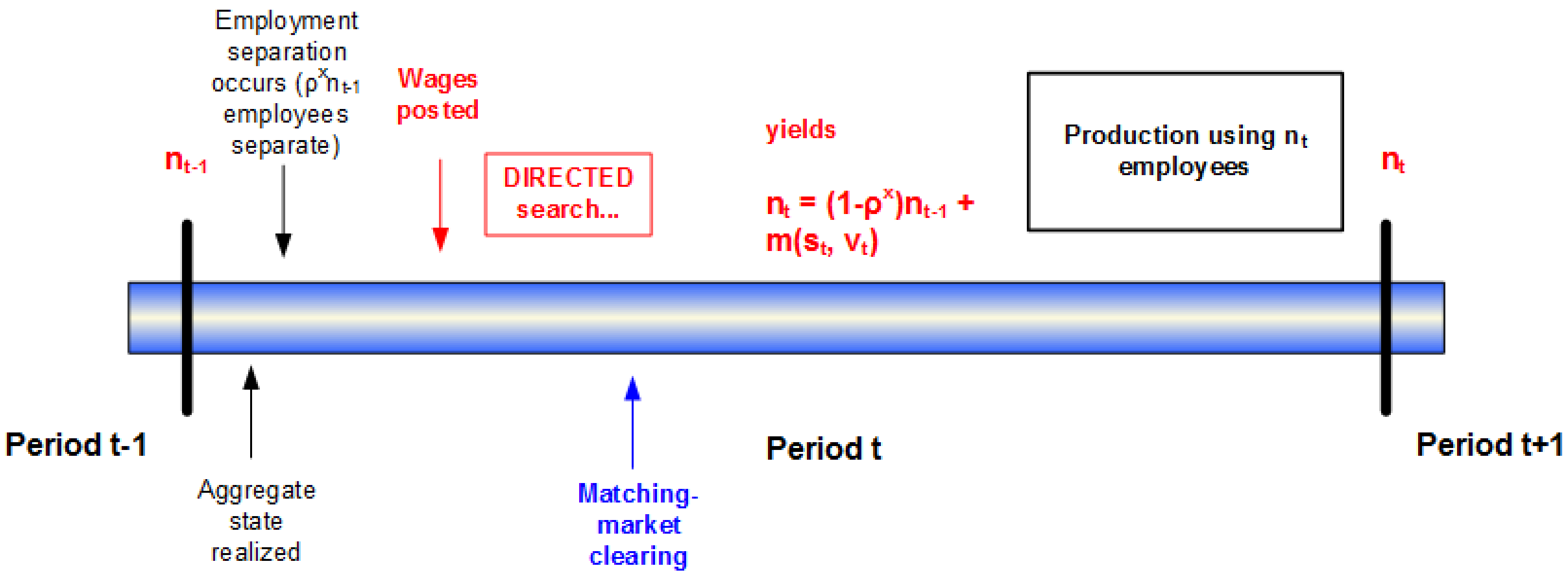
LABOR MATCHING MODELS: COMPETITIVE SEARCH EQUILIBRIUM (CSE)

MARCH 28, 2017

COMPETITIVE SEARCH EQUILIBRIUM

- Wage determination …
- … then matching market clearing



COMPETITIVE SEARCH EQUILIBRIUM (CSE)

- **Question: can a "competitive" notion of wage-setting be entertained in a search and matching model?**
  - **Would get away from the non-genericity of the Hosios bargaining parameterization**

- **May be apriori an appealing way of describing labor markets**
  - **Locating a firm or a worker is costly and time-consuming…**
  - **…but once matched, wages are more or less determined by "market forces," perhaps with little / no room for "bargaining"**

- **Moen (1997 *JPE*) and Shimer (1996) the original implementations of CSE**
  - **Static partial equilibrium labor matching models**

CSE – BASICS OF ENVIRONMENT

- **Need "many markets" and "many firms"**
  - **To rationalize "competition"**
- **Index continuum of labor "submarkets" by *j* – e.g., local labor markets**
- **Within a submarket *j*, many firms looking to hire workers**
  - **Even within a "local" labor market, coordination frictions in finding workers may exist**
  - **Index by *i***
- **Several equivalent ways to implement**
- **Perfectly-competitive "recruiting" sector**
- **Firms post wages before individuals search for jobs**
- **Individuals announce wages before firms search for workers**

