LABOR MATCHING MODELS: BASIC BUILDING BLOCKS

JANUARY 22, 2018

BASIC LABOR MARKET ISSUES

- How can production resources sit idle even when there is "high aggregate demand?"

- Coordination frictions in labor markets
  - Finding a job or an employee takes time and/or resources
  - Not articulated in basic neoclassical/Walrasian framework

- Are labor market transactions "spot" transactions?
  - Or do they occur in the context of ongoing relationships?
  - The answer implies quite different roles for prices (wages)

- "Structural" vs. "frictional" unemployment
  - **Structural:** unemployment induced by fundamental changes in technology, etc – dislocations due to insufficient job training, changing technical/educational needs of workforce, etc.

  - **Frictional:** temporarily unemployed as workers and jobs shuffle from one partner to another

BASIC BUILDING BLOCKS

- **Aggregate matching function**

$$m(s_t, v_t)$$

  - **Brings together individuals actively seeking work (*s*) and employers looking for workers (*v*)**
  - **A technology from the perspective of the economy**
  - **Black box that describes all the possible coordination, matching, informational, temporal, geographic, etc. frictions in finding workers and jobs**

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- **Employment is a state variable (several different timings)**

Churning of jobs; a job is not an absorbing state

$$n_{t+1} = \underbrace{(1-\rho_x)n_t} + \underbrace{m(s_t, v_t)}$$

Aggregate law of motion of employment

Number of existing jobs that end: ρ_x exogenous separation rate

Number of new jobs (matches) that form in *t*

ANALOGY: $k_{t+1} = (1-\delta)k_t + i_t$

BASIC BUILDING BLOCKS

- Wage models (decentralized economy)
    - Nash bargaining

BASIC BUILDING BLOCKS

- (Generalized) Nash Bargaining

Bargaining powers η and 1-η measure "strength" of each party in negotiations

$$\max_{w_t}\underbrace{\left(\mathbf{W}(w_t)-\mathbf{U}(w_t)\right)}_{\text{Net payoff to an individual of agreeing to wage } w \text{ and beginning production}}^{h}\underbrace{\left(\mathbf{J}(w_t)-\mathbf{V}(w_t)\right)}_{\text{Net payoff to a firm of agreeing to wage } w \text{ and beginning production}}^{1-h}$$

Original Nash 1950 was η = 0.5

- The unique problem whose solution satisfies three axioms (Nash 1950)
  - Pareto optimality
  - Scale invariance
  - Independence of irrelevant alternatives

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  - **The unique problem whose solution satisfies three axioms (Nash 1950)**
    - **Pareto optimality**
    - **Scale invariance**
    - **Independence of irrelevant alternatives**

- **Given an extensive-form foundation by Binmore (1980) and Binmore, Rubinstein, Wolinksy (1986)**
  - **Nash solution the limiting solution of a Rubinstein alternating-offers game (as time interval between successive offers → zero)**
  - **In which (*η*, 1- *η*) measure discount factors of each party between successive offers**

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- **Value equations**
  - ***W*(.): value to (representative) household of having one additional member employed**
  - ***U*: value to (representative) household of having one additional member unemployed and actively seeking work**
  - ***J*(.): value to (representative) firm of having one additional employee**
  - ***V*: value to (representative) firm vacancy that goes unfilled**
    - **Free entry in vacancy-posting → *V* = 0**

- **Define *W*(.) and *U* in terms of household problem**
  - **i.e., based on envelope conditions of household value function**

BASIC BUILDING BLOCKS

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- **Nash surplus-sharing condition**

$$\eta\left(\underbrace{\mathbf{W}'(w_t)}_{=1} - \underbrace{\mathbf{U}'(w_t)}_{=0} \right) \mathbf{J}(w_t) = (1-\eta)\underbrace{(-\mathbf{J}'(w_t))}_{=1}\left(\mathbf{W}(w_t) - \mathbf{U}_t \right)$$

(FOC with respect to w_t)

b/c zero proportional taxation on wage

- **For sake of ease, temporarily assume basic properties for *W*(.), *U*, *J*(.)**
  - **Specify details soon**

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Nash-bargained surplus-sharing condition

BASIC BUILDING BLOCKS

- Wage models (decentralized economy)
  - Nash bargaining
  - Competitive search equilibrium
  - Competitive equilibrium
  - Proportional bargaining
  - “Rigid” wages
  - ***MANY*** others....

- Efficient allocations – zero congestion effects

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- ❒ Efficient allocations – zero congestion effects
  - ❒ Suppose Cobb-Douglas matching technology
  - ❒ Efficient market tightness *θ* the key

$$m(s,v) = s^{\xi} v^{1-\xi}$$

$$m_v = (1-\xi)\cdot\theta^{-\xi}$$

$$m_s = \xi\cdot\theta^{1-\xi}$$

$$\theta \equiv \frac{v}{s}$$

$$\boxed{\xi\cdot\mathbf{J}(w_t) = (1-\xi)\cdot\left(\mathbf{W}(w_t) - \mathbf{U}_t\right)}$$

Efficient surplus-sharing condition

***GIVEN* C-D m(.) fct…**

CSE I

BASIC BUILDING BLOCKS

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- How does "matching" process work?
  - Random search
  - Competitive search
  - "Owners" of matching technology
  - "Price" of a match

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sheds new insights

COMPETITIVE SEARCH EQUILIBRIUM (CSE)

