OPTIMAL FISCAL POLICY

FEBRUARY 7, 2018

Basics of Ramsey Analysis

- **Ramsey (1927 *Economic Journal*)**

A CONTRIBUTION TO THE THEORY OF TAXATION

THE problem I propose to tackle is this: a given revenue is to be raised by proportionate taxes on some or all uses of income, the taxes on different uses being possibly at different rates; how should these rates be adjusted in order that the decrement of utility may be a minimum?

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- **Stiglitz (2014 NBER working paper)**
  - **Pros and cons of Ramsey taxation framework**
  - **"In Praise of Frank Ramsey's Contribution to the Theory of Taxation"**

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- **Maintained assumptions**
  - **Lack of lump-sum taxes (the starting point of Ramsey 1927)**

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  - **Household optimality conditions**

$$\frac{u_1(c_1,c_2,c_3)}{u_2(c_1,c_2,c_3)}=\frac{(1-\tau_1)\cdot p_1}{(1-\tau_2)\cdot p_2} \qquad \frac{u_2(c_1,c_2,c_3)}{u_3(c_1,c_2,c_3)}=\frac{(1-\tau_2)\cdot p_2}{(1-\tau_3)\cdot p_3}$$

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  - **Completeness of tax instruments exists if, given a Ramsey allocation**
    - **There is ≥ one tax rate on each MRS = price ratio condition and…**
    - **…there is a unique mapping from the Ramsey allocation to a set of tax rates**

Basics of Ramsey Macro Fiscal Policy

- **Ramsey problem**

$$\max_{\{c_t, n_t, k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(n_t) \right] \quad \textbf{s.t.}$$

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f\left(k_t, n_t\right)$$

<u>Sequence</u> of Lagrange multipliers $\beta^t \lambda_t$

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Present-value implementability constraint (PVIC): the PV GBC

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- **Ramsey FOCs (for $t > 0$, which sidesteps issue of taxation of $t = 0$ initial capital stock and other assets, of which A_0 is a function)**
  - **Commitment by Ramsey government to its $t > 0$ policies at $t = 0$**
  - **Discretionary Ramsey government does *not* commit to its $t > 0$ policies at $t = 0$**

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Evaluate at deterministic steady states

BASICS OF RAMSEY MACRO FISCAL POLICY

- **Ramsey FOCs (for $t > 0$) at deterministic steady state**

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- **Social Planner FOCs at deterministic steady state**

$$u'(c^{SP}) - \lambda^{SP} = 0 \qquad (4)$$

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- **(3) and (6) imply Ramsey-optimal *k/n* ratio = efficient *k/n* ratio**
  - **(Given maintained assumption of CRS production *f*(.))**
  - **A crucial result!**
  - **Second-best *k/n* ratio = first-best *k/n* ratio**
  - **Chamley (1986 *ECTA*), Judd (1985 *JPub*) seminal references**

ZERO CAPITAL INCOME TAX

- What does this imply for Ramsey-optimal tax rates?
- Recall household optimization
  - With labor income tax and capital income tax (and no lump-sum taxes)

$$\max_{\{c_t, n_t, k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(n_t) \right] \quad \textbf{s.t.} \quad c_t + k_{t+1} = (1-\tau_t^n) w_t n_t + \left[1 + (1-\tau_t^k)(r_t - \delta) \right] k_t$$

  - Steady-state consumption-labor optimality (labor supply condition)

$$\frac{h'(n)}{u'(c)} = (1-\tau^n) z \cdot f_n(k,n)$$

= *w* in equilibrium

  - Steady-state consumption-savings optimality (capital Euler condition)

= *r* in equilibrium

$$u'(c) = \beta u'(c) \left(1 + (1-\tau^k)(z \times f_k(k,n) - \delta) \right)$$

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  - **Ramsey-optimal capital income tax rate = 0!**
  - **Don't tax intertemporal margin at all in the long run…**
  - **…even though Ramsey government has to raise revenue through distortionary taxes**

Positive Labor Income Tax

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  - Ramsey-optimal capital income tax rate = 0!
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  - …even though Ramsey government has to raise revenue through distortionary taxes
- All revenue must be raised through positive labor income tax
- Two central Ramsey macro fiscal policy results

DYNAMICS OF TAX RATES

- Outside the steady state?
- Focus on labor income tax rate (simple to consider)

  - Consumption-labor optimality (labor supply condition)

$$\frac{h'(n_t)}{u'(c_t)} = (1-\tau_t^n) z_t f_n(k_t, n_t)$$

Dynamics of Tax Rates

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$$\underbrace{\frac{h'(n_t)}{u'(c_t)}}_{= MRS_t} = (1-\tau_t^n)\underbrace{z_t f_n(k_t, n_t)}_{= MPN_t}$$

$$\rightarrow MRS_t = (1-\tau_t^n) MPN_t$$

- ❑ **Labor income tax is a wedge between labor supply and labor demand**

- ❑ **Along the business cycle?**
  - ❑ **Consider utility form** $u(c_t) - h(n_t) = \ln c_t - \frac{k}{1+1/\iota} n_t^{1+1/\iota}$

ι is labor supply elasticity with respect to real wage

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  - ...which are needed to compute W_c(.) and W_n(.)
- Do some algebra combining the Ramsey FOCs ...

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- **Wedge is a (endogenous…) constant between MRS and MPN in every time period**
  - **μ = 0 (the case of lump-sum taxes) ➔ wedge = 0**
  - **μ > 0 (the Ramsey case) ➔ wedge ≠ 0**

Dynamics of Tax Rates

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= MRS_t = wedge between MRS_t and MPN_t = MPN_t

