



CALIBRATING THE (RBC + SOLOW) MODEL

JANUARY 10, 2018

CALIBRATION – PHILOSOPHY

- ❑ **An economic model is a measuring device**
- ❑ **If model makes “believable” predictions along some important dimensions (i.e., “matches some key data”)...**
- ❑ **...then maybe its predictions are “believable” along the novel dimensions of the model**
- ❑ **Getting some “partial derivatives” of the model in known directions correct...**
- ❑ **...may build credibility that its “partial derivatives” in novel directions are at least not grossly incorrect**

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- ❑ ...then maybe its predictions are “believable” along the novel dimensions of the model
- ❑ Getting some “partial derivatives” of the model in known directions correct...
- ❑ ...may build credibility that its “partial derivatives” in novel directions are at least not grossly incorrect
- ❑ Make model match some data of interest – often long-run (i.e., time-averaged data) growth facts
 - ❑ Preferably well-accepted “stylized facts”
 - ❑ Solow growth model in the background
 - ❑ Natural candidate: Kaldor growth facts

CALIBRATION OF BASELINE RBC MODEL

- ❑ **Must take a stand on three (related) points**
 - ❑ **Which data do we want model to match? (even constructing data is challenging...)**
 - ❑ **Functional forms (utility, production)**
 - ❑ **Parameter values**

- ❑ **Choose functional forms consistent with “Kaldor-plus facts”**
 - ❑ **(K1) Capital income share and labor income share of GDP are stationary**
 - ❑ **(K2) All real quantity variables grow at same rate in the long run (“great ratios”)**
 - ❑ **(K3) Real interest rate is stationary**
 - ❑ **(K4) Hours per worker are stationary**
 - ❑ **(K5) (K2) requires trend productivity to be labor-augmenting (Phelps 1966)**

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- ❑ **But “true” calibration begins with model featuring only long-run growth**
 - ❑ Puts restrictions on instantaneous utility and production forms
 - ❑ Use **(K1)-(K5)** to obtain these restrictions

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- ❑ **Richer models: more calibration targets and/or treating data differently**
 - ❑ Monopoly markups (e.g., Dixit-Stiglitz and sticky price models)
 - ❑ Probability of finding a job (e.g., labor matching models)
 - ❑ Durable consumption vs. non-durable consumption

RBC MODEL WITH GROWTH

□ Absent shocks, TFP grows at deterministic rate γ

□ Planner problem/perfect competition

$$\max E_0 \sum_{t=0}^{\infty} b^t u(C_t, n_t)$$

subject to

$$C_t + K_{t+1} - (1 - \delta)K_t = z_t F(K_t, n_t X_t)$$

$$X_{t+1} = \gamma X_t, \quad \gamma \geq 1$$

Trend productivity is labor-augmenting (Harrod-neutral) (Makes use of fact **(K5)**)

Flow resource constraint

Evolution of deterministic component of productivity

Red indicates variables or parameters that will be modified when detrending the model

given stochastic process for evolution of z_t and (K_{-1}, z_0, X_0)

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given stochastic process for evolution of z_t and (K_{-1}, z_0, X_0)

□ Suppose $z_t = 1$ always, so only deterministic growth

□ Deterministic dynamics of $(C_t, K_{t+1}, n_t, X_{t+1})$ governed by

$$(1) \quad -\frac{u_n(C_t, n_t)}{u_c(C_t, n_t)} = X_t F_2(K_t, n_t X_t)$$

Labor supply function (aka consumption-labor optimality)

$$(2) \quad \frac{u_c(C_t, n_t)}{\beta u_c(C_{t+1}, n_{t+1})} = F_1(K_{t+1}, n_{t+1} X_{t+1}) + 1 - \delta$$

Capital supply function (aka consumption-savings optimality)

$$(3) \quad C_t + K_{t+1} - (1 - \delta)K_t = F(K_t, n_t X_t)$$

$$(4) \quad X_{t+1} = \gamma X_t$$

Normalize $X_0 = 1$

RESTRICTIONS ON FUNCTIONAL FORMS

□ **Deterministic dynamics of (C_t, K_{t+1}, n_t, X_t) governed by**

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 \text{(1)} & -\frac{u_n(C_t, n_t)}{u_c(C_t, n_t)} = X_t F_2(K_t, n_t X_t) & \text{(3)} & C_t + K_{t+1} - (1-\delta)K_t = F(K_t, n_t X_t) \\
 \text{(2)} & \frac{u_c(C_t, n_t)}{b u_c(C_{t+1}, n_{t+1})} = F_1(K_{t+1}, n_{t+1} X_{t+1}) + 1 - \delta & \text{(4)} & X_{t+1} = \gamma X_t
 \end{array}$$

