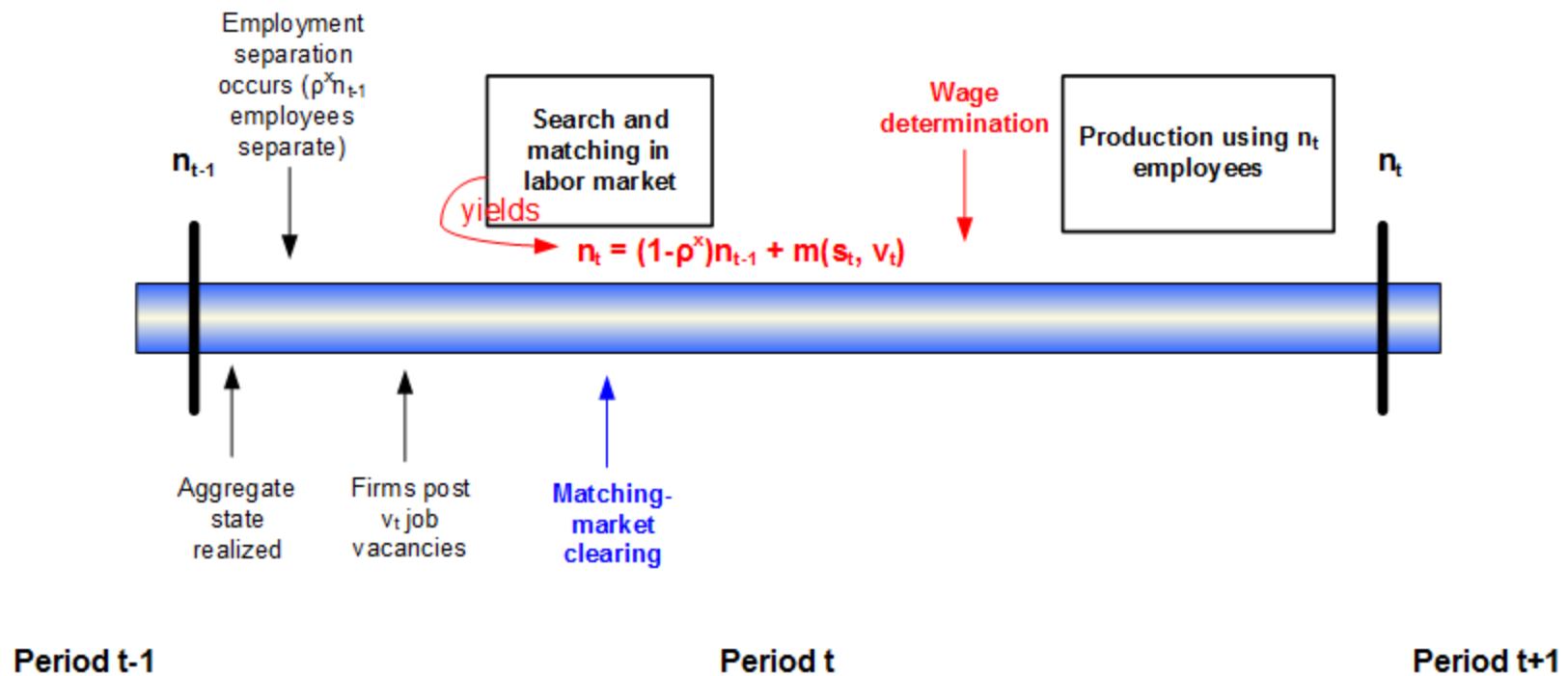

MATCHING MARKET CLEARING

MARCH 30, 2017

TIMING OF EVENTS

- Matching market clearing ...
- ... then wage determination
- Focus just on extensive margin



LABOR SUPPLY (LFP)

- Representative household

$$\max_{c_t, n_t^s, s_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h \left(\underbrace{(1 - k_t^h) s_t + n_t^s}_{\equiv lfp_t} \right) \right]$$

$$c_t = (1 - \tau_t^n) w_t n_t^s + (1 - k_t^h) s_t b$$

$$n_t^s = (1 - \rho_x) n_{t-1}^s + s_t \cdot k_t^h$$

- $lfp_t = (1 - k_t^h) s_t + n_t^s$

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- $lfp_t = (1 - k_t^h)s_t + n_t^s$