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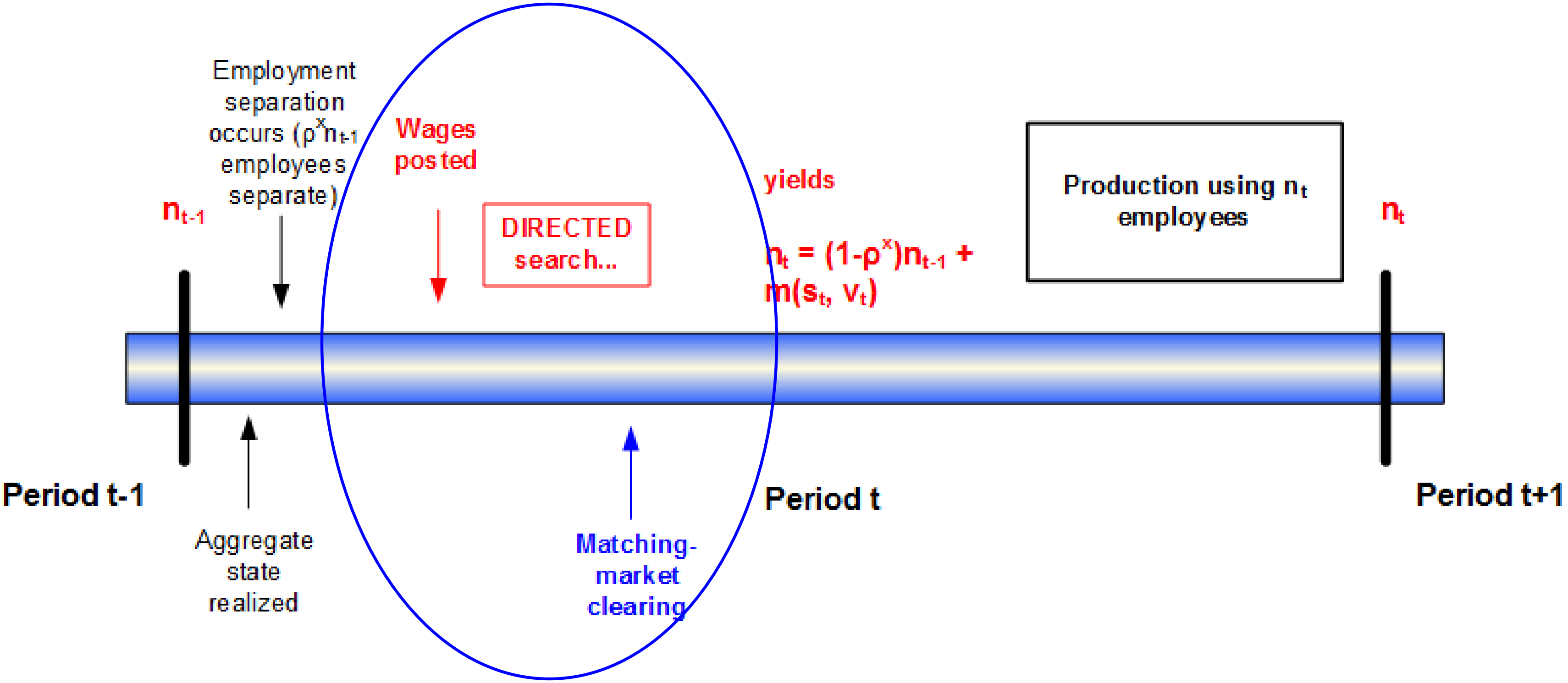
- **Question: can a "competitive" notion of wage-setting be entertained in a search and matching model?**
  - **Wages playing allocational role in determining meeting process**
  - **In contrast to wage bargaining, which plays small/no allocational role**



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- May be apriori an appealing way of describing labor markets
  - Locating a firm or a worker is costly and time-consuming…
  - …but once matched, wages are more or less determined by "market forces," perhaps with little/no room for "bargaining"

- Moen (1997 *JPE*) and Shimer (1996) the pioneers of CSE
  - Static partial equilibrium labor matching models
  - "Small firms" (one firm = one job)

- Re-explore CSE framework

- Recent burst of work extending CSE wage model in many dimensions

CSE – BASICS OF ENVIRONMENT

- **Need “many markets” and “many firms”**
- **To rationalize “competition”**

- **Index continuum of labor “submarkets” by *ij***
  - **Same geographic region**
  - **Same career, regardless of geography**
  - ***Many* ways to interpret…..**

“submarket” denotes notion of “closeness”

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- **Several equivalent ways to implement *perfectly* CSE**

- **Firms post wages before individuals search for job opportunities**
- **Perfectly-competitive recruiting sector**
- **Individuals announce wages before firms direct their vacancies**

- **Regardless of implementation, market tightness is efficient**

CSE – IMPLEMENTATION I

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CSE – IMPLEMENTATION I

- Job seekers *direct* their active search (“send an application”) to a particular submarket
  - Based on wages announced by goods-producing firms in that submarket
  - And on probability of contacting an open vacancy in that submarket
  - Directed search a key component of CSE…
  - …but match formation still subject to probabilities

- Ordering of events
  - Wages determined before search…
  - …job seekers actively direct search according to posted wages…
  - …then probability of landing a match resolved