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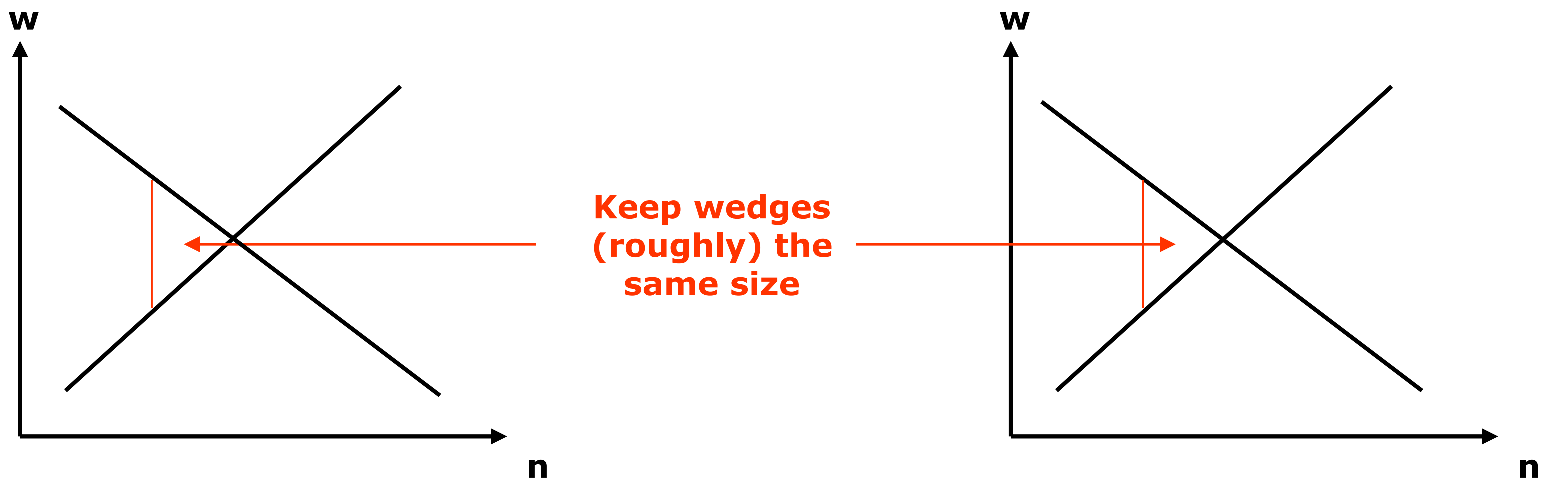
- **…thus labor income tax rate is constant over time (for this utility form)**
  - **Nearly constant if move to slightly different *h*(*n*) function**

- **Labor income tax smoothing**
  - **Key Ramsey macro fiscal policy result**
  - **Keep deadweight losses constant across markets over time**
    - **aka wedges constant**

TAX SMOOTHING VS. WEDGE SMOOTHING



Period *t*

Period *t*+1

- **Ramsey wants to keep these wedges constant**

$$MRS_t = \text{WEDGE}_t \cdot MPN_t \;\forall t$$

Wedge (Walrasian labor market)

GENERAL EQUILIBRIUM WEDGES

FEBRUARY 7, 2018

TRANSFORMATION FUNCTION

- Construct model-consistent transformation function

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"The production set is taken as a primitive datum of the theory...If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

$$MRT_{l,k}(y) = \frac{\partial F(y)/\partial y_l}{\partial F(y)/\partial y_k}$$

is called the *marginal rate of transformation (MRT) of good l for good k at vector y*...

***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

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…A single-output technology is commonly described by means of a *production function f*(*z*)…Holding the level of output fixed, we can define the *marginal rate of technical substitution* (*$MRTS_{l,k}$*) … Note that *$MRTS_{l,k}$* is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."

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- **Does one-unit decrease in 1-n_t affect c_t?**
  - **If so, how?**

- **Does one-unit decrease in c_t affect c_{t+1}?**
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- **Transformation function of RBC model**

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Goods resource constraint

- **One-unit decrease in 1-n_t → one-unit increase in n_t**

- **One-unit increase in n_t → output increases by $z_t f_n(k_t, n_t)$ units**

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- **Increase of output by $z_t f_n(k_t, n_t)$ units → c_t increases by $z_t f_n(k_t, n_t)$ units**

$$MRT_{c_t, n_t} \equiv z_t f_n(k_t, n_t)$$

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- **One-unit decrease in c_t → one-unit increase in k_{t+1}**
- **One-unit increase in k_{t+1} → output increases by $z_{t+1}f_k(k_{t+1},n_{t+1})$ units**
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TRANSFORMATION FUNCTION

- **Transformation function of RBC model (between *t* and *t* + 1)**

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t) \qquad c_{t+1} + g_{t+1} + k_{t+2} - (1-\delta)k_{t+1} = z_{t+1} f(k_{t+1}, n_{t+1})$$

- **One-unit decrease in c_t → one-unit increase in k_{t+1}**
- **One-unit increase in k_{t+1} → output increases by $z_{t+1}f_k(k_{t+1},n_{t+1})$ units**
- **Increase of output by $z_{t+1}f_k(k_{t+1},n_{t+1})$ units**
- **→ c_{t+1} increases by**

$$MRT_{c_t, c_{t+1}} \equiv 1 + z_{t+1} f_k(k_{t+1}, n_{t+1}) - \delta$$

TRANSFORMATION FUNCTION

- **Construct model-consistent transformation function**

"The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

$$MRT_{l,k}(y) = \frac{\partial F(y)/\partial y_l}{\partial F(y)/\partial y_k}$$

is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…

…A single-output technology is commonly described by means of a *production function f*(*z*)…Holding the level of output fixed, we can define the *marginal rate of technical substitution (MRTS$_{l,k}$)* … Note that *MRTS$_{l,k}$* is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."

***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

- **RBC model**

- **Does one-unit decrease in 1-n_t affect c_t?**
  - $MRT_{c_t,n_t} \equiv z_t f_n(k_t, n_t)$

- **Does one-unit decrease in c_t affect c_{t+1}?**
  - $MRT_{c_t,c_{t+1}} \equiv 1 + z_{t+1} f_k(k_{t+1}, n_{t+1}) - \delta$

Labor Search and Matching: General Equilibrium Wedges

February 7, 2018

MATCHING EFFICIENCY

- **Social Planner**

$$\max_{\{c,n_t,s_t,v_t\}} E_0 \sum_{t=0}^{\infty} b^t \left[u(c_t) - h\left(lfp_t\right) \right]$$

$lfp_t \equiv (1-p_t)s_t + n_{t-1}$

s.t.

$$c_t + g_t + \gamma v_t = z_t n_t$$

Resource constraint

$$n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

Aggregate LOM for total employment

Matching Efficiency

- **Social Planner**

$$\max_{\{c,n_t,s_t,v_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(lfp_t\right) \right]$$

$lfp_t \equiv (1-p_t)s_t + n_{t-1}$

s.t.

$$c_t + g_t + \gamma v_t = z_t n_t$$

Resource constraint

$$n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

Aggregate LOM for total employment

FOCs

(consider deterministic case)

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{\gamma m_s(s_t,v_t)}{m_v(s_t,v_t)}$$

$$= \gamma q_t \frac{\xi}{1-\xi}$$

Static Efficiency Condition.