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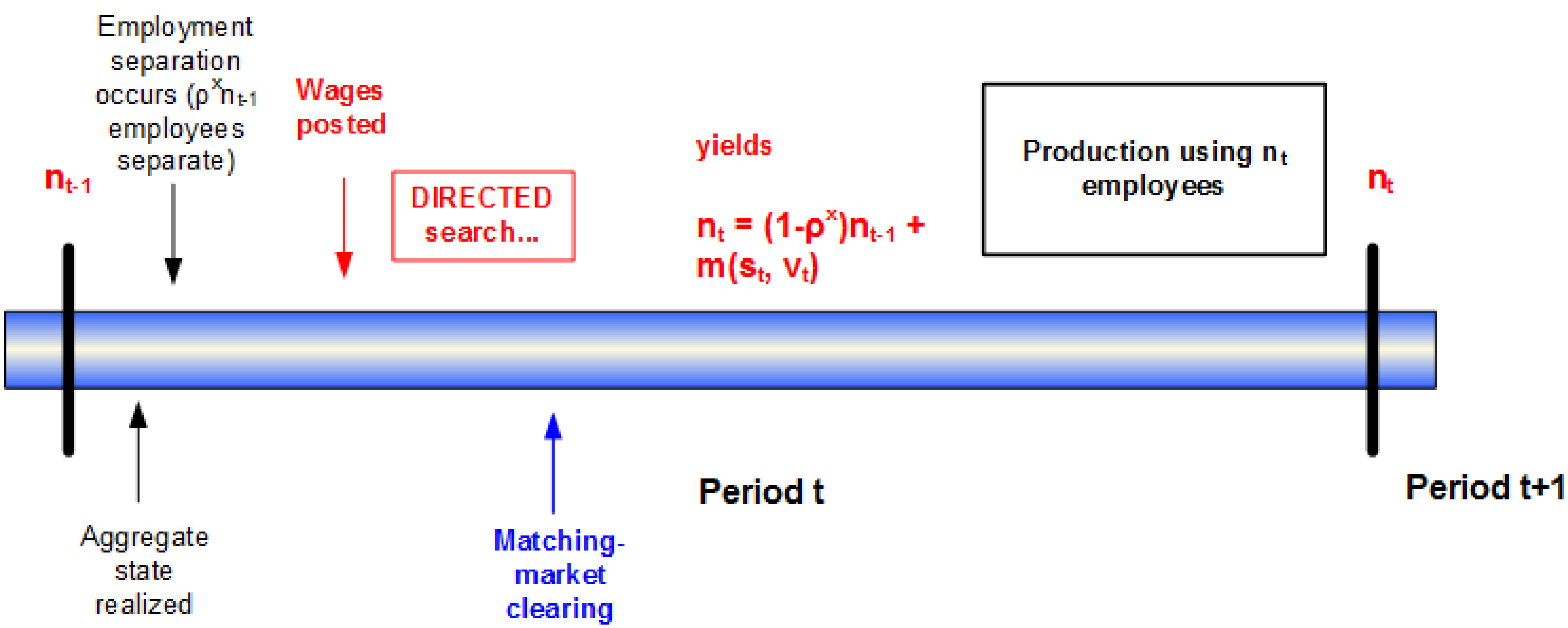
CSE – Basics of Environment

- Unemployed individuals *direct* their job search ("send an application") to a particular submarket
  - Based on wages announced by firms in that submarket
  - And on probability of getting a job in that submarket
  - Directed search a key component of CSE…
  - …but match formation still subject to probabilities
- Ordering of events
  - Wages determined before search…
  - …all parties direct search according to posted wages…
  - …then probability of landing a match resolved

TIMELINE



Competitive Search

CSE – Basics of Environment

- Idea of firm wage-posting / wage-announcement implementation
  - Define (expected) payoff function to firm *ij* of finding an additional worker
  - Define (expected) payoff function to individual searching for / applying to a job at firm *ij*
  - Firm *ij* maximizes its payoff subject to the directed search (aka reaction) function defined by the individual's payoff function
    - i.e., firm *internalizes* the effect of wages on the other side of the market…
    - …can already see how congestion *externality* issues will be taken care of…

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- Internalizing congestion externalities would also be achieved by…
  - Individuals announcing wages taking into account reactions by firms
  - "Market maker" calling out wages taking into account reactions by both sides of market

CSE – IMPLEMENTATION

- **Firm *ij* payoff function described by vacancy-posting decision**

$$\gamma = k^f(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{ \Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]$$

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of matching with a worker) x (contemporaneous payoff + continuation payoff)

Note *ij* subscripts:

Matching probability depends on tightness of "applications" at firm *ij*...