CSE – IMPLEMENTATION I

- **Firm *ij* payoff function described by vacancy-posting decision**

γ

Cost of posting a vacancy

CSE – IMPLEMENTATION I

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$$\gamma = k^f(\theta_{ijt})\left[z_t - w_{ijt} \right]$$

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CSE – IMPLEMENTATION I

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$$\underset{\uparrow}{\gamma} = \underbrace{k^{f}(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{ \Xi_{t+1|t}\left(\frac{\gamma}{k^{f}(\theta_{jt+1})} \right) \right\} \right]}$$

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of matching with a worker) x (contemporaneous payoff + continuation payoff)

Note *ij* subscripts:

Matching probability depends on tightness of "applications" at firm *ij*…

***…*but future asset value of employee depends on market *j* conditions (i.e., replacement value depends on (sub-)market conditions)**

CSE – IMPLEMENTATION I

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- **Value equations for an individual searching for a match at firm *ij***

$$\mathbf{W}(w_{ijt}) = w_{ijt} + E_t\left\{\Xi_{t+1|t}\left[(1-\rho_x)\mathbf{W}(w_{jt+1}) + \rho_x \mathbf{U}_{t+1}\right]\right\}$$

With probability $k^h(\theta_{ijt})$, individual gets this payoff

$$\mathbf{U}_t = b + E_t\left\{\Xi_{t+1|t}\left[k^h(\theta_t)\mathbf{W}(w_{t+1}) + (1-k^h(\theta_t))\mathbf{U}_{t+1}\right]\right\}$$

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CSE – IMPLEMENTATION I

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- **Individuals seeking a job optimally direct their search so that expected payoff of successful contact with a job opening at firm *ij* is**

$$k^h(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^h(\theta_{ijt}))\mathbf{U}_t = \underbrace{\mathbf{X}^H}$$

Payoff of searching at another firm or another submarket independent of *ij*

CSE – IMPLEMENTATION I

- ❒ **Firm *ij* maximizes**

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taking as constraint

$$k^h(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^h(\theta_{ijt}))\mathbf{U}_t = \mathbf{X}^H$$

- ❒ **Choice variables: w_{ijt} and θ_{ijt} (isomorphic to choosing v_{ijt} for a given number of searchers u_{ijt})**

CSE – IMPLEMENTATION I

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- **Choice variables: w_{ijt} and θ_{ijt} (isomorphic to choosing v_{ijt} for a given number of searchers u_{ijt})**

- **First-order conditions**

1)

$$-k^f(\theta_{ijt}) - \varphi_{ijt}k^h(\theta_{ijt})\mathbf{W}'(w_{ijt}) = 0$$

2)

$$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}}\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right] - \varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t\right] = 0$$

Taking into account how matching probabilities are affected by tightness is the central idea

CSE – IMPLEMENTATION I

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$$\underbrace{\frac{\partial k^{f}(\theta_{ijt})}{\partial \theta_{ijt}} \left[z_t - w_{ijt} + (1-\rho_x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^{f}(\theta_{jt+1})} \right) \right\} \right]}_{= J_{ijt}} - \varphi_{ijt} \frac{\partial k^{h}(\theta_{ijt})}{\partial \theta_{ijt}} \left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t \right] = 0$$

Recruiter

CSE – IMPLEMENTATION I

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1) $$-k^{f}(\theta_{ijt})-\varphi_{ijt}k^{h}(\theta_{ijt})\mathbf{W}'(w_{ijt})=0 \xrightarrow{W'(.)\,=\,1} \boxed{\varphi_{ijt}=-\frac{k^{f}(\theta_{ijt})}{k^{h}(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

2) $$\frac{\partial k^{f}(\theta_{ijt})}{\partial\theta_{ijt}}\underbrace{\left[z_t-w_{ijt}+(1-\rho_x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^{f}(\theta_{jt+1})}\right)\right\}\right]}_{=\,J_{ijt}}-\varphi_{ijt}\frac{\partial k^{h}(\theta_{ijt})}{\partial\theta_{ijt}}\left[\mathbf{W}(w_{ijt})-\mathbf{U}_t\right]=0$$

Cobb-Douglas matching

$$m(s,v)=s^{\xi}v^{1-\xi}$$

$$k^{h}(\theta)=\frac{m(s,v)}{s}=m(1,\theta)=\theta^{1-\xi}$$

$$k^{f}(\theta)=\frac{m(s,v)}{v}=m(\theta^{-1},1)=\theta^{-\xi}$$

AND

$$\frac{\partial k^{h}(\theta)}{\partial\theta}=(1-\xi)\theta^{-\xi}$$

$$\frac{\partial k^{f}(\theta)}{\partial\theta}=-\xi\theta^{-\xi-1}$$

Combine and rearrange

$$\boxed{\xi\cdot\mathbf{J}(w_t)=(1-\xi)\cdot\left(\mathbf{W}(w_t)-\mathbf{U}_t\right)}$$

Exactly the Nash-bargained sharing rule with knife-edge Hosios condition (η = ξ)…

efficient

CSE – IMPLEMENTATION I

$$m(s,v) = \qquad s^{\xi} \cdot v^{1-\xi}$$

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CSE – IMPLEMENTATION I

$$m(s,v) = m^{EFF} \cdot s^{\xi} \cdot v^{1-\xi}$$

Change m^{EFF} to ensure probabalitites lie within [0, 1] boundaries

Exactly the Nash-bargained sharing rule with knife-edge Hosios condition ($\eta = \xi$)…

