LABOR MATCHING MODELS: FURTHER EQUILIBRIUM CONCEPTS

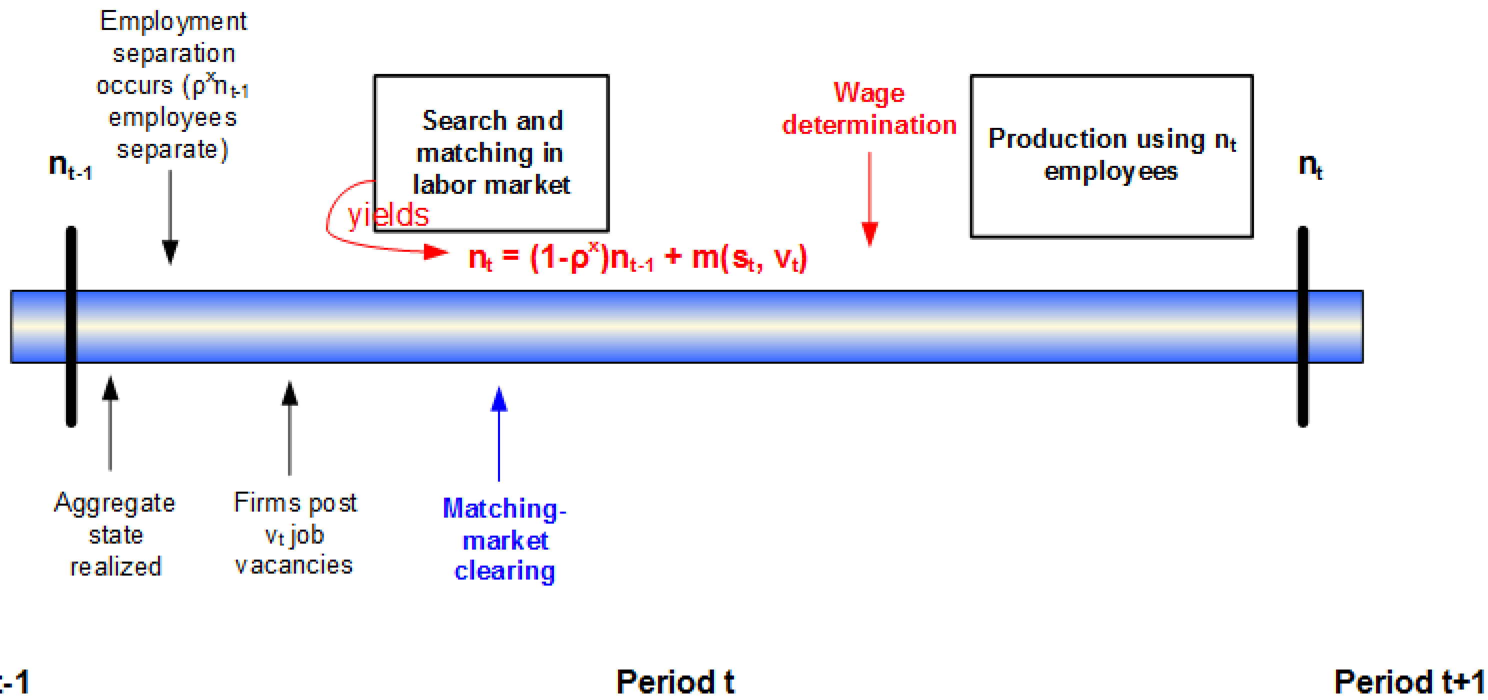
MARCH 30, 2017

GENERALIZE MATCHING MODEL

- Equilibrium concepts
- Search Equilibrium (undirected search + post-match wage determination)
  - Extensive margin generically inefficient