...but future asset value of employee depends on market *j* conditions (i.e., replacement value depends on (sub-)market conditions)

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- **Value equations for an individual searching for a match at firm *ij***

$$\mathbf{W}(w_{ijt}) = w_{ijt} + E_t\left\{\Xi_{t+1|t}\left[(1-\rho_x)\mathbf{W}(w_{jt+1}) + \rho_x \mathbf{U}_{t+1}\right]\right\}$$

With probability $k^h(\theta_{ijt})$, individual gets this payoff

$$\mathbf{U}_{it} = b_i + E_t\left\{\Xi_{t+1|t}\left[k^h(\theta_t)\mathbf{W}(w_{t+1}) + (1-k^h(\theta_t))\mathbf{U}_{t+1}\right]\right\}$$

With probability $1-k^h(\theta_{ijt})$, individual gets this payoff

- **With individuals (households) optimally directing their search, the expected payoff of searching for / applying to a job at firm *ij* is**

$$k^h(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^h(\theta_{ijt}))\mathbf{U}_t = X$$

Payoff of searching at another firm or another submarket independent of *ij*

CSE – IMPLEMENTATION

- **Firm *ij* maximizes**

$$\gamma = k^{f}(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{ \Xi_{t+1|t}\left(\frac{\gamma}{k^{f}(\theta_{jt+1})} \right) \right\} \right]$$

taking as constraint

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- **Choice variables: w_{ijt} and θ_{ijt} (isomorphic to choosing v_{ijt} for a given number of searchers u_{ijt})**

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- **First-order conditions**

1)

$$-k^f(\theta_{ijt}) - \varphi_{ijt}k^h(\theta_{ijt})\mathbf{W}'(w_{ijt}) = 0$$

2)

$$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}}\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{ \Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right] - \varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\left[\mathbf{W}(w_{ijt}) - \mathbf{U}_{it} \right] = 0$$

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Taking into account how matching probabilities are affected by tightness is the central idea

CSE – IMPLEMENTATION

- First-order conditions

1)
$$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) \mathbf{W}'(w_{ijt}) = 0 \quad \xrightarrow{W'(.)=1} \quad \varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}$$

2)
$$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \underbrace{\left[z_t - w_{ijt} + (1-\rho_x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]}_{= J_{ijt} \text{ if firms are optimizing}} - \varphi_{ijt} \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \left[\mathbf{W}(w_{ijt}) - \mathbf{U}_{it} \right] = 0$$

= J_{ijt} if firms are optimizing

CSE – IMPLEMENTATION

- First-order conditions

1) $$-k^f(\theta_{ijt})-\varphi_{ijt}k^h(\theta_{ijt})\mathbf{W}'(w_{ijt})=0 \xrightarrow{W'(.)=1} \varphi_{ijt}=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}$$

2) $$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}}\left[z_t-w_{ijt}+(1-\rho_x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]-\varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\left[\mathbf{W}(w_{ijt})-\mathbf{U}_{it}\right]=0$$

= J_{ijt} if firms are optimizing

Cobb-Douglas matching

$$m(u,v)=u^{\alpha}v^{1-\alpha}$$

$$k^h(\theta)=\frac{m(u,v)}{u}=m(1,\theta)=\theta^{1-\alpha}$$

$$k^f(\theta)=\frac{m(u,v)}{v}=m(\theta^{-1},1)=\theta^{-\alpha}$$

AND

$$\frac{\partial k^h(\theta)}{\partial \theta}=(1-\alpha)\theta^{-\alpha}$$

$$\frac{\partial k^f(\theta)}{\partial \theta}=-\alpha\theta^{-\alpha-1}$$

Combine and rearrange

$$\boxed{(1-\alpha)\left(\mathbf{W}(w_t)-\mathbf{U}_t\right)=\alpha\mathbf{J}(w_t)}$$

Exactly the Nash-bargaining sharing rule with endogenous emergence of Hosios condition ($\eta = \alpha$)…

CSE – Implementation

- First-order conditions

1) $-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) \mathbf{W}'(w_{ijt}) = 0 \xrightarrow{W'(.)=1} \varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}$