CSE – IMPLEMENTATION I

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What if different matching function?

$$m(s,v) = \frac{s \cdot v}{\left(s^\epsilon + v^\epsilon\right)^{1/\epsilon}}$$

denHaan, Ramey, Watson (2000 *AER*)

CSE – IMPLEMENTATION I

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$$m(.) = \frac{s \cdot \theta}{\left(1+\theta^{\epsilon}\right)^{1/\epsilon}}$$

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$$\frac{\partial k^h(\theta)}{\partial \theta} = \frac{1}{\left(1+\theta^\epsilon\right)^{1/\epsilon}} \cdot \left[1 - \frac{\theta^\epsilon}{1+\theta^\epsilon}\right] \qquad \frac{\partial k^f(\theta)}{\partial \theta} = -\frac{\theta^\epsilon}{\left(1+\theta^\epsilon\right)^{\frac{1+\epsilon}{\epsilon}} \cdot \theta}$$

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dHRW matching

$$m(.) = \frac{s\cdot\theta}{\left(1+\theta^\epsilon\right)^{1/\epsilon}}$$

$$k^h(\theta) = \frac{m(s,v)}{s} = m(1,\theta) = \frac{\theta}{\left[1+\theta^\epsilon\right]^{1/\epsilon}}$$

$$k^f(\theta) = \frac{m(s,v)}{v} = m(\theta^{-1},1) = \frac{1}{\left[1+\theta^\epsilon\right]^{1/\epsilon}}$$

$$\frac{\partial k^h(\theta)}{\partial\theta} = \frac{1}{\left(1+\theta^\epsilon\right)^{1/\epsilon}}\cdot\left[1-\frac{\theta^\epsilon}{1+\theta^\epsilon}\right] \qquad \frac{\partial k^f(\theta)}{\partial\theta} = -\frac{\theta^\epsilon}{\left(1+\theta^\epsilon\right)^{\frac{1+\epsilon}{\epsilon}}\cdot\theta}$$

CSE – IMPLEMENTATION I

❑ **First-order conditions**

1) $$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt})\mathbf{W}'(w_{ijt}) = 0 \xrightarrow{\mathbf{W}'(.) = 1} \boxed{\varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}^{-1}}$$

2) $$\underbrace{\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]}_{= J_{ijt}} - \varphi_{ijt}\underbrace{\left(\frac{\partial k^h(\theta_{ijt})/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}\right)}_{= \theta^{-\epsilon}}\left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t\right] = 0$$

dHRW matching

$$m(.) = \frac{s \cdot \theta}{\left(1+\theta^{\epsilon}\right)^{1/\epsilon}}$$

$$k^h(\theta) = \frac{m(s,v)}{s} = m(1,\theta) = \frac{\theta}{\left[1+\theta^{\epsilon}\right]^{1/\epsilon}}$$

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CSE – IMPLEMENTATION I

- ❑ **First-order conditions**

1)

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(Competition within submarket *j* and symmetry across submarkets: drop *ij* indices)

Combine and rearrange

$$\boxed{\theta_t^{-\epsilon}\cdot\left(\mathbf{W}(w_t) - \mathbf{U}_t\right) = \mathbf{J}(w_t)}$$

CSE surplus-sharing rule for dHRW m(.)

CSE VS. NASH

- ☐ **Optimal Policy Analysis?...**

CSE surplus-sharing rule for dHRW m(.)

$$\boxed{\theta_t^{-\epsilon} \cdot \left(\mathbf{W}(w_t) - \mathbf{U}_t \right) = \mathbf{J}(w_t)}$$

Nash surplus-sharing rule for *ANY* m(.)

$$\boxed{(1-\eta) \cdot \left(\mathbf{W}(w_t) - \mathbf{U}_t \right) = \eta \cdot \mathbf{J}(w_t)}$$

Nash-Hosios surplus-sharing rule for C-D m(.)

$$\boxed{(1-\xi)\left(\mathbf{W}(w_t) - \mathbf{U}_t \right) = \xi \mathbf{J}(w_t)}$$

CSE VS. NASH

- ☐ **Optimal Policy Analysis?...**

Nash Bargaining Problem....

$$\max_{w_t} \left(\mathbf{W}(w_t) - \mathbf{U}_t \right)^{\omega_t} \cdot \mathbf{J}(w_t)^{1-\omega_t}$$

CSE surplus-sharing rule for dHRW m(.)

$$\theta_t^{-\epsilon} \cdot \left(\mathbf{W}(w_t) - \mathbf{U}_t \right) = \mathbf{J}(w_t)$$

Nash surplus-sharing rule for (some…?) m(.)….

$$\left(\frac{1-\omega_t}{\omega_t} \right) \cdot \left(\mathbf{W}(w_t) - \mathbf{U}_t \right) = \mathbf{J}(w_t)$$

….that requires particular *endogenous* time variation …..