"Efficient Participation Condition"

Can instead derive directly off transformation frontier of model.

Matching Efficiency

- **Social Planner**

$$\max_{\{c,n_t,s_t,v_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(lfp_t\right) \right]$$

$lfp_t \equiv (1-p_t)s_t + n_{t-1}$

s.t.

$$c_t + g_t + \gamma v_t = z_t n_t$$

Resource constraint

$$n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

Aggregate LOM for total employment

FOCs

(consider deterministic case)

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{\gamma m_s(s_t,v_t)}{m_v(s_t,v_t)}$$

$$= \gamma q_t \frac{\xi}{1-\xi}$$

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(\frac{\gamma}{m_v(s_{t+1},v_{t+1})}\right)\left(1-m_s(s_{t+1},v_{t+1})\right)}{\frac{\gamma}{m_v(s_t,v_t)} - z_t}$$

Static Efficiency Condition.

"Efficient Participation Condition"

Can instead derive directly off transformation frontier of model.

Intertemporal Efficiency Condition.

"Efficient Vacancies Condition"

Can instead derive directly off transformation frontier of model.

TRANSFORMATION FRONTIER

- **Construct model-consistent transformation function**

"The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

$$MRT_{l,k}(y) = \frac{\partial F(y)/\partial y_l}{\partial F(y)/\partial y_k}$$

is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…

…A single-output technology is commonly described by means of a *production function f*(*z*)…Holding the level of output fixed, we can define the *marginal rate of technical substitution* (*$MRTS_{l,k}$*) … Note that *$MRTS_{l,k}$* is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."

***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

TRANSFORMATION FRONTIER

- **Ceteris paribus…**
- **Does one-unit decrease in (1 – lfp_t) affect c_t?**
  - **If so, how?**
- **Does one-unit decrease in c_t affect c_{t+1}?**
  - **If so, how?**

TRANSFORMATION FRONTIER

- **Transformation function**

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho) n_{t-1} + m(s_t, v_t)$$

TRANSFORMATION FRONTIER

- **Transformation function**

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho) n_{t-1} + m(s_t, v_t)$$

- → $v_t = \dfrac{z_t n_t - c_t}{\gamma}$

- **Insert into LOM for *n*$_t$ to construct** $n_t - (1-\rho) n_{t-1} - m\left(s_t, \dfrac{z_t n_t - c_t}{\gamma}\right) = 0$

- **Use *lfp*$_t$ = (1-*ρ*)*n*$_{t-1}$ + *s*$_t$ to construct within-period transformation frontier**

$$\Gamma\left(c_t, lfp_t, n_t; \cdot\right) \equiv n_t - (1-\rho) n_{t-1} - m\left(lfp_t - (1-\rho) n_{t-1}, \frac{z_t n_t - c_t}{\gamma}\right) = 0$$

TRANSFORMATION FRONTIER

- **Transformation function**

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

- → $v_t = \dfrac{z_t n_t - c_t}{\gamma}$

- **Insert into LOM for n_t to construct** $n_t - (1-\rho)n_{t-1} - m\left(s_t, \dfrac{z_t n_t - c_t}{\gamma}\right) = 0$

- **Use $lfp_t = (1\text{-}\rho)n_{t\text{-}1} + s_t$ to construct within-period transformation frontier**

$$\Gamma\left(c_t, lfp_t, n_t; \cdot\right) \equiv n_t - (1-\rho)n_{t-1} - m\left(lfp_t - (1-\rho)n_{t-1}, \frac{z_t n_t - c_t}{\gamma}\right) = 0$$

- **Use IFT to obtain static MRT (participation margin)**

$$MRT_{c_t, lfp_t} = -\frac{\Gamma_{lfp_t}}{\Gamma_{c_t}} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

STATIC **MRT between LFP and Walrasian good**

TRANSFORMATION FRONTIER – INTUITION

- **One-unit decrease in (1 – lfp_t) …**
- **→ increases s_t by one unit …**
- **→ increases n_t by $m_s(s_t, v_t)$ units …**

Transformation Frontier – Intuition

- **One-unit decrease in (1 – lfp_t) …**
- **→ increases s_t by one unit …**
- **→ increases n_t by** $m_s(s_t, v_t)$ **units …**
- **→ increases $z_t n_t$ by** $z_t m_s(s_t, v_t)$ **units …**

TRANSFORMATION FRONTIER – INTUITION

- One-unit decrease in $(1 - lfp_t)$ …
- → increases s_t by one unit …
- → increases n_t by $m_s(s_t, v_t)$ units …
- → increases $z_t n_t$ by $z_t m_s(s_t, v_t)$ units …
- To hold n_t constant, v_t must ***decrease*** by $m_v(s_t, v_t)$ …
- … which decreases $z_t n_t$ by $\frac{z_t m_v(s_t, v_t)}{\gamma}$ units

$$\Rightarrow MRT_{c_t, lfp_t} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

TRANSFORMATION FRONTIER

- **Ceteris paribus…**
- **Does one-unit decrease in (1 – lfp_t) affect c_t?**

$$MRT_{c_t,lfp_t} = -\frac{\Gamma_{lfp_t}}{\Gamma_{c_t}} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

- **Does one-unit decrease in c_t affect c_{t+1}?**
  - **If so, how?**

TRANSFORMATION FRONTIER

- **Transformation function**

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

- → $v_t = \dfrac{z_t n_t - c_t}{\gamma}$ **, then insert into LOM for n_t**
- → $n_t - (1-\rho)n_{t-1} - m\left(s_t, \dfrac{z_t n_t - c_t}{\gamma}\right) = 0$
- **Use $lfp_t = (1-\rho)n_{t-1} + s_t$ to construct within-period transformation frontier**

$$\Gamma\left(c_t, lfp_t, n_t; \cdot\right) \equiv n_t - (1-\rho)n_{t-1} - m\left(lfp_t - (1-\rho)n_{t-1}, \frac{z_t n_t - c_t}{\gamma}\right) = 0$$