2) $\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \left[z_t - w_{ijt} + (1-\rho_x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right] - \varphi_{ijt} \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \left[\mathbf{W}(w_{ijt}) - \mathbf{U}_{it} \right] = 0$

= J_{ijt} if firms are optimizing

Cobb-Douglas matching

$m(u,v) = u^{\alpha} v^{1-\alpha}$

$$k^h(\theta) = \frac{m(u,v)}{u} = m(1,\theta) = \theta^{1-\alpha}$$

$$k^f(\theta) = \frac{m(u,v)}{v} = m(\theta^{-1},1) = \theta^{-\alpha}$$

AND

$$\frac{\partial k^h(\theta)}{\partial \theta} = (1-\alpha)\theta^{-\alpha}$$

$$\frac{\partial k^f(\theta)}{\partial \theta} = -\alpha \theta^{-\alpha-1}$$

Combine and rearrange

(Competition within submarket *j* and symmetry across submarkets: drop *ij* indices)

$$(1-\alpha)\left(\mathbf{W}(w_t) - \mathbf{U}_t\right) = \alpha \mathbf{J}(w_t)$$

Exactly the Nash-bargaining sharing rule with endogenous emergence of Hosios condition ($\eta = \alpha$)...

Inserting value equations and solving explicitly for wage obviously gives same outcome as Nash-bargained wage with $\eta = \alpha$...

CSE – INTERPRETATIONS

- Mortensen and Pissarides (1999 *Handbook Chapter* p. 2589-2592)
  - "Price of time" priced efficiently by markets in ex-ante CSE
  - "Price of time" generically mispriced if ex-post wage determination
  - ("Price of time" = matching probabilities, which reflect congestion externalities)
  - Bargaining equilibrium features a particular type of market incompleteness: workers and firms cannot contract on efficient surplus sharing before meeting

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  - Bargaining equilibrium features a particular type of market incompleteness: workers and firms cannot contract on efficient surplus sharing before meeting
- CSE effectively fills in this missing market…
  - …provided we're willing to assume/believe the strong degree of commitment built into CSE model
  - (i.e., each side of a job-match would have an incentive to try to "renegotiate" the "posted" wage once they actually meet)
  - An open (?) question in search theory

RELEVANCE OF HOSIOS CONDITION IN DSGE

- Optimal policy (monetary and/or fiscal) will depend on whether or not $\eta = \alpha$
  - Yet another distortion (if $\eta = \alpha$ not satisfied) for policy to respond to
  - New Keynesian models: monetary policy (absent Pigouvian fiscal policy…) can be used to correct search externalities (Cooley and Quadrini (2004 *JET*), Arseneau and Chugh (2008 *JME*), Faia (2008 *JEDC*), Blanchard and Gali (2010 *AEJ:Macro*), Ravenna and Walsh (2011 *AEJ:Macro*))

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- Model dynamics can depend (noticeably) on whether or not $\eta = \alpha$
  - Positive analysis: Walsh (2005 *RED*) the first to demonstrate this, many others since
  - Optimal policy analysis: Arseneau and Chugh (2012 *JPE*), Chugh, Merkl, and Lechthaler (2016)

- Hosios issues arise in any DGE model with any type of search and matching market
  - New monetarist models (aka, "money search" models)
    - Rocheteau and Wright (2005 *Econometrica*)
    - Aruoba and Chugh (2010 *JET*)
  - Product search models
    - Gourio and Rudanko (2014 *ReStud*)
    - Arseneau, Chahrour, Chugh, and Finkelstein (2015 *JMCB*)

DSGE (LABOR) SEARCH MODELS

- ❒ Search models articulate trading frictions – cannot instantaneously or costlessly find trading partners
  - ❒ An appealing description of labor markets
  - ❒ Maybe of other markets also
- ❒ Tractable to incorporate in DSGE models because of assumption of aggregate matching function
- ❒ Too ad-hoc or “reduced-form” because of assumption of (black box) aggregate matching friction?
- ❒ The Shimer Puzzle and attempted answers continue(?)…
- ❒ …as do New Keynesian modelers’ incorporation of labor matching structure
  - ❒ Perhaps enables talking meaningfully about the tradeoffs between inflation and unemployment…
  - ❒ …i.e., seemingly resuscitates the original Phillips Curve, not the NK Phillips Curve (which links inflation to marginal costs…)