- ☐ **Implementation of efficiency via Nash bargaining with SOME weight requires**

$$\frac{\omega_t}{1-\omega_t} = \theta_t^{\epsilon}$$

CSE VS. NASH

- ☐ **Optimal Policy Analysis?...**

Nash Bargaining Problem….

$$\max_{w_t} \left(\mathbf{W}(w_t) - \mathbf{U}_t \right)^{\omega_t} \cdot \mathbf{J}(w_t)^{1-\omega_t}$$

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- ☐ **Proposal: If worker Nash bargaining power is** $\omega_t = \dfrac{\theta_t^{\epsilon}}{1+\theta_t^{\epsilon}}$

CSE VS. NASH

- ☐ **Optimal Policy Analysis?...**

Nash Bargaining Problem....

$$\max_{w_t} \left(\mathbf{W}(w_t) - \mathbf{U}_t\right)^{\omega_t} \cdot \mathbf{J}(w_t)^{1-\omega_t}$$

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$$\frac{\omega_t}{1-\omega_t} = \theta_t^{\epsilon}$$

- ☐ **Proposal: If worker Nash bargaining power is** $\omega_t = \dfrac{\theta_t^{\epsilon}}{1+\theta_t^{\epsilon}}$

(**.... several steps of algebra (Assignment 1) ...**)

$$\boxed{\Rightarrow \frac{\omega_t}{1-\omega_t} = \theta_t^{\epsilon}}$$

Plausibility?...

Realism?...

What happened to Rubinstein??...

CSE – IMPLEMENTATION II

- Need "many markets" and "many firms"
- To rationalize "competition"

- Index continuum of labor "submarkets" by *ij*
  - Same geographic region
  - Same career, regardless of geography
  - *Many* ways to interpret…..

"submarket" denotes notion of "closeness"

- Several equivalent ways to implement *perfectly* CSE

- Firms post wages before individuals search for job opportunities
- Perfectly-competitive recruiting sector
- Individuals announce wages before firms search for workers

- **Regardless of implementation, market tightness is efficient**

CSE – IMPLEMENTATION II

- ❑ **Recruiting sector intermediates labor markets**

CSE – IMPLEMENTATION II

- ❑ **Recruiting sector intermediates labor markets**

glassdoor

ZipRecruiter

onTargetjobs

healthecareers NETWORK

monster

USAJOBS
"WORKING FOR AMERICA"

indeed
one search. all jobs.

COG(n)IUS

CAREER BUILDER

LiveCareer

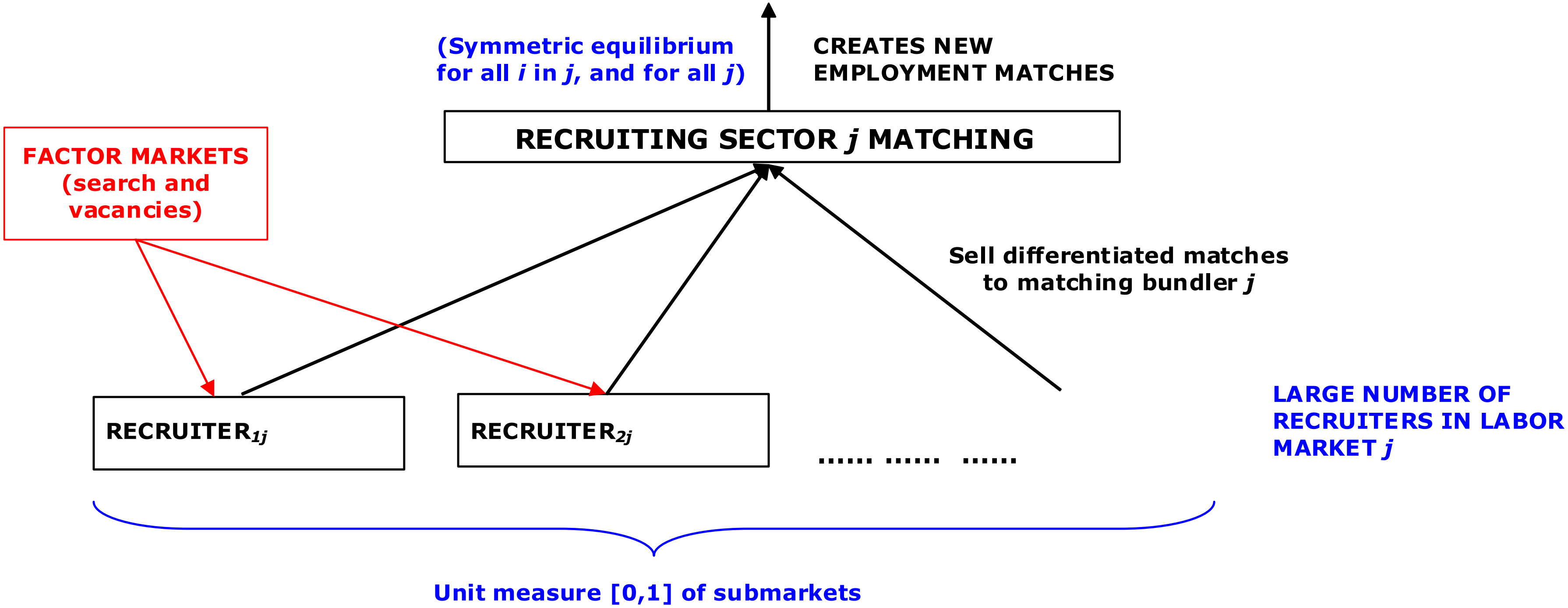
LinkedIn

EJM

LAW CROSSING

CSE – IMPLEMENTATION II

- ☐ **Recruiting sector intermediates labor markets**
- ☐ **Measure [0, 1] of recruiting markets**
  - ☐ **Perfectly-competitive – index by *j***
- ☐ **Measure [0, 1] of submarkets in recruiting market *j***
  - ☐ **Index by *ij***



(Symmetric equilibrium for all *i* in *j*, and for all *j*)

CREATES NEW EMPLOYMENT MATCHES

RECRUITING SECTOR *j* MATCHING

FACTOR MARKETS (search and vacancies)

Sell differentiated matches to matching bundler *j*

RECRUITER_{1j}

RECRUITER_{2j}

....