- **Use IFT to obtain static MRT (participation margin)**

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STATIC **MRT between LFP and Walrasian good**

$$\frac{\partial n_t}{\partial c_t} = -\frac{\Gamma_{c_t}}{\Gamma_{n_t}} = -\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$$

Marginal effect on n_t of a change in c_t
…which has *intertemporal* consequences

Transformation Frontier

- **Transformation function across periods**

$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho)n_t - m\left(lfp_{t+1} - (1-\rho)n_t, \frac{z_{t+1}n_{t+1} - c_{t+1}}{\gamma}\right) = 0$$

- **Use IFT to obtain intertemporal MRT**

$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}}$$

Transformation Frontier

- **Transformation function across periods**

$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho)n(c_t) - m\left(lfp_{t+1} - (1-\rho)n(c_t), \frac{z_{t+1}n_{t+1} - c_{t+1}}{\gamma}\right) = 0$$

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TRANSFORMATION FRONTIER

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$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho)n(c_t) - m\left(lfp_{t+1} - (1-\rho)n(c_t), \frac{z_{t+1}n_{t+1} - c_{t+1}}{\gamma}\right) = 0$$

- **Use IFT to obtain intertemporal MRT**

$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}} \qquad G_{c_{t+1}} = \frac{m_v(s_{t+1}, v_{t+1})}{\gamma}$$

Transformation Frontier

- **Transformation function across periods**

$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho) n(c_t) - m\left(lfp_{t+1} - (1-\rho) n(c_t), \frac{z_{t+1} n_{t+1} - c_{t+1}}{\gamma}\right) = 0$$

- **Use IFT to obtain intertemporal MRT**

$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}} \qquad G_{c_{t+1}} = \frac{m_v(s_{t+1}, v_{t+1})}{\gamma} \qquad G_{c_t} = -(1-\rho)\frac{\partial n_t}{\partial c_t} + (1-\rho) m_s(s_{t+1}, v_{t+1}) \frac{\partial n_t}{\partial c_t}$$

TRANSFORMATION FRONTIER

- **Transformation function across periods**

$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho) n(c_t) - m\left(lfp_{t+1} - (1-\rho) n(c_t), \frac{z_{t+1} n_{t+1} - c_{t+1}}{\gamma} \right) = 0$$

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$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}} \qquad G_{c_{t+1}} = \frac{m_v(s_{t+1}, v_{t+1})}{\gamma} \qquad G_{c_t} = -(1-\rho)\frac{\partial n_t}{\partial c_t} + (1-\rho) m_s(s_{t+1}, v_{t+1}) \frac{\partial n_t}{\partial c_t}$$

$$\frac{\partial n_t}{\partial c_t} = -\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$$

And convert back into consumption units …

$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}} = \frac{(1-\rho)\left(\dfrac{\gamma}{m_v(s_{t+1}, v_{t+1})}\right)\left(1 - m_s(s_{t+1}, v_{t+1})\right)}{\dfrac{\gamma}{m_v(s_t, v_t)} - z_t}$$

TRANSFORMATION FRONTIER – INTUITION

- One unit reduction in c_t …
- → increases v_t by $1/\gamma$ units
- → increases n_t by $\dfrac{m_v(s_t, v_t)}{\gamma}$ units

Transformation Frontier – Intuition

- **One unit reduction in c_t …**
- **→ increases v_t by $1/\gamma$ units**
- **→ increases n_t by $\dfrac{m_v(s_t, v_t)}{\gamma}$ units**
- **→ *<u>increases</u>* c_t by $\dfrac{z_t m_v(s_t, v_t)}{\gamma}$ units**

Transformation Frontier – Intuition

- **One unit reduction in c_t …**
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- **→ *increases* c_t by $\dfrac{z_t m_v(s_t,v_t)}{\gamma}$ units**
- **… so resulting change in c_t is …**

Must be netted out…

…in order to hold period-*t* output constant

$$\frac{\gamma - z_t m_v(s_t,v_t)}{\gamma}(<1)$$

Transformation Frontier – Intuition

- **One unit reduction in c_t …**
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- **… so resulting change in c_t is …**

Must be netted out…

…in order to hold period-*t* output constant

$$\frac{\gamma - z_t m_v(s_t,v_t)}{\gamma} (<1)$$

- **→ increase in v_t by $\dfrac{1}{\gamma - z_t m_v(s_t,v_t)}$ units for *ONE-UNIT* DECREASE IN c_t**

Transformation Frontier – Intuition

- Increase in v_t by $\dfrac{1}{\gamma - z_t m_v(s_t, v_t)}$ units …
- → increase in $m(s_t, v_t)$ by $\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ units …

Transformation Frontier – Intuition

- **Increase in v_t by** $\dfrac{1}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_t, v_t)$ by** $\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_{t+1}, v_{t+1})$ by** $(1-\rho)\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**

Transformation Frontier – Intuition

- **Increase in v_t by** $\frac{1}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_t,v_t)$ by** $\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
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- **To hold n_{t+1} constant, s_{t+1} must *decrease* by $m_s(s_{t+1},v_{t+1})$ …**

TRANSFORMATION FRONTIER – INTUITION

- **Increase in v_t by** $\dfrac{1}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_t,v_t)$ by** $\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
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- **To hold n_{t+1} constant, s_{t+1} must *decrease* by $m_s(s_{t+1},v_{t+1})$ …**
- **→ increase in v_{t+1} by** $(1-\rho)\left(\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}\right)\left(1 - m_s(s_{t+1}, v_{t+1})\right)$ **units …**

TRANSFORMATION FRONTIER – INTUITION

- **Increase in v_{t+1} by** $(1-\rho)\left(\dfrac{m_v(s_t,v_t)}{\gamma - z_t m_v(s_t,v_t)}\right)\left(1-m_s(s_{t+1},v_{t+1})\right)$ **units …**

- **→ increases by c_{t+1}** $\dfrac{(1-\rho)\left(\dfrac{\gamma}{m_v(s_t,v_t)}\right)\left(1-m_s(s_{t+1},v_{t+1})\right)}{\dfrac{\gamma}{m_v(s_t,v_t)}-z_t}$ **units**

Transformation Frontier

- **Ceteris paribus…**
- **Does one-unit decrease in (1 – lfp_t) affect c_t?**

$$MRT_{c_t, lfp_t} = -\frac{\Gamma_{lfp_t}}{\Gamma_{c_t}} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

- **Does one-unit decrease in c_t affect c_{t+1}?**

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Matching Efficiency

- Efficiency characterized by

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{gm_s(s_t,v_t)}{m_v(s_t,v_t)}$$

$$= g q_t \frac{x}{1-x}$$

= Static MRT$_t$

Static Efficiency Condition.