- Consumption-LFP optimality condition

$$\frac{h'(lfp_t)}{u'(c_t)} = k_t^h (1 - \tau_t^n)w_t + (1 - k_t^h) \cdot b + k_t^h (1 - \rho_x) E_t \left\{ \Xi_{t+1} \left(\frac{h'(lfp_{t+1}) - u'(c_{t+1})b}{u'(c_{t+1})} \right) \cdot \left(\frac{1 - k_{t+1}^h}{k_{t+1}^h} \right) \right\}$$

VALUE EQUATIONS

- ❑ Endogenous entry into matching market for workers?
- ❑ **Endogenous LFP allows for richer analysis**
 - ❑ How LFP responds to changes in ue benefits
 - ❑ How LFP responds to changes in taxes
 - ❑ **Nests exogenous LFP model**
- ❑ **Endogenize labor force participation (aka "labor supply")**
- ❑ Value expressions for an atomistic individual's potential new job/potential new employee

NOTE: APART
from their effects
on wages

$$W_t = w_t + E_t \left\{ \Xi_{t+1|t} \left[(1 - \rho_x) W_{t+1} + \rho_x U_{t+1} \right] \right\}$$

$$U_t = b$$

- ❑ Employment a state variable from household perspective
- ❑ Unemployment not a state variable from household perspective

LABOR SUPPLY (LFP)

- **Definition: Household optimality is a set of state-contingent functions for $\{c_t, s_t, n_t^s\}$ that satisfy**

- **Consumption-LFP optimality condition**

$$\frac{h'(lfp_t)}{u'(c_t)} = k_t^h (1 - \tau_t^n) w_t + (1 - k_t^h) \cdot b + k_t^h (1 - \rho_x) E_t \left\{ \Xi_{t+1} \left(\frac{h'(lfp_{t+1}) - u'(c_{t+1})b}{u'(c_{t+1})} \right) \cdot \left(\frac{1 - k_{t+1}^h}{k_{t+1}^h} \right) \right\}$$

- **Household budget constraint**

$$c_t = (1 - \tau_t^n) w_t n_t^s + (1 - k_t^h) s_t b$$

- **Perceived law of motion of employment (aka job-finding constraint)**

$$n_t^s = (1 - \rho_x) n_{t-1}^s + s_t \cdot k_t^h$$

taking as given exogenous processes $\{k_t^h, w_t, \tau_t^n\}$ and n_{-1}

LABOR DEMAND (JC)

□ Representative firm

$$\max_{v_t, n_t^D} E_0 \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t f(n_t^D) - w_t n_t^f - \gamma v_t \right) \right]$$

$$\text{s.t. } n_t^D = (1 - \rho_x) n_{t-1}^D + v_t k_t^f$$

□ Job-creation condition

$$\frac{\gamma}{k_t^f} = z_t f'(n_t^D) - w_t n_t^D + (1 - \rho_x) E_t \left\{ \Xi_{t+1|t} \frac{\gamma}{k_{t+1}^f} \right\}$$

LABOR DEMAND (JC)

- **Definition:** Firm optimality is a set of state-contingent functions $\{v_t, n_t^D\}$ that satisfy

- **Job-creation condition**

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$$n_t^D = (1 - \rho_x) n_{t-1}^D + v_t \cdot k_t^f$$

taking as given exogenous processes $\{k_t^f, w_t\}$ and n_{-1}

MATCHING MARKET CLEARING

- **Definition:** Equilibrium optimality is a set of state-contingent functions $\{\theta_t, v_t, s_t, n_t, c_t\}$ that satisfies aggregate RC,