SEARCH EQUILIBRIUM

- **Matching market clearing …**
- **… then wage determination**



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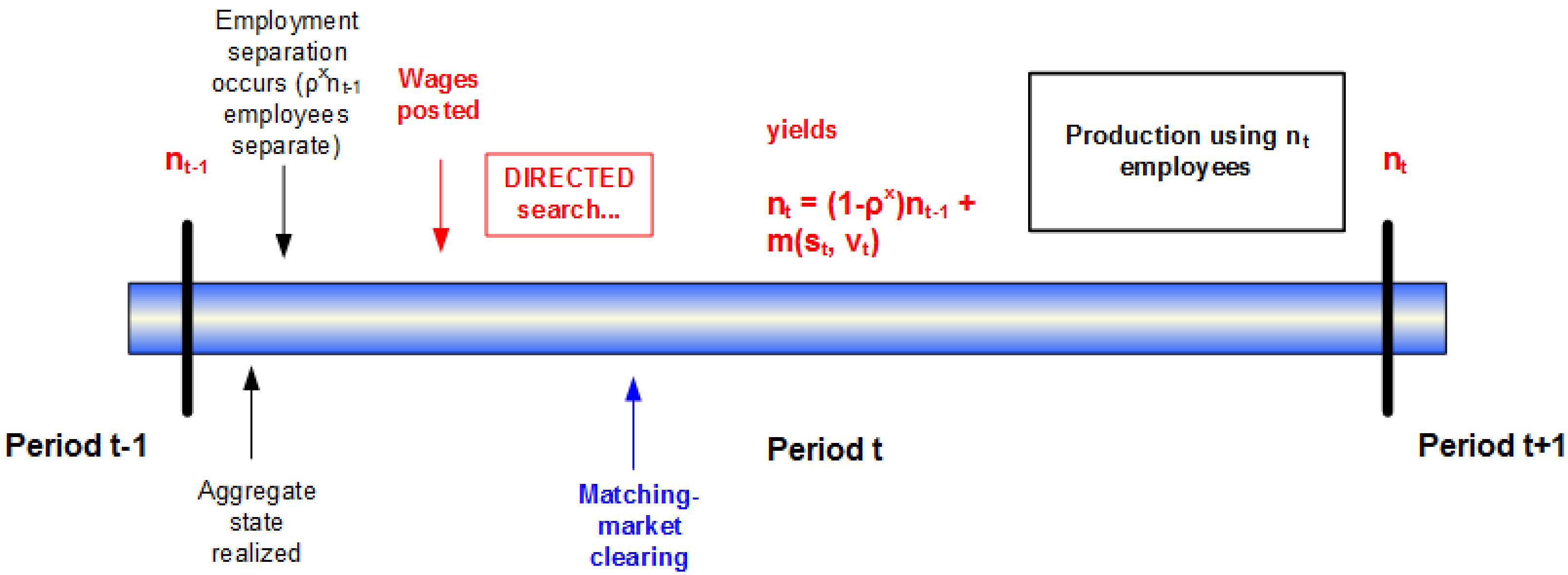
- Equilibrium concepts
- Search Equilibrium (undirected search + post-match wage determination)
  - Extensive margin generically inefficient
- Competitive Search Equilibrium (wage posting + directed search)
  - Extensive margin efficient

COMPETITIVE SEARCH EQUILIBRIUM

- ❒ Wage determination …
- ❒ … then matching market clearing



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  - **Extensive margin generically inefficient**
- **Competitive Search Equilibrium (wage posting + directed search)**
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- **Competitive Equilibrium**
  - **Requires intensive margin**