□ **(K1) Capital income share and labor income share of GDP are stationary
And viewing economic profits as zero**

$$\Rightarrow F(K, nX) = K^\alpha (nX)^{1-\alpha} \quad (\alpha \approx 0.4)$$

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□ **(K2) All real quantity variables grow at same rate in the long run**

$$\begin{aligned}
 & \Rightarrow \frac{Y_{t+1}}{Y_t} = \frac{C_{t+1}}{C_t} = \frac{K_{t+1}}{K_t} = \frac{X_{t+1}}{X_t} = \gamma, \quad \forall t \\
 & \Rightarrow y_t \equiv \frac{Y_t}{X_t} = \bar{y}, \quad k_t \equiv \frac{K_t}{X_t} = \bar{k}, \quad c_t \equiv \frac{C_t}{X_t} = \bar{c}, \quad \forall t
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□ **And scale (3) by X_t to make stationary**

$$\begin{aligned}
 (1) \quad & -\frac{u_n(C_t, n_t)}{u_c(C_t, n_t)} = (1-\alpha)X_t \left(\frac{k_t}{n_t} \right)^\alpha & (3) \quad & c_t + \gamma k_{t+1} - (1-\delta)k_t = k_t^\alpha n_t^{1-\alpha} \\
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Note long run growth rate affects capital accumulation even in stationary representation!



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□ **(K4) Hours per worker are stationary**

$$\Rightarrow n_t = \bar{n}$$

along deterministic path.
BUT \bar{n} is endogenous...

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□ **Observations**

□ **Optimal choice of labor (\bar{n}) must be independent of X_t (from (1))**

□ **Requires offsetting income and substitution effects of wages (long-run productivity) on labor supply**

□ **IMRS can only depend on C_{t+1}/C_t (from (2)), which in turn = γ**

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- **Two requirements together imply**

$$u(C_t, n_t) = \begin{cases} \frac{[C_t v(n_t)]^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma > 0, \sigma \neq 1 \\ \ln C_t + v(n_t) & \text{if } \sigma = 1 \end{cases}$$

King, Plosser, Rebelo (1988 JME)

STEADY STATE

- **Steady state $(\bar{c}, \bar{n}, \bar{k})$ solves (1), (2), (3)**
- **A dynamic phenomenon!**
 - **Not static!**
 - **Economy is moving exactly along its long-run (i.e., deterministic) growth path**
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 - ❑ **Provide calibration targets**
 - ❑ **e.g., $\bar{c} / \bar{y} = 0.70$, $\bar{k} / \bar{y} = 2.5$ (if annual measurement)**

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 - ❑ e.g., $\bar{c} / \bar{y} = 0.70$, $\bar{k} / \bar{y} = 2.5$ (if annual measurement)
- ❑ **Time use and intertemporal price outcomes within model are interpretable**
 - ❑ Provide calibration targets
 - ❑ \bar{n} is fraction of time spent in paid market work
 - ❑ Empirical: $n \approx 0.30$
 - ❑ Return on capital $\alpha \left(\frac{\bar{k}}{\bar{n}} \right)^{\alpha-1} + 1 - \delta$
- ❑ **Use ss calibration targets to set parameter values, given functional forms**

RBC MODEL WITHOUT GROWTH

□ Often start instead with

$$c_t = C_t/X_t \quad \uparrow \quad u(c_t, n_t) = \begin{cases} \frac{[c_t v(n_t)]^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma > 0, \sigma \neq 1 \\ \ln c_t + v(n_t) & \text{if } \sigma = 1 \end{cases}, \quad \text{and} \quad \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

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- For this transformed model to deliver same steady state (relative quantities, time use, and r), require

- Resource constraint $c_t + \gamma k_{t+1} - (1-\delta)k_t = z_t k_t^\alpha n_t^{1-\alpha}$

- Subjective discount factor $\beta \equiv b\gamma^{1-\sigma}$ (King and Rebelo, p. 945)

- Typical assumption $\gamma = 1$ omits growth altogether

- What if trend growth rate fluctuates, γ_t ?