- **Consumption-LFP optimality condition**

$$\frac{h'((1-k^h(\theta_t))s_t + n_t^s)}{u'(c_t)} = k^h(\theta_t)(1-\tau_t^n)w_t + (1-k^h(\theta_t)) \cdot b$$

$$+ k^h(\theta_t)(1-\rho_x)E_t \left\{ \Xi_{t+1} \left(\frac{h'(lfp_{t+1}) - u'(c_{t+1})b}{u'(c_{t+1})} \right) \cdot \left(\frac{1-k^h(\theta_{t+1})}{k^h(\theta_{t+1})} \right) \right\}$$

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$$\frac{\gamma}{k^f(\theta_t)} = z_t f'(n_t^D) - w_t n_t^D + (1-\rho_x)E_t \left\{ \Xi_{t+1|t} \frac{\gamma}{k^f(\theta_{t+1})} \right\}$$

- **Aggregate LOM for employment**

$$n_t = (1-\rho_x)n_{t-1} + m(s_t, v_t)$$

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$$n_t = (1-\rho_x)n_{t-1} + m(s_t, v_t)$$

- **Matching-market clearing** $k^h(\theta_t) \cdot s_t = k^f(\theta_t) \cdot v_t = m(s_t, v_t)$

LABOR SUPPLY (LFP)

- Consider $\rho_x = 1, \mathbf{b} = 0$
- Definition: Household optimality is a set of state-contingent functions for $\{c_t, s_t, n_t^s\}$ that satisfy

- Consumption-LFP optimality condition

$$\frac{h'((1-k_t^h)s_t + n_t^s)}{u'(c_t)} = k_t^h(1-\tau_t^n)w_t$$

- Household budget constraint

$$c_t = (1-\tau_t^n)w_t n_t^s$$

- Perceived “law of motion” of employment (aka job-finding constraint)

$$n_t^s = s_t \cdot k_t^h$$

taking as given exogenous processes $\{k_t^h, w_t, \tau_t^n\}$ and n_{-1}

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$$n_t^s = s_t \cdot k_t^h \quad \Leftrightarrow \quad s_t = \frac{n_t^s}{k_t^h}$$

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- Consumption-LFP optimality condition

$$\frac{h'\left(\frac{n_t^s}{k^h(\theta_{nt})}\right)}{u'(c_t)} = k(\theta_{nt})(1 - \tau_t^n)w_t$$

- Household budget constraint

$$c_t = (1 - \tau_t^n)w_t n_t^s$$

taking as given exogenous processes $\{k^h(\theta_{nt}), w_t, \tau_t^n\}$ and n_{-1}

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- **Definition:** Firm optimality is a set of state-contingent functions $\{v_t, n_t^D\}$ that satisfy

- **Job-creation condition**

$$\frac{\gamma}{k_t^f} = z_t f'(n_t^D) - w_t$$

- **Perceived law of motion of employment (aka job hiring constraint)**

$$n_t^D = v_t \cdot k_t^f$$

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- **Aggregate RC** $c_t + \gamma \cdot v_t = z_t f(n_t)$

- **Consumption-LFP optimality condition**

$$\frac{h'\left(\frac{n_t}{k^h(\theta_{nt})}\right)}{u'(c_t)} = k^h(\theta_{nt})(1 - \tau_t^n)w_t$$

Solve for θ using hh job finding constraint



- **Job-creation condition**

$$\frac{\gamma}{k^f(\theta_{nt})} = z_t f'(n_t) - w_t$$



Solve for θ using firm job hiring constraint

- **Aggregate LOM for employment**

$$n_t = m(s_t, v_t)$$

- **Matching-market clearing** $k^h(\theta_{nt}) \cdot s_t = k^f(\theta_{nt}) \cdot v_t = m(s_t, v_t)$

