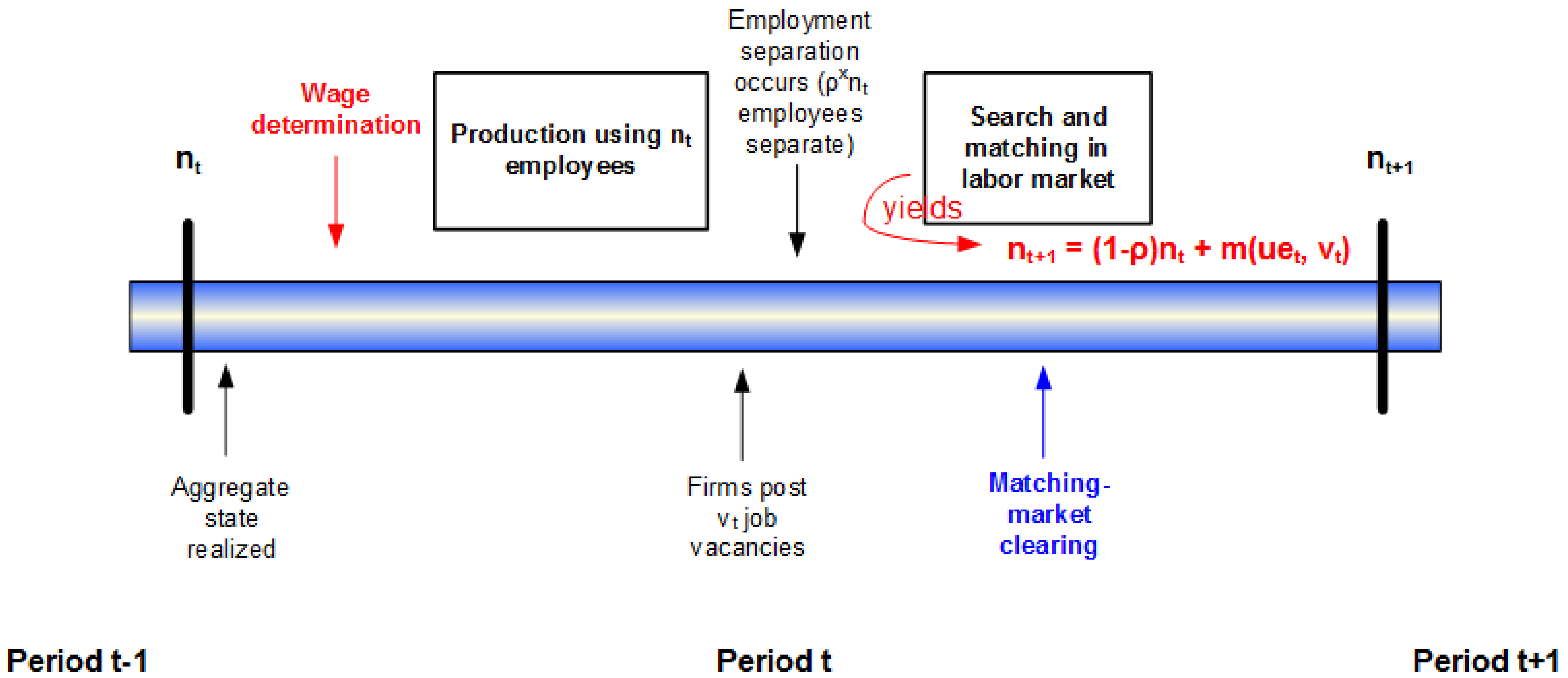
LABOR MATCHING MODELS: BASIC DSGE IMPLEMENTATION

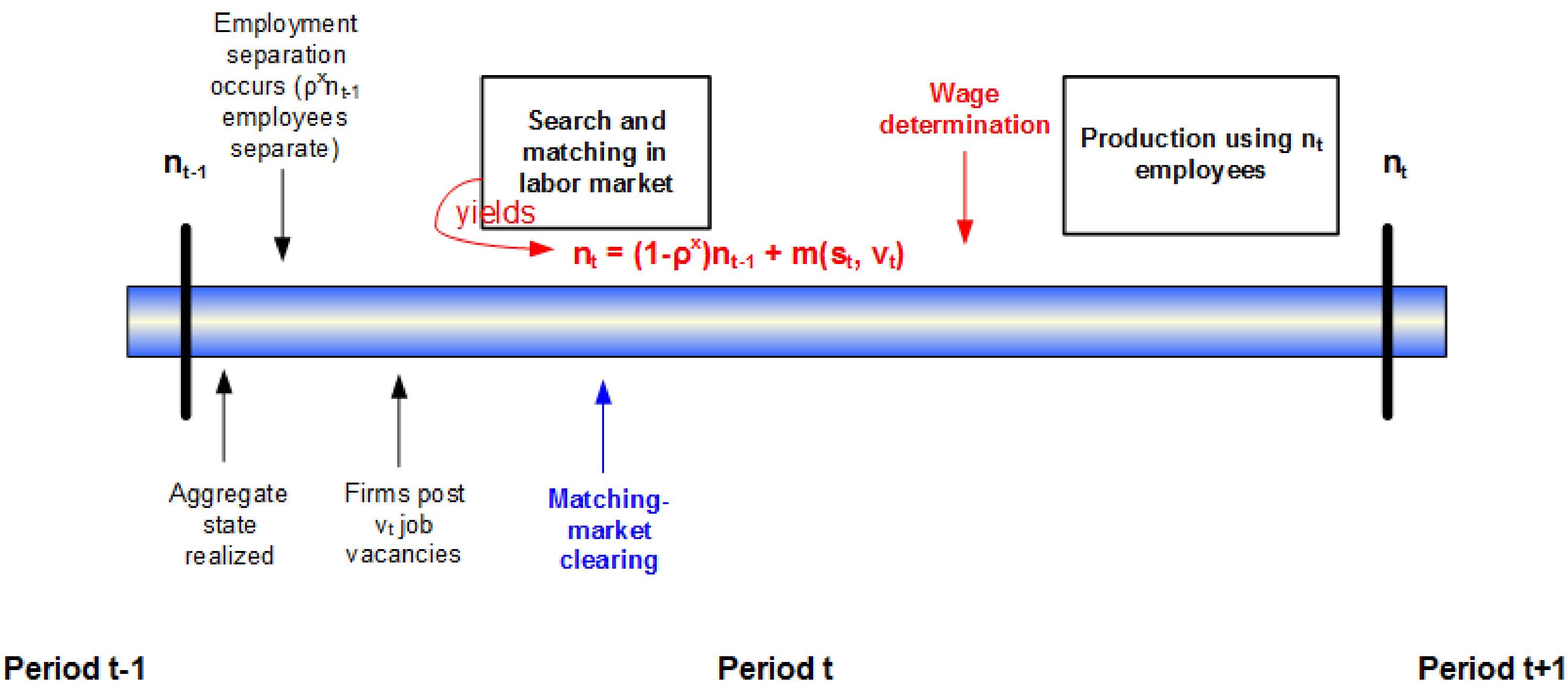
MARCH 23, 2017

TIMELINE



(Stick with this "lagged production" timing for now…)

TIMELINE



("Instantaneous production" timing soon…)

FIRM VACANCY-POSTING PROBLEM

- Dynamic firm profit-maximization problem

$$\max_{v_t, n_{t+1}^f} \left[E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f f(h_t) - w_t n_t^f h_t - \gamma g(v_t) \right) \right]$$

Discount factor between time 0 and *t* because ***dynamic*** firm problem; in equilibrium, = household stochastic discount factor

Number of vacancies to post (how many job advertisements)

Desired target ***future*** firm employment

Total output – sold in perfectly-competitive goods market

Total wage bill depends on both extensive and intensive employment

Total cost of posting *v* vacancies

  - Subject to (perceived) law of motion for firm's employment stock

- For starters
  - Shut down intensive margin: $h_t = 1$
  - Linear posting costs: $g(v) = v$
  - Firm production function: $y_t = z_t n_t$

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- Two "market-determined" prices taken as given
- Wage-setting (process) taken as given

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$$\text{s.t. } n_{t+1}^f = (1-\rho_x) n_t^f + v_t k^f(\theta_t)$$

Perceived law of motion for evolution of employment stock

Number of existing jobs that remain intact: ρ_x exogenous separation rate, but can also endogenize

Each vacancy has probability $k^f(\theta)$ of attracting a prospective employee: depends on a *market* variable, θ, so taken as given

- Market-determined probability k^f taken as given

Firm Vacancy-Posting Problem

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_{t+1}^f} \left[E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f - w_t n_t^f - \gamma v_t \right) \right]$$

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Each vacancy has probability $k^f(\theta)$ of attracting a prospective employee: depends on a *market* variable, θ, so taken as given

FOCs with respect to v_t, n_{t+1}

$$-\gamma + \mu_t k^f(\theta_t) = 0$$

$$-\mu_t + E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + (1-\rho_x) \mu_{t+1} \right) \right\} = 0$$