LARGE NUMBER OF RECRUITERS IN LABOR MARKET *j*

Unit measure [0,1] of submarkets

CSE – IMPLEMENTATION II

- Recruiting sector intermediates labor market
- Recruiter (aka, submarket) *ij* optimally chooses (w_{ij}, θ_{ij})
- Firms *direct* their vacancies ("post a job opening") to a particular submarket
  - Based on wages determined by recruiting firms in that submarket
  - And on probability of successfully contacting a worker in that submarket
- Job seekers actively *direct* their search ("send an application") to a particular submarket
  - Based on wages determined by recruiting firms in that submarket
  - And on probability of successfully contacting an open job in that submarket
- Directed search and directed vacancies the key components
- …but match formation still subject to probabilities

CSE – IMPLEMENTATION II

- ❒ Recruiting agency *ij* operates matching technology

$$\rho_{ijt} \cdot m(s_{ijt}, v_{ijt})$$

Question: In context of bargained wage models, who own/operates matching technology?….

CSE – IMPLEMENTATION II

- **Recruiting agency *ij* operates matching technology**

$$\rho_{ijt} \cdot m(s_{ijt}, v_{ijt})$$

Question: In context of bargained wage models, who own/operates matching technology?….

- **Profit function of recruiting firm *ij***

$$\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - p_{s_{jt}} s_{ijt} - p_{v_{jt}} v_{ijt}$$

- **Recruiting firm *ij***
  - **Pays p_{sjt} to s_{ijt} searchers**
  - **Pays p_{vjt} to v_{ijt} vacancies posted**

Question: In context of bargained wage models, do people get paid for their search effort?….

CSE – IMPLEMENTATION II

- **Recruiting agency *ij* operates matching technology**

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  - **Pays p_{vjt} to v_{ijt} vacancies posted**

Question: In context of bargained wage models, do people get paid for their search effort?….

- **Recruiter *ij* must incentivize labor suppliers seeking new jobs**
- **Recruiter *ij* must incentivize labor demanders to post new job openings**

CSE – IMPLEMENTATION II

- ❒ **Recruiter *ij* total profit function**

$$\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - p_{s_{jt}} s_{ijt} - p_{v_{jt}} v_{ijt}$$

CSE – IMPLEMENTATION II

- **Recruiter *ij* total profit function**

$$\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - p_{s_{jt}} s_{ijt} - p_{v_{jt}} v_{ijt}$$

- **Zero fixed costs of creating new job match**
- **Operates a constant-returns-to-scale (CRS) matching technology**
- **Marginal cost of creating a match**
  - **= average cost of creating a match**
  - **is invariant to the quantity of matches created**
- **→ mc is NOT a function mc(quantity of matches)**

CSE – IMPLEMENTATION II

- **Recruiter *ij* total profit function**

$$\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - p_{s_{jt}} s_{ijt} - p_{v_{jt}} v_{ijt}$$

- **Zero fixed costs of creating new job match**
- **Operates a constant-returns-to-scale (CRS) matching technology**
- **Marginal cost of creating a match**
  - **= average cost of creating a match**
  - **is invariant to the quantity of matches created**
- **→ mc is NOT a function mc(quantity of matches)**
- **Re-express recruiter *ij* total profit function**

$$\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m(s_{ijt}, v_{ijt})$$

CSE – IMPLEMENTATION II

- **Recruiter *ij* total profit function**

$$\left(\rho_{ijt} - mc_{jt}\right) \cdot m(s_{ijt}, v_{ijt})$$

CSE – IMPLEMENTATION II

- ❒ **Recruiter *ij* marginal profit function**

$$\left(\rho_{ijt} - mc_{jt}\right) \cdot m_{v_{ijt}}$$

CSE – IMPLEMENTATION II

- **Recruiter *ij* marginal profit function**

$$\left(\rho_{ijt} - mc_{jt}\right)\cdot m_{v_{ijt}}$$

- **Moen (1997)**
  - **ONE firm = ONE worker ("small" firms)**
  - **"Marginal" profit term vacuous in ONE-worker firm**

- **Micro origins of search and matching framework: ONE firm = ONE worker**
  - **Pissarides (1985), Mortensen and Pissarides (1994)**

CSE – IMPLEMENTATION II

- **Recruiter *ij* marginal profit function**

$$\left(\rho_{ijt} - mc_{jt}\right) \cdot m_{v_{ijt}}$$

- **Moen (1997)**
  - **ONE firm = ONE worker (“small” firms)**
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- **Micro origins of search and matching framework: ONE firm = ONE worker**
  - **Pissarides (1985), Mortensen and Pissarides (1994)**

- **Macro: Goods-producing firms that “need” / hire “many” workers**
  - **CRTS f(k,n): lack of IO structure means number of workers can be ANYTHING**
  - **DRTS f(k,n): IO structure that limits number of workers, though still large**