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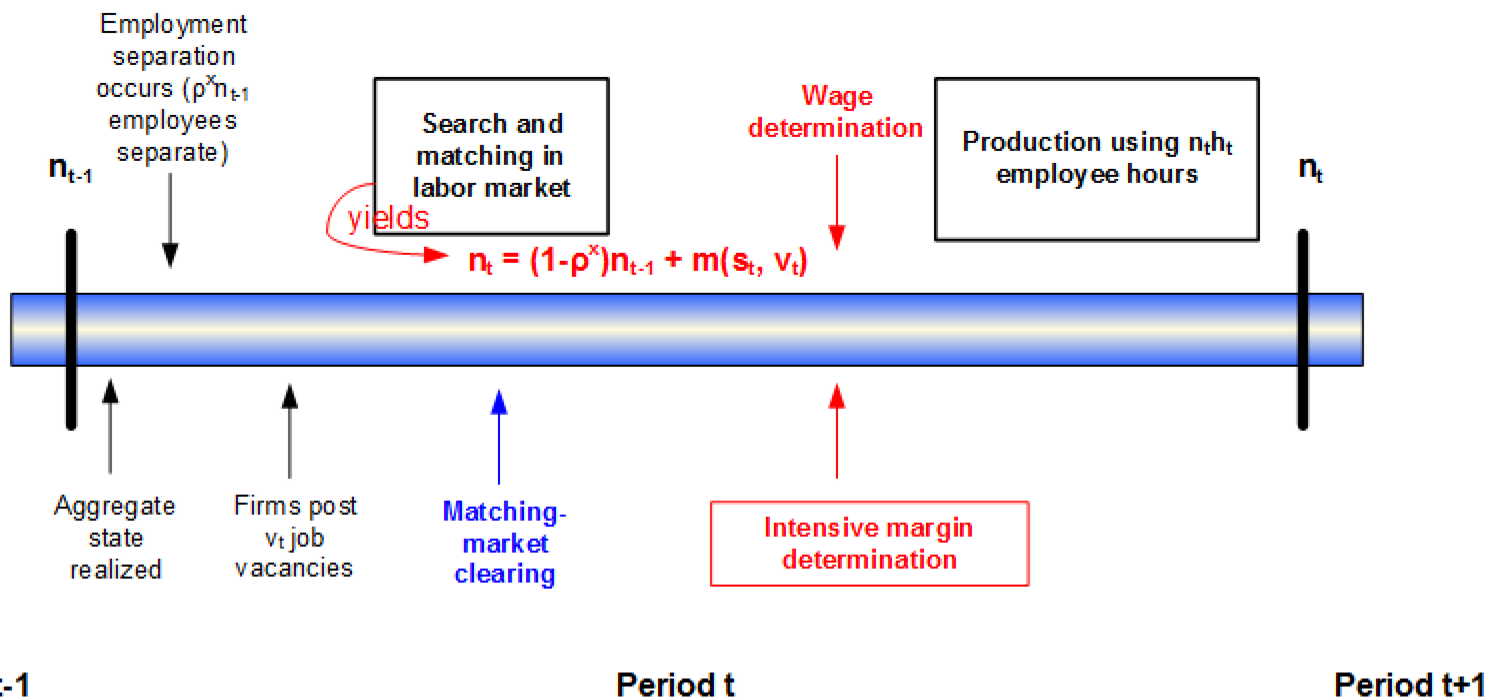