Combine

FIRM VACANCY-POSTING PROBLEM

- **Vacancy posting condition (aka job creation condition)**

$$\gamma = k^f(\theta_t) E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of attracting a worker) x (expected future benefit of an additional worker)

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γ / k^f is capital value of an existing employee – because one *less* worker firm has to find in the future

EMPLOYEES ARE ASSETS

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- Vacancy-posting is a type of investment decision
  - Intertemporal dimension makes discount factor potentially important
    - Makes general equilibrium effects potentially important

- Two prices affect posting decision (aside from intertemporal price)
  - Wage
  - Matching probability k^f (which depends on the market variable θ)

HOUSEHOLD PROBLEM

- Dynamic household utility-maximization problem
  - A continuum [0, 1] of households (standard assumption)
  - A continuum [0, 1] of atomistic individuals live in each household
  - Representative household has continuum of "family members"

$A > 0$

$$\max_{c_t, n_t, a_t} \left[E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - A \cdot n_t \right) \right]$$

$$\text{s.t.} \quad c_t + a_t = n_t w_t h_t + (1 - n_t) b + R_t a_{t-1}$$

An (arbitrary) asset to make pricing interest rates explicit

Wage (-setting process) taken as given by household

Measure n_t of family members earn labor income (because they work) (and recall we've normalized h = 1)

Measure 1-n_t of family members receive unemployment benefits and / or engaged in home production

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Measure 1-n_t of family members receive unemployment benefits and / or engaged in home production

- **Consumption-savings optimality condition:** $1 = R_t E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}$

Stochastic discount factor

- **No LFP margin in starter model**
  - **Each family member either works or is looking for work**

WAGE BARGAINING

- (Generalized) Nash Bargaining

$$\max_{w_t}\left(\mathbf{W}(w_t)-\mathbf{U}(w_t)\right)^{\eta}\left(\mathbf{J}(w_t)-\mathbf{V}(w_t)\right)^{1-\eta}$$

Bargaining over how to divide the surplus

Net payoff to an individual / household of agreeing to wage *w* and beginning production

Net payoff to a firm of agreeing to wage *w* and beginning production

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- Value equations
  - ***W***: value to (representative) household of having one additional member employed
  - ***U***: value to (representative) household of having one additional member unemployed and searching for work
  - ***J***: value to (representative) firm of having one additional employee
  - ***V***: value to (representative) firm of having a vacancy that goes unfilled
    - Free entry in vacancy-posting → ***V*** = 0

- Define ***W*** and ***U*** in terms of household problem
  - i.e., based on envelope conditions of household value function

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Bargaining over how to divide the surplus

Net payoff to an individual / household of agreeing to wage *w* and beginning production

Net payoff to a firm of agreeing to wage *w* and beginning production

- Nash surplus-sharing rule

$$\eta\left(\mathbf{W}'(w_t)-\mathbf{U}'(w_t)\right)\mathbf{J}(w_t)=(1-\eta)(-\mathbf{J}'(w_t))\left(\mathbf{W}(w_t)-\mathbf{U}(w_t)\right)$$

(FOC with respect to w_t)

- Must specify value equations *W*(.), *U*(.), *J*(.)

- (For ease: set parameter *A* = 0 for remainder of analysis)

VALUE EQUATIONS

- **Individual / household value equations (constructed from household problem)**

Each searching individual has probability $k^h(\theta)$ of finding a job opening: depends on a *market* variable, θ, so taken as given

$$\mathbf{W}(w_t) = \underbrace{w_t}_{\text{Contemporaneous return is wage}} + E_t\left\{\underbrace{\Xi_{t+1|t}\left[(1-\rho_x)\mathbf{W}(w_{t+1}) + \rho_x \mathbf{U}(w_{t+1})\right]}_{\text{Expected future return takes into account transition probabilities}}\right\}$$

Value to household of having the marginal individual employed

$$\mathbf{U}(w_t) = \underbrace{b}_{\text{Contemporaneous return is unemployment benefit / home production}} + E_t\left\{\underbrace{\Xi_{t+1|t}\left[k^h(\theta_t)\mathbf{W}(w_{t+1}) + (1-k^h(\theta_t))\mathbf{U}(w_{t+1})\right]}_{\text{Expected future return takes into account transition probabilities}}\right\}$$

Value to household of having the marginal individual unemployed and searching

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Contemporaneous return is wage