- **Stole and Zweibel (1996 *ReStud*) and Brugemann et al (2017 *ReStud*)**
  - **Wage bargaining model for firms with “large” number of workers and DRTS goods-production f(k,n)**

CSE – IMPLEMENTATION II

- **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right) \cdot m_{v_{ijt}}$$

subject to

$$\gamma - p_{v_{jt}} - k^{f}(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^{F} = 0$$

$$p_{s_{jt}} + k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1 - k^{h}(\theta_{ijt}))\mathbf{U}_{t} - \mathbf{X}^{H} = 0$$

m_s

CSE – IMPLEMENTATION II

- ❒ **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right) \cdot (1-\xi) \cdot k^f(\theta_{ijt})$$

Suppose

$m(s,v) = s^{\xi} v^{1-\xi}$

subject to

$$\gamma - p_{v_{jt}} - k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^F = 0$$

$$p_{s_{jt}} + k^h(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^h(\theta_{ijt}))\mathbf{U}_t - \mathbf{X}^H = 0$$

CSE – IMPLEMENTATION II

Suppose

$m(s,v) = s^{\xi} v^{1-\xi}$

- **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right) \cdot (1-\xi) \cdot k^{f}(\theta_{ijt})$$

subject to

multipliers

$$\gamma - p_{v_{jt}} - k^{f}(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^{F} = 0$$

1

$$p_{s_{jt}} + k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1 - k^{h}(\theta_{ijt}))\mathbf{U}_{t} - \mathbf{X}^{H} = 0$$

$\boldsymbol{\kappa_{ijt}}$

(given CRS m(.), only one multiplier needed)

$\downarrow$

FOCs wrt *w*$_{ijt}$ and *θ*$_{ijt}$

CSE – IMPLEMENTATION II

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1) $$-k^{f}(\theta_{ijt})\cdot\frac{\partial\mathbf{J}(w_{ijt})}{\partial w_{ijt}}+\kappa_{ijt}^{H}\cdot k^{h}(\theta_{ijt})\cdot\frac{\partial\mathbf{W}(w_{ijt})}{\partial w_{ijt}}=0$$

2) $$\left(\rho_{ijt}-mc_{jt}\right)\cdot(1-\xi)\cdot\frac{\partial k^{f}(\theta_{ijt})}{\partial\theta_{ijt}}-\frac{\partial k^{f}(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\kappa_{ijt}^{H}\cdot\frac{\partial k^{h}(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_{t}\right)=0$$

CSE – IMPLEMENTATION II

- ❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa^H_{ijt}\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \longrightarrow \boxed{\kappa^H_{ijt}=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta^{-1}_{ijt}}$$

b/c zero proportional taxation on wage

2)

$$\left(\rho_{ijt}-mc_{jt}\right)\cdot(1-\xi)\cdot\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}-\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\kappa^H_{ijt}\cdot\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

Firm Posting

CSE – IMPLEMENTATION II

❐ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot \underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1} + \kappa_{ijt}^H \cdot k^h(\theta_{ijt})\cdot \underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1} = 0 \longrightarrow \boxed{\kappa_{ijt}^H = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}^{-1}}$$

b/c zero proportional taxation on wage

2)

$$\underbrace{\left(\rho_{ijt} - mc_{jt}\right)}_{=0}\cdot(1-\xi)\cdot\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} - \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}}\cdot \mathbf{J}(w_{ijt}) + \kappa_{ijt}^H \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt}) - \mathbf{U}_t\right) = 0$$

PERFECTLY competitive recruiting sector

CSE – IMPLEMENTATION II

❐ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^{f}(\theta_{ijt})\cdot\underbrace{\frac{\partial\mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^{H}\cdot k^{h}(\theta_{ijt})\cdot\underbrace{\frac{\partial\mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0\longrightarrow\boxed{\kappa_{ijt}^{H}=-\frac{k^{f}(\theta_{ijt})}{k^{h}(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

b/c zero proportional taxation on wage

2)

$$-\frac{\partial k^{f}(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\kappa_{ijt}^{H}\cdot\frac{\partial k^{h}(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_{t}\right)=0$$

CSE – IMPLEMENTATION II

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$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa^H_{ijt}\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \longrightarrow \boxed{\kappa^H_{ijt}=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

b/c zero proportional taxation on wage

2)

$$-\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}}\cdot \mathbf{J}(w_{ijt})+\kappa^H_{ijt}\cdot\frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

Cobb-Douglas matching

$$m(s,v)=s^{\xi}v^{1-\xi}$$

$$k^h(\theta)=\frac{m(s,v)}{s}=m(1,\theta)=\theta^{1-\xi}$$

$$k^f(\theta)=\frac{m(s,v)}{v}=m(\theta^{-1},1)=\theta^{-\xi}$$

AND

$$\frac{\partial k^h(\theta)}{\partial\theta}=(1-\xi)\theta^{-\xi}$$

$$\frac{\partial k^f(\theta)}{\partial\theta}=-\xi\theta^{-\xi-1}$$

Combine and rearrange

$$\boxed{(1-\xi)\left(\mathbf{W}(w_t)-\mathbf{U}_t\right)=\xi\mathbf{J}(w_t)}$$

Exactly the Nash-bargained sharing rule with knife-edge Hosios condition (η = ξ)…

CSE – IMPLEMENTATION III

- Need "many markets" and "many firms"
- To rationalize "competition"

- Index continuum of labor "submarkets" by *ij*
  - Same geographic region
  - Same career, regardless of geography
  - *Many* ways to interpret…..