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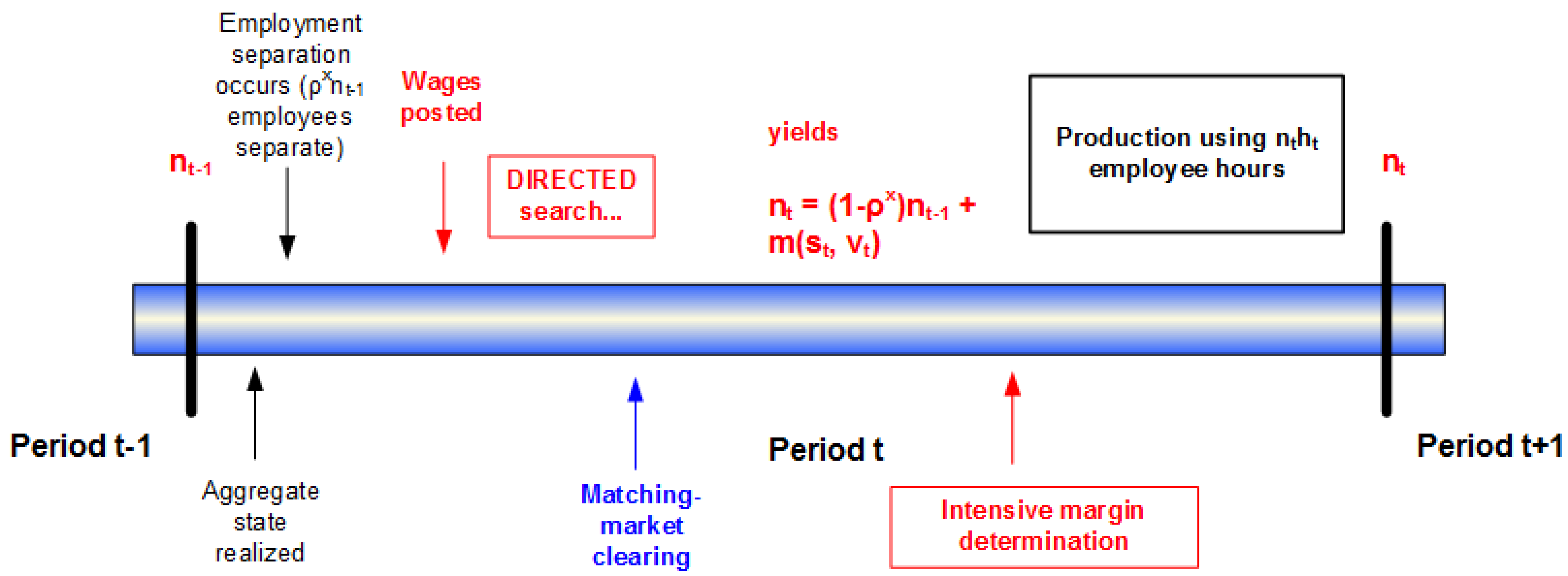

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- **Search Equilibrium (undirected search + post-match wage determination)**
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- **Competitive Search Equilibrium (wage posting + directed search)**
  - **Extensive margin efficient**
- **Competitive Equilibrium**
  - **Requires intensive margin**
  - **Intensive margin determined ex-post of match**
  - **Whether or not extensive margin is efficient**
- **Matching Market Equilibrium**

INTENSIVE MARGIN

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_t^f} E_0 \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f f(h_t) - w_t n_t^f h_t - \gamma v_t \right) \right]$$

$$\text{s.t. } n_t^f = (1-\rho_x) n_{t-1}^f + v_t k^f(\theta_t)$$

- **Total output produced by all employees =** $z \cdot n \cdot f(h)$

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- **Vacancy posting condition**

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- **How is h determined?**

INTENSIVE MARGIN

- **Dynamic household utility maximization problem**

$$\max_{c_t} E_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-h\left(n_t^h\cdot e(h_t)\right)\right)\right]$$