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(\frac{\gamma}{m_v(s_{t+1},v_{t+1})}\right)(1-m_s(s_{t+1},v_{t+1}))}{\frac{\gamma}{m_v(s_t,v_t)} - z_t}$$

= Intertemporal MRT$_t$

Intertemporal Efficiency Condition.

MATCHING EFFICIENCY

- **Efficiency characterized by**

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{g m_s(s_t,v_t)}{m_v(s_t,v_t)} = g q_t \frac{x}{1-x}$$

= Static MRT$_t$

Static Efficiency Condition.

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(\frac{\gamma}{m_v(s_{t+1},v_{t+1})}\right)(1-m_s(s_{t+1},v_{t+1}))}{\frac{\gamma}{m_v(s_t,v_t)} - z_t}$$

= Intertemporal MRT$_t$

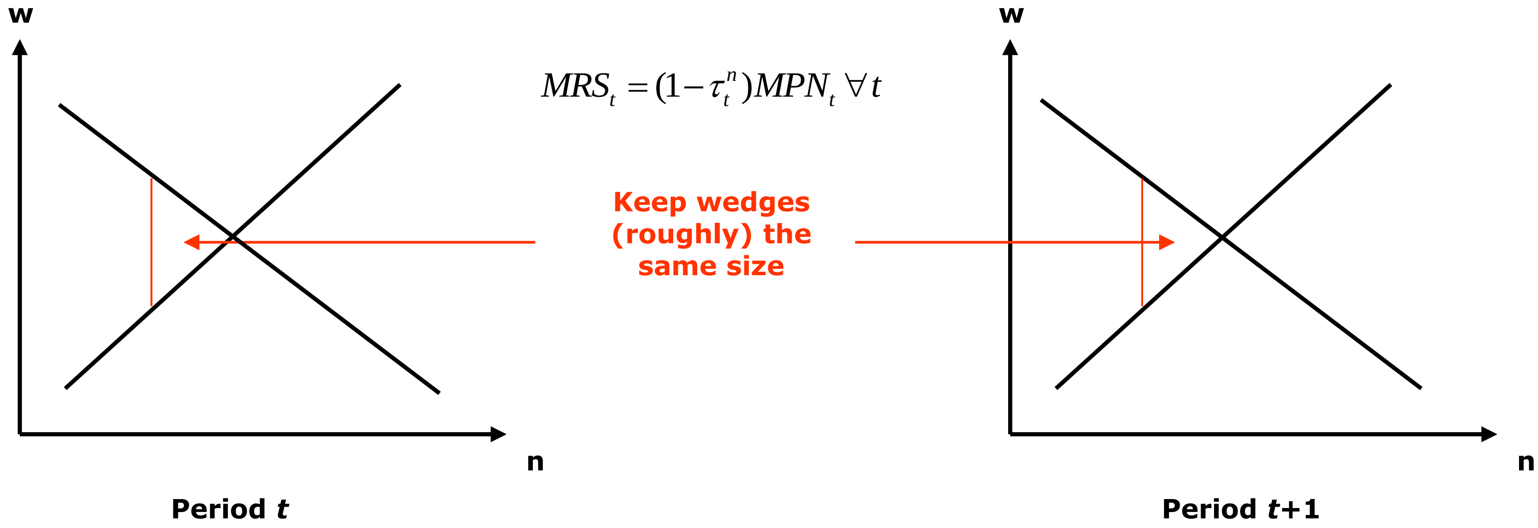
Intertemporal Efficiency Condition.

- **Ramsey theory: stabilizing THESE wedges is optimal**
  - **MRTs in DSGE search and matching model: Arseneau and Chugh (2012 *JPE*)**
- **Contribution to understanding efficiency in DGE models with "entry" margins**
  - **MRTs in new monetarist models: Aruoba and Chugh (2010 *JET*)**
  - **MRTs in customer market models: Arseneau, Chahrour, Chugh, and Finkelstein Shapiro (2015 *JMCB*)**
  - **MRTs in endogenous product variety framework: Chugh and Ghironi (2015)**

Tax Smoothing in Frictional Labor Markets

February 7, 2018

TAX SMOOTHING



- Ramsey wants to keep these wedges constant
- Result and intuition depend on neoclassical view of labor markets
  - Labor tax is the only wedge → tax-smoothing is wedge-smoothing

- Question: Is tax smoothing optimal in search and matching labor markets?

Overview of Model

- Infinitely-lived representative household, measure one of members
  - Employed members
  - Unemployed members
  - Members outside the labor force ("leisure")

Full consumption insurance – standard in DSGE labor search models

Incompleteness of government debt markets NOT driving our results (Aiyagari et al (2002 *JPE*))

- Exogenous stochastic government spending
  - Financed via labor income taxation and one-period real state-contingent debt
  - Government provides unemployment benefits
  - Government provides vacancy subsidies
    - For completeness of tax instruments (Ramsey issue)