MATCHING MARKET CLEARING

□ Functional forms

$$m(s_t, v_t) = s_t^{\alpha_n} v_t^{1-\alpha_n} \Rightarrow m_s(s_t, v_t) = \alpha_n s_t^{\alpha_n-1} v_t^{1-\alpha_n} = \alpha_n \theta_{nt}^{1-\alpha_n} \quad m_v(s_t, v_t) = (1-\alpha_n) s_t^{\alpha_n} v_t^{-\alpha_n} = (1-\alpha_n) \theta_{nt}^{-\alpha_n}$$

$$k^h(\theta_{nt}) = \theta_{nt}^{1-\alpha_n} \quad k^f(\theta_{nt}) = \theta_{nt}^{-\alpha_n}$$

$$f(n_t) = n_t^\alpha \Rightarrow f'(n_t) = \alpha n_t^{\alpha-1}$$

$$h(lfp_t) = \frac{\psi_n}{1+1/\varphi_n} lfp_t^{1+1/\varphi_n} \Rightarrow h'(lfp_t) = \psi_n lfp_t^{1/\varphi_n} \quad u(c_t) = c_t \Rightarrow u'(c_t) = 1$$

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$$\frac{h'\left(\frac{n_t}{k^h(\theta_{nt})}\right)}{u'(c_t)} = k^h(\theta_{nt}) w_t$$

$$\frac{\gamma}{k^f(\theta_{nt})} = z_t f'(n_t) - w_t$$

Substitute...(and set tax rate = 0)

MATCHING MARKET CLEARING

- In (θ, n) space

$$\psi_n \cdot (n_t \cdot \theta_{nt}^{\alpha_n - 1})^{1/\varphi_n} = \theta_{nt}^{1 - \alpha_n} w_t$$

$$\gamma = \theta_{nt}^{-\alpha_n} (z_t \alpha n_t^{\alpha - 1} - w_t)$$

...and solve for θ^{LFP} and θ^{JC}

$$\theta_{nt}^{LFP} = \left[\frac{w_t}{\psi_n \cdot n_t^{1/\varphi_n}} \right]^{-\frac{1}{(1 - \alpha_n)(1 + 1/\varphi_n)}}$$

$$\theta_{nt}^{JC} = \left[\frac{z_t \alpha n_t^{\alpha - 1} - w_t}{\gamma} \right]^{\frac{1}{\alpha_n}}$$

MATCHING MARKET CLEARING

- In (θ, n) space

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$$\theta_{nt}^{JC} = \left[\frac{z_t \alpha n_t^{\alpha - 1} - w_t}{\gamma} \right]^{\frac{1}{\alpha_n}}$$

- Arseneau and Chugh (2012 *JPE*, Proposition 1 (p. 950))

- Efficient allocations characterized by

$$\left(MRS_{c_t, lfp_t} \equiv \right) \frac{h'(lfp_t)}{u'(c_t)} = \frac{\gamma \cdot m_s(s_t, v_t)}{m_v(s_t, v_t)} \left(\equiv MRT_{c_t, lfp_t} \right)$$

- Given functional forms, stated in (θ, n) space

$$\theta_{nt}^{LFP, EFF} = \theta_{nt}^{JC, EFF}$$

Obvious in (decentralized equilibrium that supports...) efficient allocations....