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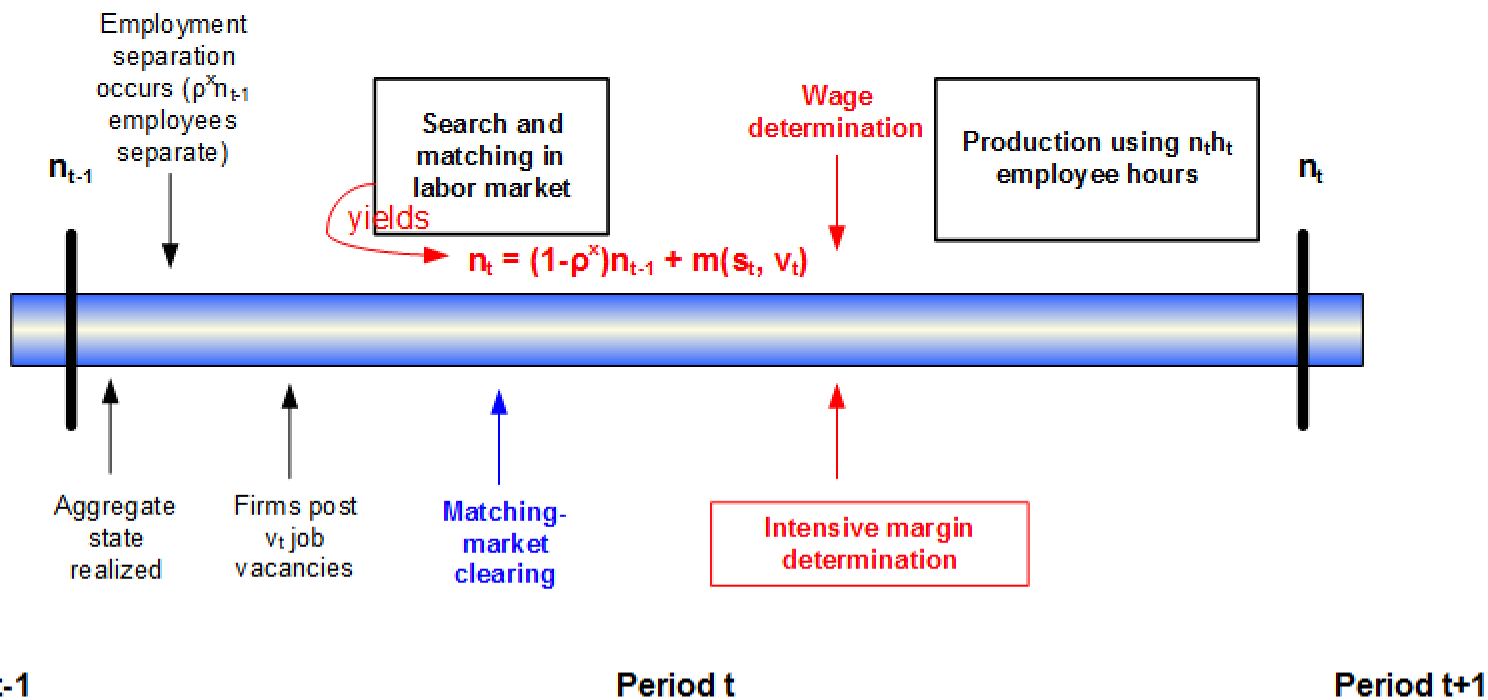

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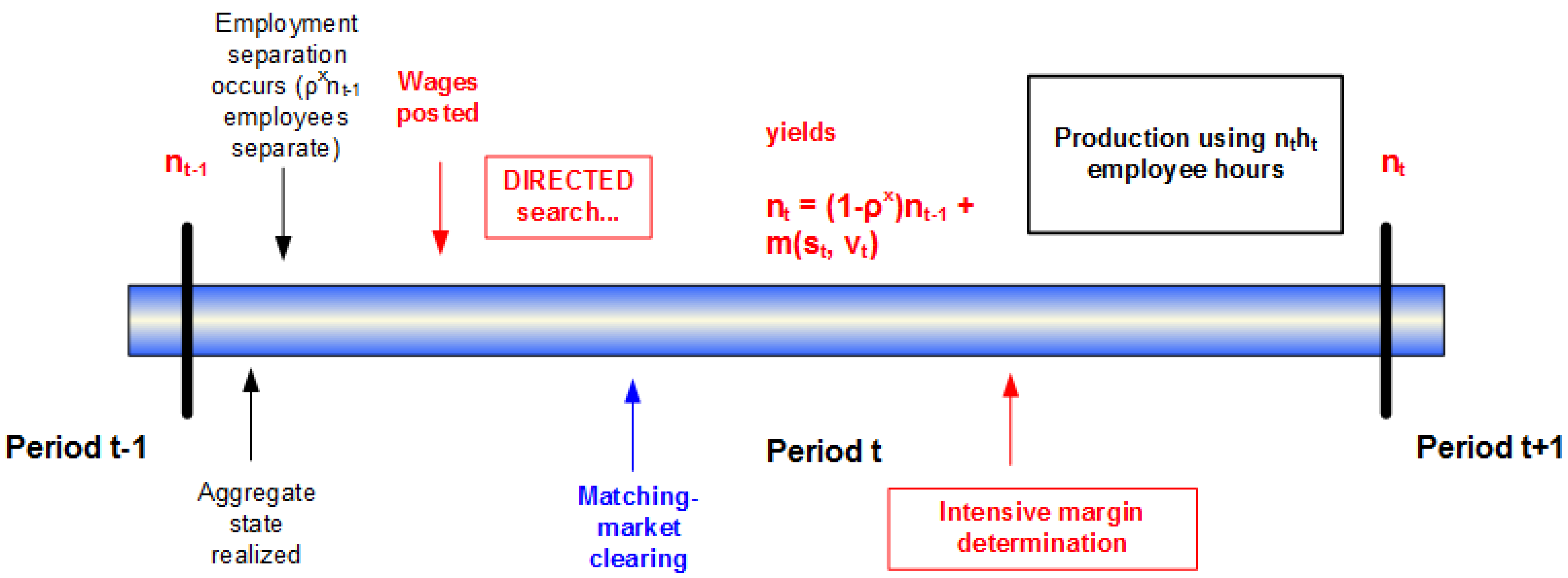

