j}b_{t+1}^j + \sum_{t=0}^{\infty}\beta^t u'(c_t)k_{t+1}$$

$$= \sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot(1-\tau_t^n)w_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot b_t$$

$$u'(c_t) = \beta R_t^j u'(c_{t+1}^j) \quad \forall j$$

$$\frac{h'(n_t)}{u'(c_t)} = (1-\tau_t^n)w_t$$

$$u'(c_t) = \beta E_t\left\{u'(c_{t+1})\left(1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right)\right\}$$

PVIC

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot c_t+\sum_{t=0}^{\infty}\sum_{j}\beta^{t+1}u'(c_{t+1}^j)\cdot\frac{R_t^j}{R_t^j}b_{t+1}^j+\sum_{t=0}^{\infty}\beta^t u'(c_t)k_{t+1}$$

$$=\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot(1-\tau_t^n)w_t n_t+\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t+\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot b_t$$

$$\frac{h'(n_t)}{u'(c_t)}=(1-\tau_t^n)w_t$$

$$u'(c_t)=\beta E_t\left\{u'(c_{t+1})\left(1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right)\right\}$$

PVIC

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot c_t+\sum_{t=0}^{\infty}\sum_{j}\beta^{t+1}u'(c_{t+1}^j)\cdot b_{t+1}^j+\sum_{t=0}^{\infty}\beta^t u'(c_t)k_{t+1}$$

$$=\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot\frac{h'(n_t)}{u'(c_t)}\cdot n_t+\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t+\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot b_t$$

$$u'(c_t)=\beta E_t\left\{u'(c_{t+1})\left(1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right)\right\}$$

PVIC

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot c_t+\sum_{t=0}^{\infty}\sum_j \beta^{t+1}u'(c_{t+1}^j)\cdot b_{t+1}^j+\sum_{t=0}^{\infty}\beta^{t+1}u'(c_{t+1})\cdot\left(1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right)\cdot k_{t+1}$$

$$=\sum_{t=0}^{\infty}\beta^t h'(n_t)\cdot n_t+\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t+\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot b_t$$

PVIC

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot c_t+\sum_{t=0}^{\infty}\sum_{j}\beta^{t+1}u'(c_{t+1}^j)\cdot b_{t+1}^j+\sum_{t=0}^{\infty}\beta^{t+1}u'(c_{t+1})\cdot\left(1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right)\cdot k_{t+1}$$

$$=\sum_{t=0}^{\infty}\beta^t h'(n_t)\cdot n_t+\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t+\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot b_t$$

↓ **Cancel summations in b_t**

PVIC

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot c_t+\sum_{t=0}^{\infty}\beta^{t+1}u'(c_{t+1})\cdot\left(1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right)\cdot k_{t+1}$$

$$=\sum_{t=0}^{\infty}\beta^t h'(n_t)\cdot n_t+\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t+u'(c_0)\cdot b_0$$

PVIC

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) \cdot c_t + \sum_{t=0}^{\infty} \beta^{t+1} u'(c_{t+1}) \cdot \left(1 + (1 - \tau_{t+1}^k)(r_{t+1} - \delta)\right) \cdot k_{t+1}$$

$$= \sum_{t=0}^{\infty} \beta^t h'(n_t) \cdot n_t + \sum_{t=0}^{\infty} \beta^t u'(c_t) \left[1 + (1 - \tau_t^k)(r_t - \delta)\right] k_t + u'(c_0) \cdot b_0$$

Cancel summations in k_{t+1}

Rearrange terms

PVIC

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)\cdot c_t+\sum_{t=0}^{\infty}\beta^{t+1}u'(c_{t+1})\cdot\left(1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right)\cdot k_{t+1}$$

$$=\sum_{t=0}^{\infty}\beta^t h'(n_t)\cdot n_t+\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t+u'(c_0)\cdot b_0$$

↓

Cancel summations in k_{t+1}

Rearrange terms

$$\sum_{t=0}^{\infty}\beta^t\left[u'(c_t)\cdot c_t-h'(n_t)\cdot n_t\right]=u'(c_0)\left[1+(1-\tau_0^k)(r_0-\delta)\right]k_0+u'(c_0)\cdot b_0$$

PVIC

$$\sum_{t=0}^{\infty} \beta^t u'(c_t)\cdot c_t + \sum_{t=0}^{\infty} \beta^{t+1} u'(c_{t+1})\cdot\left(1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right)\cdot k_{t+1}$$

$$= \sum_{t=0}^{\infty} \beta^t h'(n_t)\cdot n_t + \sum_{t=0}^{\infty} \beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t + u'(c_0)\cdot b_0$$

Cancel summations in k_{t+1}

Rearrange terms

$$\sum_{t=0}^{\infty} \beta^t \left[u'(c_t)\cdot c_t - h'(n_t)\cdot n_t\right] = \underbrace{u'(c_0)\left[1+(1-\tau_0^k)(r_0-\delta)+b_0\right]k_0}_{\equiv A_0}$$

back