Overview of Model

- Infinitely-lived representative household, measure one of members
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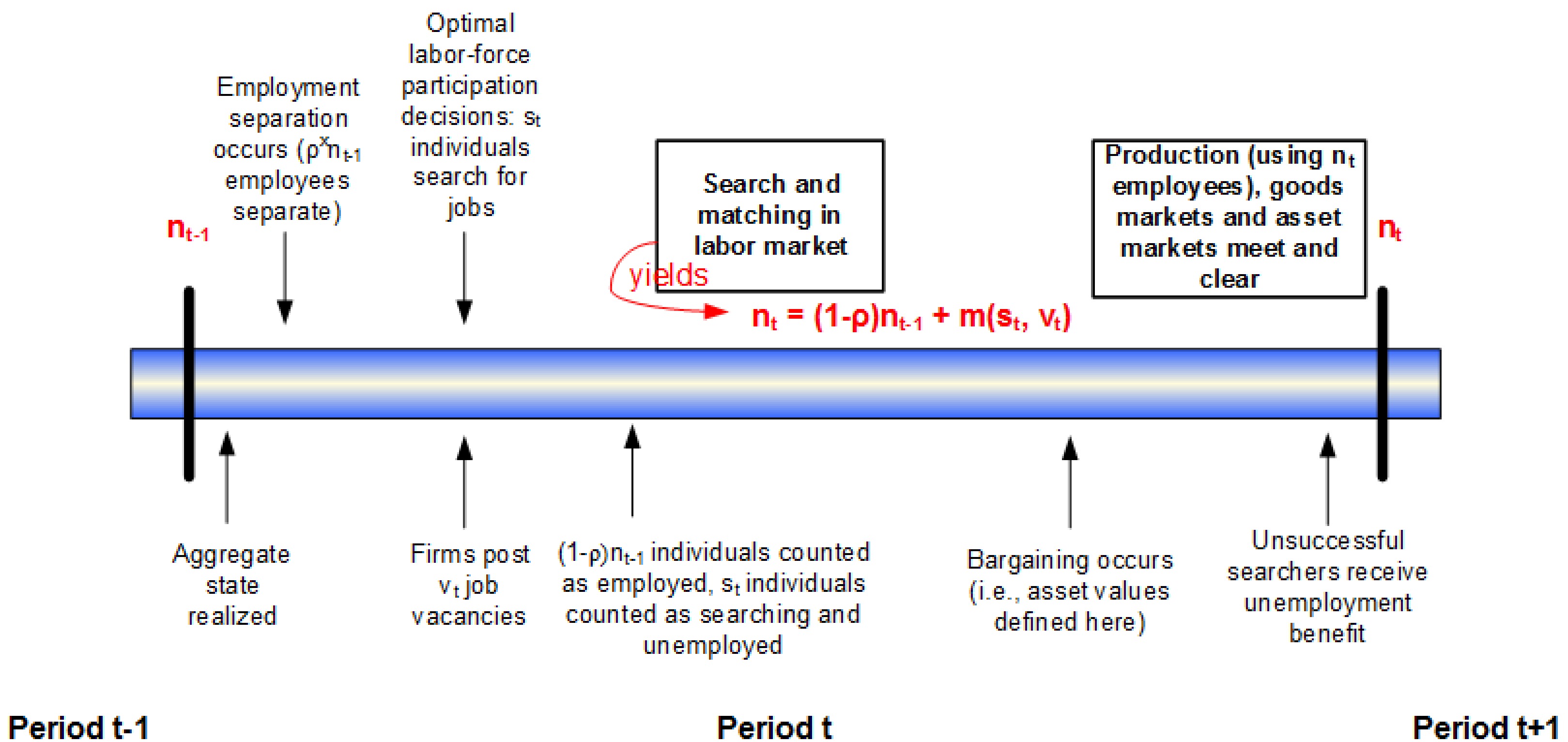
Incompleteness of government debt markets NOT driving our results (Aiyagari et al (2002 *JPE*))

- Exogenous stochastic government spending
  - Financed via labor income taxation and one-period real state-contingent debt
  - Government provides unemployment benefits
  - Government provides vacancy subsidies
    - For completeness of tax instruments (Ramsey issue)
- Labor market with matching frictions and wage-setting frictions
- Only an extensive labor margin, no intensive labor margin
- Timing: "instantaneous production"

Overview of Model



- **Unemployed are the unsuccessful searchers: $ue_t = (1-p_t)s_t$**
  - **p_t = probability an individual finds a job and begins working immediately**

Household Optimization

- Maximize expected lifetime utility

$$\max_{\{c, n_t, s_t, b_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left((1-p_t)s_t + n_t\right) \right]$$

disutility of employment + unsuccessful search

Household Optimization

- Maximize expected lifetime utility

$$\max_{\{c,n_t,s_t,b_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\big((1-p_t)s_t + n_t\big) \right]$$

disutility of employment + unsuccessful search

s.t.

$$c_t + b_t = n_t(1-\tau_t^n)w_t + (1-p_t)s_t\chi + R_t b_{t-1} + (1-\tau^d)d_t$$

Flow budget constraint

measure *n* earn after-tax wage income

measure *ue* = (1-*p*)*s* receive ue benefit χ (government financed)

Baseline analysis: set $\tau^d = 1$ → no *profit*-taxation issues driving results

Household Optimization

- Maximize expected lifetime utility

$$\max_{\{c,n_t,s_t,b_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left((1-p_t)s_t + n_t\right) \right]$$

disutility of employment + unsuccessful search

s.t.

$$c_t + b_t = n_t(1-\tau_t^n)w_t + (1-p_t)s_t\chi + R_t b_{t-1} + (1-\tau^d)d_t$$

Flow budget constraint

measure *n* earn after-tax wage income

measure *ue* = (1-*p*)*s* receive ue benefit χ (government financed)

Baseline analysis: set $\tau^d = 1$ → no *profit*-taxation issues driving results

$$n_t = (1-\rho)n_{t-1} + s_t p_t$$

Perceived LOM for employment ("instantaneous production")

(exogenous) measure of pre-existing employment relationships terminate

flow of new employment relationships = measure of searchers s_t x probability a searcher successfully lands a job

↓ FOCs with respect c_t, n_t, s_t, b_t

HOUSEHOLDS

- **Household LFP condition (the labor supply condition!)**

$$\frac{h'(lfp_t)}{u'(c_t)} = p_t\left[(1-\tau_t^n)w_t + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{1-p_{t+1}}{p_{t+1}}\right)\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})}-\chi\right)\right\}\right] + (1-p_t)\chi$$

  - **MRS between *lfp_t* and *c_t* = expected payoff of searching**
    - **Unemployment benefit (with probability 1 – *p_t*)**
    - **After-tax wage + continuation value (with probability *p_t*)**

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To recover standard labor supply function (e.g., RBC)