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- Common setup for post-match wages
  - Simultaneous Nash bargaining over *w* and *h*

$$\max_{w_t, h_t} \left(\mathbf{W}_t - \mathbf{U}_t \right)^{\eta} \mathbf{J}_t^{1-\eta}$$

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- Value expressions for an atomistic individual's potential new job/potential new employee

$$\mathbf{W}_t = w_t h_t - \frac{e(h_t)}{u'(c_t)} + E_t \left\{ \Xi_{t+1|t} \left[(1-\rho_x)\mathbf{W}_{t+1} + \rho_x \mathbf{U}_{t+1} \right] \right\}$$

$$\mathbf{U}_t = b + E_t \left\{ \Xi_{t+1|t} \left[k^h(\theta_{t+1})\mathbf{W}_{t+1} + (1-k^h(\theta_{t+1}))\mathbf{U}_{t+1} \right] \right\}$$

$$\mathbf{J}_t = z_t f(h_t) - w_t h_t + E_t \left\{ \Xi_{t+1|t} (1-\rho_x) \mathbf{J}_{t+1} \right\}$$

INTENSIVE MARGIN

- Compute FOCs wrt ***w*** and ***h***
- FOC wrt ***w*** yields

$$w_t h_t = \eta\left[z_t f(h_t) + \gamma\theta_t\right] + (1-\eta)b$$

Identical algebra to the ***h*** = 1 case

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$$\eta \mathbf{J}_t \left(\frac{\partial \mathbf{W}_t}{\partial h_t} - \frac{\partial \mathbf{U}_t}{\partial h_t} \right) = (1-\eta)(-1)\left(\mathbf{W}_t - \mathbf{U}_t\right)\frac{\partial \mathbf{J}_t}{\partial h_t}$$

(VERIFY THE DERIVATION)

Insert marginal values and rearrange
(a key observation is that….)

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

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PRIVATE BILATERAL EFFICIENCY

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

(…given simultaneous bargaining over *w* and *h*….)