- Typical representation cannot accommodate **trend shocks** because $\gamma = 1$

- **Trend shocks** fairly common in small-open-economy models

- Affects discount factor and capital accumulation equation

BASELINE RBC MODEL

- ❑ Assuming $\gamma = 1$...
- ❑ ...complete calibration?
- ❑ **Data:** long-run labor income share of GDP ≈ 0.60
 - ❑ Cobb-Douglas $F(\cdot)$ implies

$$\frac{wn}{F(k,n)} = \frac{(1-\alpha)k^\alpha n^{1-\alpha}}{k^\alpha n^{1-\alpha}} = 1 - \alpha \quad \Rightarrow \quad \alpha = 0.40$$

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- **Data:** long-run ratio of (annual) gross investment to capital stock ≈ 0.07

$$\frac{k - (1-\delta)k}{k} = \frac{\delta k}{k} = 0.07 \quad \Rightarrow \quad \delta = 0.07 \text{ (annual) or } 0.018 \text{ (quarterly)}$$

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- **Data:** long-run ratio of (annual) output to capital stock ≈ 0.4
 - Steady-state Euler equation

$$1 = \beta \left[\frac{\alpha k^\alpha n^{1-\alpha}}{k} + 1 - \delta \right] = \beta \left[\frac{\alpha F(k,n)}{k} + 1 - \delta \right] \quad \Rightarrow \quad \beta = 0.95 \text{ (annual) or } 0.99 \text{ (quarterly)}$$

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- OR
- **Data:** avg. net real return on capital $\approx 5\%$ per year (e.g, return on S&P500)
 - Steady-state Euler equation

$$f_k(k,n) = \frac{1}{\beta} - 1 + \delta \quad \Rightarrow \quad \beta = 0.96 \text{ (annual) or } 0.99 \text{ (quarterly)}$$

BASELINE RBC MODEL

□ Utility parameters

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□ **Data:** IES is around unity(?) or lower

- **Implies $\sigma > 1$**
- **(Recall: IES = $1/\sigma$ for time-separable CRRA utility)**
- **$\sigma = 1$ a conventional value**

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Labor subutility

- Common form

$$v(n) = -\frac{\psi}{1+1/\eta} n^{1+1/\eta}$$

- η measures Frisch elasticity of labor supply (use C-L optimality condition)
- Calibrate ψ to hit $\bar{n} \approx 0.3$
- Empirical evidence on Frisch elasticity?

BASELINE RBC MODEL

- ❑ Labor supply elasticity “controversial”
- ❑ **Micro evidence:** very low – η (substantially) smaller than one
- ❑ **Macro evidence:** very high – η (substantially) larger than one

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- ❑ **Macro evidence:** very high – η (substantially) larger than one

- ❑ “Tension” between macro and micro evidence not useful way to frame the “controversy”

- ❑ Micro studies pick up **intensive** margin of labor supply
- ❑ Macro studies pick up (mostly) **extensive** margin of labor supply
 - ❑ And other frictions in allocation of workers to jobs...

- ❑ Chetty (2012 *Econometrica*): uses both macro and micro evidence to put bounds on labor supply elasticity

- ❑ Common in DSGE models: $\eta > 1$

BASELINE RBC MODEL

- **Exogenous process for TFP (deviations from long-run trend productivity)**

$$\ln z_{t+1} = (1 - \rho_z) \ln \bar{z} + \rho_z \ln z_t + \varepsilon_{t+1}$$

ε_{t+1} distributed iid $N(0, \sigma_z^2)$

- **Normalize $\bar{z} = 1$**
 - **Only governs absolute scale of model, which is arbitrary**
 - **What does, e.g., $\bar{c} = 1.56$ mean?**

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- **Construct time-series for z_t using**
 - **Data on labor, (detrended) capital, and (detrended) output**
- **AR(1) estimation**
 - **Quarterly frequency**

$$\Rightarrow \rho_z = 0.95 \quad \text{and} \quad \sigma_z = 0.007$$

USING THE RBC (OR ANY DSGE) MODEL

1. **Dream up/construct/write fully-articulated model**
 - Ideally to answer questions motivated by data and with hypotheses**
2. **Define equilibrium!**
3. **Choose parameter values**
 - Perhaps extremely rigorously, if goal is to match certain empirical facts very precisely**
 - Perhaps adopting generally-accepted values, if goal is to illustrate some insight**
4. **Solve for deterministic steady state (balanced growth path)**
5. **Solve for dynamic decision rules (e.g., linear approximation, second-order approximation, global approximation)**
6. **Conduct informative battery of experiments (impulse responses, simulations, etc.) to try to falsify hypotheses**
7. **Tabulate results, write a (good!) paper, get it published**