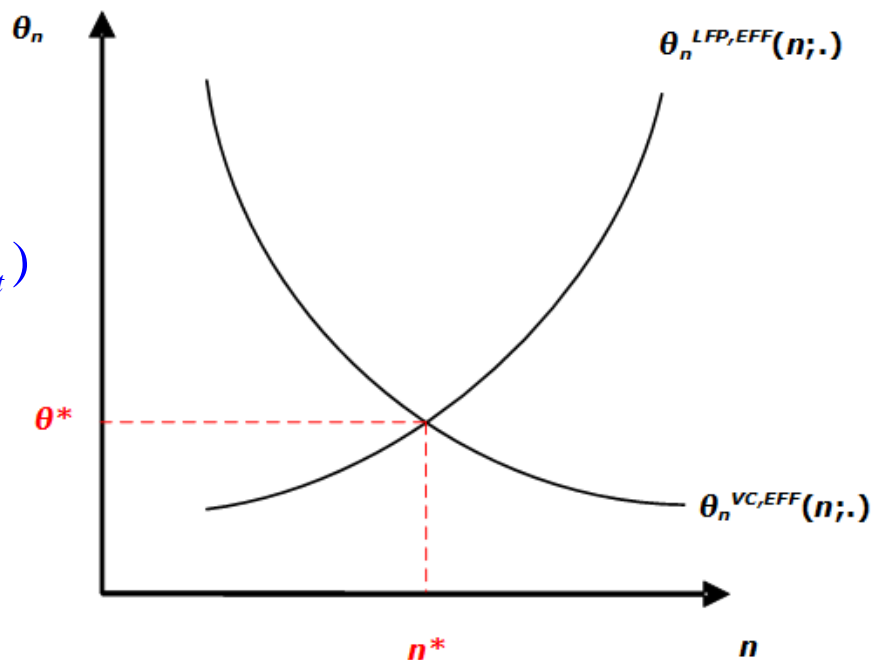
MATCHING MARKET CLEARING

- Aggregate matching-market clearing

Aggregate matching
market clearing

$$k_t^h \cdot s_t = k_t^f \cdot v_t = m(s_t, v_t)$$

\equiv new employees_t



MATCHING MARKET CLEARING

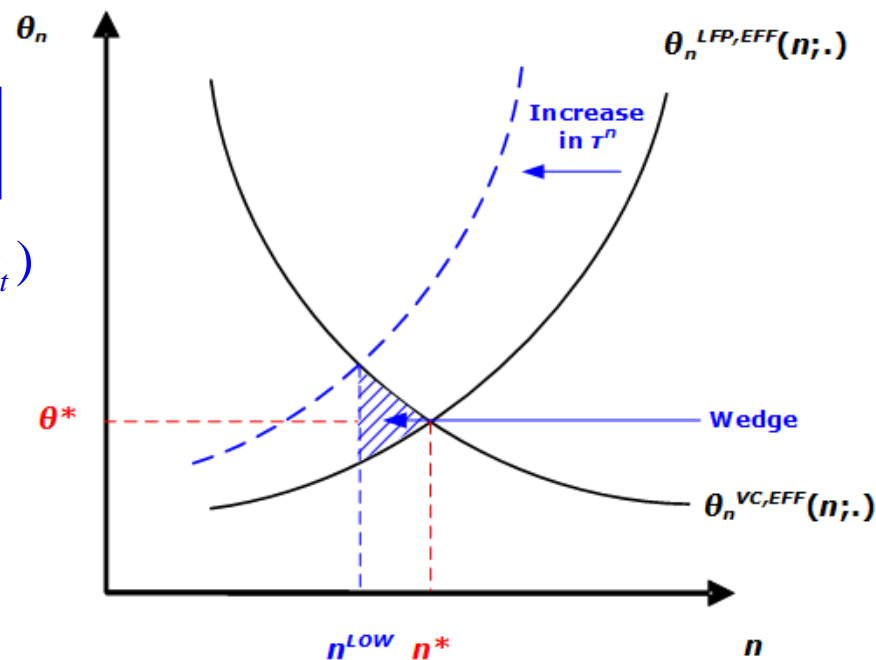
- ❑ Aggregate matching-market clearing
- ❑ Experiment 1: labor income tax rate $\tau^n > 0$

$$WEDGE_{n,\tau^n} \equiv \left| \int_{n^{LOW}}^{n^*} [\theta_n^{LFP,EFF}(n_i;\cdot) - \theta_n^{VC,EFF}(n_i;\cdot)] dn_i \right|$$

Aggregate matching market clearing

$$k_t^h \cdot s_t = k_t^f \cdot v_t = m(s_t, v_t)$$

\equiv new employees_t



MATCHING MARKET CLEARING

- ❑ Aggregate matching-market clearing
- ❑ Experiment 2: unemployment benefit $b > 0$

$$WEDGE_{n,b} \equiv \left| \int_{n^*}^{n^{HIGH}} \left[\theta_n^{LFP,EFF}(n_i; \cdot) - \theta_n^{VC,EFF}(n_i; \cdot) \right] dn_i \right|$$

Units of wedge?....

Aggregate matching market clearing

$$k_t^h \cdot s_t = k_t^f \cdot v_t = m(s_t, v_t)$$

≡ new employees_t