- **Interpretation: $mrs_t = mpn_t$ for each given worker**
  - **Private bilateral efficiency on the hours margin**
  - **Whether or not Hosios efficiency holds on extensive margin**

COMPETITIVE EQUILIBRIUM

- **Competitive Equilibrium**

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- **Where?**
  - **In a particular industry…**
  - **In a particular submarket…**
  - **In a particular "island"…**

- **For whom?…**

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  - **In a particular submarket…**
  - **In a particular "island"…**

- **For whom?…**
- **Those who have already entered the submarket**
- **But entering a submarket / finding a match could be (temporarily?) inefficient due to congestion externalities in market *ij***
  - **Inefficient θ_{ij}**

FREE-ENTRY CONDITIONS

- Set $h = 1$
- Free entry into matching market for firms?
- Job-creation condition

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- **Job-creation condition (now with diminishing marg. product in *n*)**

$$\frac{\gamma}{k^f(\theta_t)} = z_t f'(n_t) - w_t n_t + (1-\rho_x) E_t \left\{ \Xi_{t+1|t} \frac{\gamma}{k^f(\theta_{t+1})} \right\}$$

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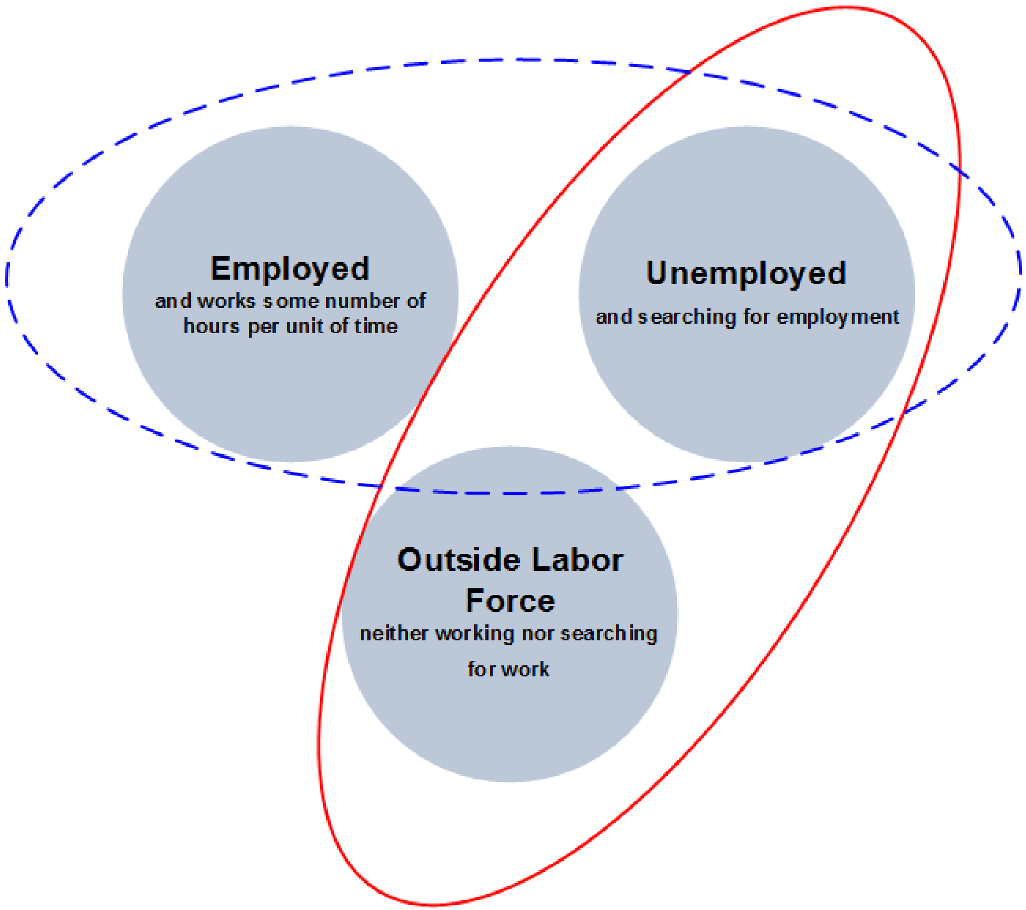
- Free entry into matching market for workers?
- Not required given CRS matching and free entry by firms…
- …but will lead to neat results
- Endogenize labor force participation (aka "labor supply")

LABOR FORCE PARTICIPATION



Allows for optimal search and optimal "rest" (outside the LFP, aka "leisure")