"submarket" denotes notion of "closeness"

- Several equivalent ways to implement *perfectly* CSE

- Firms post wages before individuals search for job opportunities
- Perfectly-competitive recruiting sector
- Individuals announce wages before firms direct their vacancies

- **Regardless of implementation, market tightness is efficient**

CSE – IMPLEMENTATION II

- ❒ **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}}\left(\rho_{ijt}-mc_{jt}\right)\cdot m_{s_{ijt}}$$

subject to

$$\gamma - p_{v_{jt}} - k^{f}(\theta_{ijt})\cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^{F} = 0$$

$$p_{s_{jt}} + k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^{h}(\theta_{ijt}))\mathbf{U}_{t} - \mathbf{X}^{H} = 0$$

m_v

CSE – IMPLEMENTATION II

Suppose

$m(s,v)=s^{\xi}v^{1-\xi}$

- **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}}\left(\rho_{ijt}-mc_{jt}\right)\cdot\xi\cdot k^{h}(\theta_{ijt})$$

subject to

$$\gamma-p_{v_{jt}}-k^{f}(\theta_{ijt})\cdot\mathbf{J}(w_{ijt})-\mathbf{X}^{F}=0$$

$$p_{s_{jt}}+k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt})+(1-k^{h}(\theta_{ijt}))\mathbf{U}_{t}-\mathbf{X}^{H}=0$$

CSE – IMPLEMENTATION II

- ❒ **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}}\left(\rho_{ijt}-mc_{jt}\right)\cdot\xi\cdot k^{h}(\theta_{ijt})$$

Suppose

$$m(s,v)=s^{\xi}v^{1-\xi}$$

subject to

multipliers

$$\gamma-p_{v_{jt}}-k^{f}(\theta_{ijt})\cdot\mathbf{J}(w_{ijt})-\mathbf{X}^{F}=0$$

κ_{ijt}

$$p_{s_{jt}}+k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt})+(1-k^{h}(\theta_{ijt}))\mathbf{U}_{t}-\mathbf{X}^{H}=0$$

1

(given CRS m(.), only one multiplier needed)

FOCs wrt w_{ijt} and θ_{ijt}

CSE – IMPLEMENTATION II

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-\kappa_{ijt}^{F}\cdot k^{f}(\theta_{ijt})\cdot\frac{\partial\mathbf{J}(w_{ijt})}{\partial w_{ijt}}+k^{h}(\theta_{ijt})\cdot\frac{\partial\mathbf{W}(w_{ijt})}{\partial w_{ijt}}=0$$

2)

$$\left(\rho_{ijt}-mc_{jt}\right)\cdot\xi\cdot\frac{\partial k^{h}(\theta_{ijt})}{\partial\theta_{ijt}}-\kappa_{ijt}^{F}\cdot\frac{\partial k^{f}(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\frac{\partial k^{h}(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_{t}\right)=0$$

CSE – IMPLEMENTATION II

- ❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-\kappa^F_{ijt}\cdot k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \longrightarrow \boxed{\kappa^H_{ijt}=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}}$$

b/c zero proportional taxation on wage

2)

$$\left(\rho_{ijt}-mc_{jt}\right)\cdot\xi\cdot\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}-\kappa^F_{ijt}\cdot\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

CSE – IMPLEMENTATION II

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-\kappa^F_{ijt}\cdot k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \longrightarrow \boxed{\kappa^H_{ijt}=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}}$$

b/c zero proportional taxation on wage

2)

$$\underbrace{\left(\rho_{ijt}-mc_{jt}\right)}_{=0}\cdot\xi\cdot\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}-\kappa^F_{ijt}\cdot\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

PERFECTLY competitive recruiting sector

CSE – IMPLEMENTATION II

- ❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-\kappa^F_{ijt}\cdot k^f(\theta_{ijt})\cdot \underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1} + k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1} = 0 \longrightarrow \boxed{\kappa^H_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}}$$

b/c zero proportional taxation on wage

2)

$$-\kappa^F_{ijt}\cdot\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}\cdot \mathbf{J}(w_{ijt}) + \frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

CSE – IMPLEMENTATION II

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-\kappa^F_{ijt}\cdot k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \longrightarrow \boxed{\kappa^H_{ijt}=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}}$$

b/c zero proportional taxation on wage

2)

$$-\kappa^F_{ijt}\cdot\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

Cobb-Douglas matching

$$m(s,v)=s^{\xi}v^{1-\xi}$$

$$k^h(\theta)=\frac{m(s,v)}{s}=m(1,\theta)=\theta^{1-\xi}$$

$$k^f(\theta)=\frac{m(s,v)}{v}=m(\theta^{-1},1)=\theta^{-\xi}$$

AND

$$\frac{\partial k^h(\theta)}{\partial\theta}=(1-\xi)\theta^{-\xi}$$

$$\frac{\partial k^f(\theta)}{\partial\theta}=-\xi\theta^{-\xi-1}$$

Combine and rearrange

$$\boxed{(1-\xi)\left(\mathbf{W}(w_t)-\mathbf{U}_t\right)=\xi\mathbf{J}(w_t)}$$

Exactly the Nash-bargained sharing rule with knife-edge Hosios condition (η = ξ)…