Expected future return takes into account transition probabilities

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Value to household of having the marginal individual unemployed and searching

Contemporaneous return is unemployment benefit/home production

Expected future return takes into account transition probabilities

- **Firm value equation**

$$\mathbf{J}(w_t) = z_t - w_t + E_t\left\{\Xi_{t+1|t}(1-\rho_x)\mathbf{J}(w_{t+1})\right\}$$

Value to firm of the marginal employee

Contemporaneous return is marginal output net of wage payment

Expected future return takes into account transition probabilities

WAGE BARGAINING

- **The Nash surplus-sharing rule**

$$\eta\Big(\mathbf{W}'(w_t)-\mathbf{U}'(w_t)\Big)\mathbf{J}(w_t)=(1-\eta)(-\mathbf{J}'(w_t))\Big(\mathbf{W}(w_t)-\mathbf{U}(w_t)\Big)$$

(FOC with respect to w_t)

↓ **Insert marginal values**

$$\eta\mathbf{J}(w_t)=(1-\eta)\Big(\mathbf{W}(w_t)-\mathbf{U}(w_t)\Big)$$

Firm's surplus *J* a constant fraction of household's surplus *W – U*

NOTE: NOT a general property of Nash bargaining; here due to the linearity of *W, U,* and *J* with respect to wage

WAGE BARGAINING

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Using definitions of *W, U,* and *J,* the job-creation condition, and some algebra

$$w_t=\eta\left[z_t+\gamma\theta_t\right]+(1-\eta)b$$

Bargained wage a convex combination of gains from consummating the match and the gains from walking away from the match

Contemporaneous marginal output…

…plus term that captures savings on future posting costs if match continues

NOTE: With CRS matching function,

$\theta = k^h(\theta) / k^f(\theta)$

LABOR MARKET MATCHING

- **Aggregate matching function displays CRS**

$$m(u_t, v_t)$$

u_t = 1 – n_t is measure of individuals searching for work

- **For any given individual vacancy or individual (partial equilibrium), matching probabilities depend only on *v / u***

$$\theta_t \equiv \frac{v_t}{u_t}$$

Market tightness: measures relative number of traders on opposite sides of market

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$$\frac{m(u_t, v_t)}{v_t} = m\left(\frac{u_t}{v_t}, 1\right) = m\left(\theta_t^{-1}, 1\right) \equiv k^f(\theta_t)$$

Probability a given vacancy / job posting attracts a worker

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Probability a given individual finds a job opening

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Labor Market Matching

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$$m(u_t, v_t)$$

$u_t = 1 - n_t$ is measure of individuals searching for work

- **For any given individual vacancy or individual (partial equilibrium), matching probabilities depend only on *v / u***

NOTE: With CRS matching function,

$\theta = k^h(\theta) / k^f(\theta)$

In matching models, θ is key driving force of efficiency and thus optimal policy prescriptions (Mortensen 1982 *AER* and Hosios 1990 *ReStud* key references)

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Probability a given individual finds a job opening

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Market tightness: measures relative number of traders on opposite sides of market

- **Market tightness an allocational signal**
  - **Because matching probabilities depend on it**
  - **e.g., the higher (lower) is *v / u*, the easier (harder) it is for a given individual to find a job opening**

LABOR MARKET EQUILIBRIUM

- **Aggregate law of motion of employment**

$$n_{t+1} = (1-\rho_x)n_t + m(u_t, v_t)$$

- **Matching-market equilibrium**

$$m(u_t, v_t) = u_t \cdot k^h(\theta_t) = v_t \cdot k^f(\theta_t)$$

- **Vacancy-posting (aka job-creation) condition**

$$\gamma = k^f(\theta_t) E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

- **Wage determination (Nash bargaining)**

$$w_t = \eta \left[z_t + \gamma\theta_t \right] + (1-\eta)b$$

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- **Wage determination (Nash bargaining)**

$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

- **Basic labor-theory literature: impose ss, comparative static exercises, etc. (exogenous real interest rate)**
  - **Pissarides Chapter 1, RSW 2005 *JEL***

GENERAL EQUILIBRIUM

- ❒ **Aggregate law of motion for employment**
- ❒ **Vacancy-posting (aka job-creation) condition**
- ❒ **Wage determination**