1. **ρ = 1 (all employment relationships terminate at end of every period)**
2. **p = 1 (probability a searcher finds a job)**
3. **χ = 0 (no ue benefit because no notion of "ue")**

$$\frac{h'(lfp_t)}{u'(c_t)} = (1-\tau_t^n)w_t$$

FIRMS

- **Production**
  - **Requires a matched job-worker pair: posting cost γ per vacancy**
  - **Individual job *i* produces $y_{it} = z_t$**
  - **Aggregate output $y_t = n_t z_t$ (symmetry across jobs)**

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- **Dynamic profit-maximization problem**

Ensures completeness of tax instruments

$$\max_{\{n_t, v_t\}} \sum_{t=0}^{\infty} \Xi_{t|0} \left[z_t n_t - w_t n_t - (1 - \tau_t^s) \gamma v_t \right]$$

$$n_t = (1-\rho) n_{t-1} + v_t q_t$$

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- **Vacancy-creation condition**

$$\underbrace{\frac{\gamma(1-\tau_t^s)}{q_t}}_{\text{cost of posting vacancy (inclusive of subsidy or tax)}} = \underbrace{z_t - w_t + (1-\rho)E_t\left[\Xi_{t+1|t}\frac{\gamma(1-\tau_{t+1}^s)}{q_{t+1}}\right]}_{\text{benefit of posting vacancy}}$$

cost of posting vacancy (inclusive of subsidy or tax)

benefit of posting vacancy

Labor Market

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  - Important aggregate variable in matching-based models
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- Matching function $m(s_t, v_t) = \psi s_t^{\xi} v_t^{1-\xi}$
- LOM for aggregate employment $n_t = (1-\rho) n_{t-1} + m(s_t, v_t)$

- Nash bargaining over wage payment solves

$$\max_{w_t} \left(\mathbf{W}_t - \mathbf{U}_t\right)^{\eta} \mathbf{J}_t^{1-\eta} \longrightarrow \frac{\mathbf{W}_t - \mathbf{U}_t}{1-\tau_t^n} = \frac{\eta}{1-\eta} \mathbf{J}_t$$

Gain to household of successfully forming another employment relationship

Value to firm of hiring another worker

$$\longrightarrow w_t = \eta z_t + (1-\eta) \frac{c}{1-\tau_t^n} + \eta(1-\rho) E_t \left\{ \Xi_{t+1|t} \left[1 - (1-p_{t+1}) \frac{1-\tau_{t+1}^n}{1-\tau_t^n} \right] \frac{\gamma(1-\tau_{t+1}^s)}{q_{t+1}} \right\}$$

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$\left(\mathbf{W}_t - \mathbf{U}_t \right)$: Gain to household of successfully forming another employment relationship

$\mathbf{J}_t$: Value to firm of hiring another worker

Although main results also hold if we discard Nash bargaining and assume ad-hoc real wage rigidity:

w_t = *wbar* in every period t

$$\longrightarrow w_t = \eta z_t + (1-\eta)\frac{c}{1-\tau_t^n} + \eta(1-\rho) E_t \left\{ \Xi_{t+1|t} \left[1 - (1-p_{t+1}) \frac{1-\tau_{t+1}^n}{1-\tau_t^n} \right] \frac{\gamma(1-\tau_{t+1}^s)}{q_{t+1}} \right\}$$

GOVERNMENT AND RESOURCE FRONTIER

- **Exogenous government spending financed via**
  - **Labor income tax**
  - **One-period state contingent real debt**

$$\tau_t^n w_t n_t + b_t + \tau^d d_t = g_t + R_t b_{t-1} + (1 - \pi_t) s_t \chi + \tau_t^s \gamma v_t$$

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$$c_t + g_t + \gamma v_t = z_t n_t$$

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    - **Makes model more comparable to existing Ramsey models**

- **Precise nature of χ (ue benefit? home production? value of leisure?) not typically specified in DSGE matching models**
  - **Our model articulates both ue benefit and value of leisure**

PRIVATE-SECTOR EQUILIBRIUM

- Stochastic processes $\left\{c_t, n_t, s_t, w_t, \theta_t, R_t, b_t\right\}_{t=0}^{\infty}$ that satisfy
  - Household's bond Euler equation
  - Vacancy-creation condition
  - Labor force participation condition
  - Nash wage outcome
  - Law of motion for employment $n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$
  - Government budget constraint (key condition in Ramsey models)
  - Resource constraint $c_t + g_t + \gamma v_t = z_t n_t$
  - Given processes $\left\{g_t, z_t, \tau_t^n, \tau_t^s\right\}_{t=0}^{\infty}$

Standard conditions in basic Ramsey models

CALIBRATION

- **Baseline calibration**
  - **So that exogenous policy (non-Ramsey) equilibrium broadly matches U.S. labor market fluctuations**

  - **Preferences and key parameters**

$$u(c_t) - h(lfp_t) = \ln c_t - \frac{\kappa}{1+1/\iota} lfp_t^{1+1/\iota}$$

  - **Participation (labor supply) elasticity (ι = 0.18)**
  - **Low worker bargaining power (η = 0.05)**
  - **High unemployment benefit (98% of real wage)**

The two key parameters of HM calibration

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The two key parameters of HM calibration

- **Rest of parameters, matching-related and otherwise, standard**
  - **β = 0.99**
  - **ρ = 0.10**
  - **ξ = 0.40**
  - **AR(1) parameters for LOMs for TFP and government spending**
  - **Etc.**

DYNAMICS

		Ramsey		Exogenous Policy Benchmark		Data
		Calibration		Calibration		
		HM	0% and Hosios	HM		
Labor Tax Rate	Mean					22%
	Rel SD					1.4
Market tightness (θ)	Rel SD					11.3
Vacancies	Rel SD					6.3
Unemployment	Rel SD					5.2
LFP	Rel SD					0.20
Real wage	Rel SD					0.52
Static wedge	SD (%)					
Intertemporal wedge	SD (%)					

Data ← Gertler and Trigari (2009 *JPE*)

Dynamics

Ramsey | Exogenous Policy Benchmark | Data ← Gertler and Trigari (2009 *JPE*)

