
LABOR MATCHING MODELS: EFFICIENCY PROPERTIES

MARCH 28, 2017

LABOR-MATCHING EFFICIENCY

□ Social Planning problem

□ Social Planner also subject to matching **TECHNOLOGY**

$$\max_{c_t, v_t, n_{t+1}} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$c_t + \gamma v_t = z_t n_t f(h_t) + (1 - n_t)b$$

Fix $h = 1$

$$n_{t+1} = (1 - \rho_x)n_t + m(1 - n_t, v_t)$$

And $n = 1 - u$

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Fix $h = 1$

Multipliers

 λ_t

$$n_{t+1} = (1 - \rho_x)n_t + m(1 - n_t, v_t)$$

And $n = 1 - u$ μ_t

□ FOCs

$$u'(c_t) - \lambda_t = 0$$

$$-\lambda_t \gamma + \mu_t m_2(1 - n_t, v_t) = 0$$

$$-\mu_t + \beta E_t \left\{ \lambda_{t+1} [z_{t+1} - b] \right\} + \beta E_t \left\{ \mu_{t+1} [(1 - \rho_x) - m_1(1 - n_t, v_t)] \right\} = 0$$



Eliminate multipliers

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$$\frac{\gamma}{m_2(1-n_t, v_t)} = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[z_{t+1} - b - \frac{\gamma m_1(1-n_t, v_t)}{m_2(1-n_t, v_t)} + \frac{(1-\rho_x)\gamma}{m_2(1-n_t, v_t)} \right] \right\}$$

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Cobb-Douglas
matching

$$m(u, v) = u^\alpha v^{1-\alpha}$$

$$m_1(u, v) = \alpha u^{\alpha-1} v^{1-\alpha} = \alpha \theta^{1-\alpha}$$

AND

$$k^h(\theta) = \frac{m(u, v)}{u} = m(1, \theta) = \theta^{1-\alpha}$$

$$m_2(u, v) = (1-\alpha) u^\alpha v^{-\alpha} = (1-\alpha) \theta^{-\alpha}$$

$$k^f(\theta) = \frac{m(u, v)}{v} = m(\theta^{-1}, 1) = \theta^{-\alpha}$$

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Cobb-Douglas matching

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Combine and rearrange

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KEY IDEAS

Taking the pricing kernel as given, the only unknown process here is θ_t

Efficiency in job-postings is governed by efficient market tightness

LABOR-MATCHING EFFICIENCY

- **Socially-efficient vacancy posting characterized by**

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- **Decentralized vacancy posting characterized by**

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- **Efficiency in vacancy posting requires $\eta = \alpha$!**

MORTENSEN-HOSIOS CONDITION

- Cobb-Douglas matching technology + Nash bargaining
 - Efficient level of job-creation requires $\eta = \alpha$
 - Mortensen (1982 *AER*), Hosios (1990 *ReStud*)

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- ❑ Intuition: search activity generates externalities
 - ❑ One extra **individual (firm)** searching for a **job (worker)** **LOWERS** the probability that **all other individuals (firms)** will find a match...
 - ❑ ...but **RAISES** the probability that **all other firms (individuals)** will find a match
 - ❑ **Congestion externality** – search imposes both positive and negative externalities (on opposite sides of the market)

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- ❑ Nash bargaining: η governs the **private** returns to search
 - ❑ Share of total match surplus kept by **individual**
- ❑ Cobb-Douglas matching: α governs the **social** returns to search
 - ❑ Elasticity of **aggregate** number of matches with respect to u

- ❑ Efficiency requires equating private and social returns: $\eta = \alpha$

HOSIOS CONDITION

- ❑ Also holds under some more general conditions
 - ❑ Endogenous search intensity
 - ❑ Endogenous “vacancy posting intensity” (Pissarides Chapter 5)

- ❑ Pissarides (2000, p. 198): “..we are not likely to find intuition for it...”

- ❑ RSW (2005 *JEL* p. 982): “...genuinely surprising result...”

- ❑ Is the Hosios condition empirically relevant?
 - ❑ Who knows?...it’s a **nongeneric** parameterization...
 - ❑ ...but valuable because eliminates **wage-determination frictions** but retains matching technology

- ❑ **Is Nash bargaining empirically relevant?**

HOW ARE WAGES DETERMINED?

- ❑ **Nash bargaining**
 - ❑ Underlying alternating offers bargaining game
 - ❑ The relevant outside option as bargaining is occurring?
 - ❑ Value of outside market opportunities?
 - ❑ Value of continuing negotiations? (Hall and Milgrom 2008 *AER*)

- ❑ **Proportional bargaining**

- ❑ **Rigid real wages (completely rigid or partially rigid)**

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- ❑ Bargaining theoretic wages are **ex-post of match formation**
- ❑ Seems very different from taking wages as **given (ex-ante of match formation)**

- ❑ **Competitive search equilibrium**
 - ❑ Moen (1997 *JPE*): basic static partial labor search model