} **The labor market equilibrium (*partial* equilibrium)**

- ❒ **Consumption-savings optimality condition (endogenizes real interest rate)**

$$1 = R_t E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}$$

- ❒ **Aggregate resource constraint**

Often interpreted as the output of a home production sector – only the unemployed produce in the home sector

$$c_t + g_t + \gamma v_t = z_t n_t h_t + (1 - n_t) b$$

Vacancy posting costs and "outside option" are real uses of resources

- ❒ **Exogenous LOMs for any driving processes (TFP, etc)**

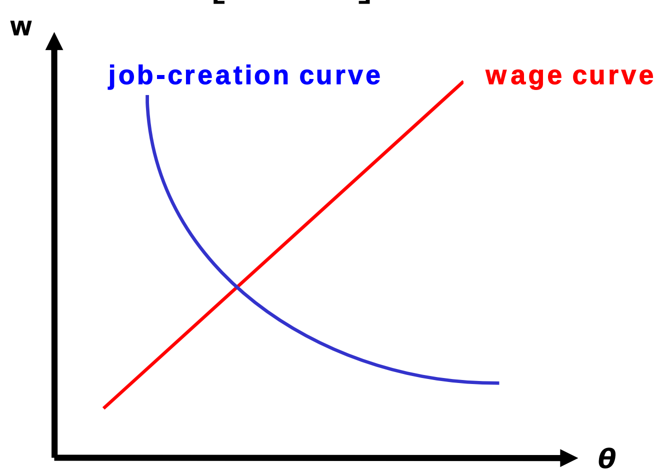
Steady State of Labor Market

- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1) $$1-u=(1-\rho_x)(1-u)+m(u,v)$$ (using $n = 1 - u$)

(2) $$\gamma=\beta k^f(\theta)\left(z-w+\frac{(1-\rho_x)\gamma}{k^f(\theta)}\right)$$ w negatively and nonlinearly related to θ (given CRS matching function)

(3) $$w=\eta\left[z+\gamma\theta\right]+(1-\eta)b$$ w positively and linearly related to θ



Pissarides 2000, Figure 1.1

STEADY STATE OF LABOR MARKET

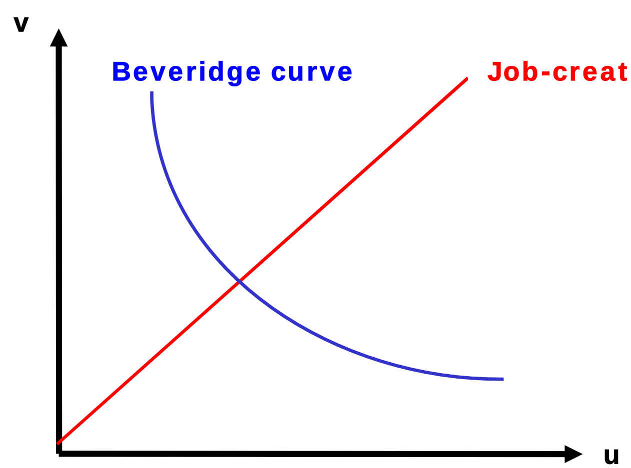
- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1) $$u = \frac{\rho_x - m(u,v)}{\rho_x}$$

For a given (w,θ), v and u negatively related (given CRS matching function)

(2) $$\gamma = \beta k^f\left(\frac{v}{u}\right)\left(z - w + \frac{(1-\rho_x)\gamma}{k^f\left(\frac{v}{u}\right)}\right)$$

For a given (w,θ), v and u positively related (given CRS matching function)



Pissarides 2000, Figure 1.2

BEVERIDGE CURVE: Empirical relationship in both long run and short run (i.e., cyclical)

STEADY STATE OF LABOR MARKET

- **Labor-market equilibrium is (*w, u, θ*) satisfying (1), (2), (3)**

- **Comparative statics**
  - **A rise in *b*…**
    - **…raises *w***
    - **…lowers *θ***
    - **…lowers *v* and raises *u***

Higher value (outside option) of unemployment requires a higher wage to induce individuals to work, which reduces firm incentives to create jobs

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    Higher value (outside option) of unemployment requires a higher wage to induce individuals to work, which reduces firm incentives to create jobs

  - **A fall in β (or a rise in ρ_x)…**
    - **…lowers w**
    - **…lowers θ**
    - **…raises u**
    - **…ambiguous effect on v**

    Higher real rate and / or faster job separations (i.e., "faster depreciation of employment stock") makes posting vacancies (FOR FIXED u) less attractive for firms (both erode firm profits)

- **See Pissarides Chapter 1 and RSW (2005 *JEL*) for more**

- **Next: dynamic stochastic partial equilibrium (Shimer 2005 *AER*, Hall 2005 *AER*, Hagedorn and Manovskii 2008 *AER*)**