		Calibration		Calibration		
		HM	0% and Hosios	HM		
Labor Tax Rate	Mean			22%		22%
	Rel SD			1.4		1.4
Market tightness (θ)	Rel SD			10.9		11.3
Vacancies	Rel SD			6.9		6.3
Unemployment	Rel SD			5.4		5.2
LFP	Rel SD			0.20		0.20
Real wage	Rel SD			0.28		0.52
Static wedge	SD (%)					
Intertemporal wedge	SD (%)					

DYNAMICS

		Ramsey		Exogenous Policy Benchmark		Data
		Calibration		Calibration		
		HM	0% and Hosios	HM		
Labor Tax Rate	Mean	11%		22%		22%
	Rel SD	5.6		1.4		1.4
Market tightness (θ)	Rel SD	1.1		10.9		11.3
Vacancies	Rel SD	1.3		6.9		6.3
Unemployment	Rel SD	1.4		5.4		5.2
LFP	Rel SD	0.13		0.20		0.20
Real wage	Rel SD	0.50		0.28		0.52
Static wedge	SD (%)					
Intertemporal wedge	SD (%)					

Data: Gertler and Trigari (2009 *JPE*)

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- **Ramsey fluctuations IDENTICAL to efficient fluctuations for ANY (η, χ) pair**
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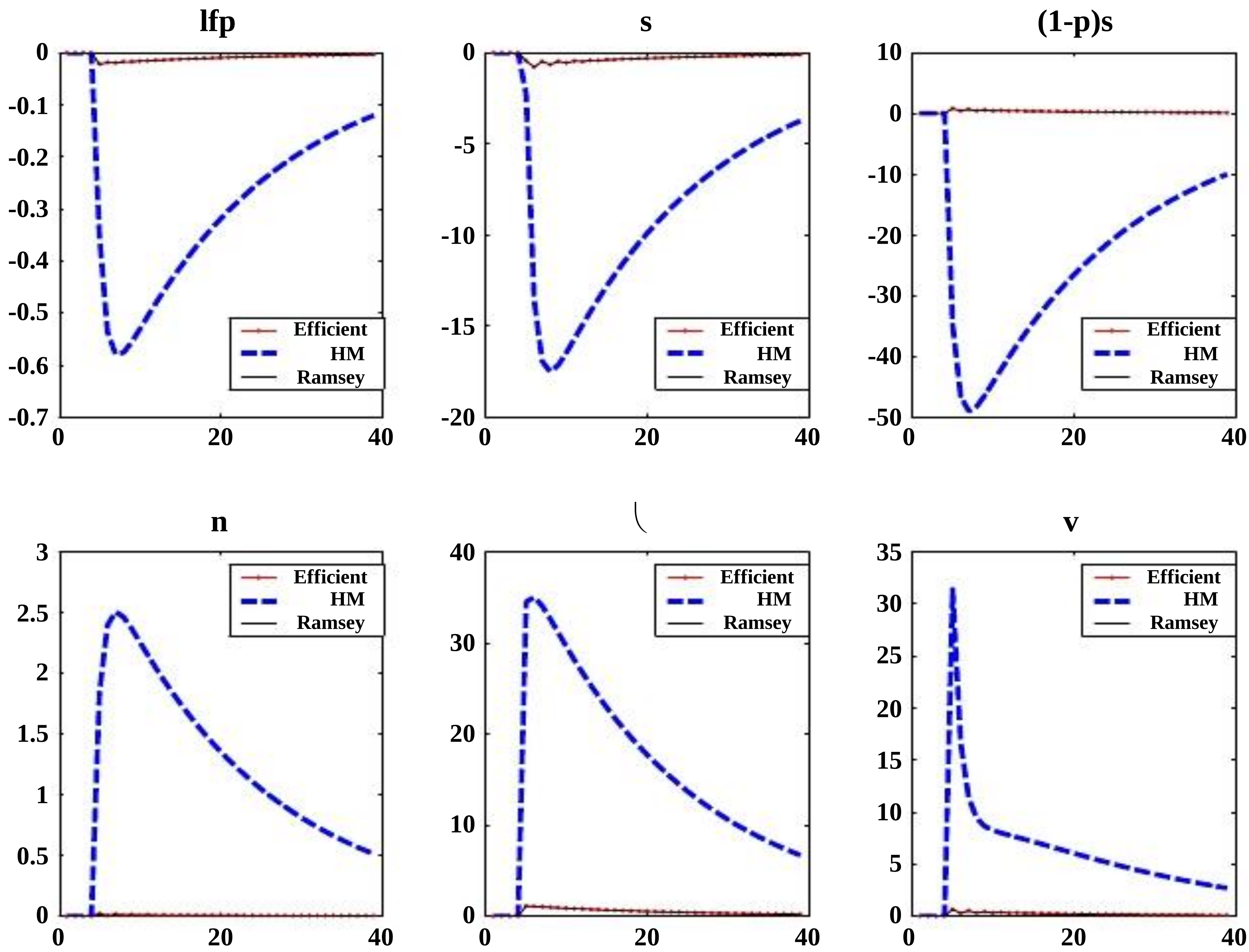
lfp
s
(1-p)s
n
v
Efficient
HM
Ramsey
0
-0.1
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- **Wedge dynamics?**
  - **Ramsey smooths both static wedge....**
  - **...and intertemporal wedge**

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Real wage	Rel SD	0.50	1.1	0.28		0.52
Static wedge	SD (%)	0.08	0	22.9	0.66	
Intertemporal wedge	SD (%)	0	0	12.3	0.63	

Static and Intertemporal Conditions

- **Efficiency characterized by**

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{gm_s(s_t,v_t)}{m_v(s_t,v_t)} = gq_t\frac{x}{1-x}$$

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(\frac{\gamma}{m_v(s_{t+1},v_{t+1})}\right)(1-m_s(s_{t+1},v_{t+1}))}{\frac{\gamma}{m_v(s_t,v_t)} - z_t}$$

- **Decentralized equilibrium conditions characterized by**

$$\frac{h'(lfp_t)}{u'(c_t)} = \left[\frac{\chi(1-\xi)}{\gamma\cdot\xi\cdot\theta_t} + (1-\tau_t^n)(1-\tau_t^s)\frac{\eta(1-\xi)}{\xi(1-\eta)}\right]\gamma\theta_t\frac{\xi}{1-\xi}$$

= wedge between static MRS$_t$ and static MRT$_t$

To obtain zero static wedge in every period, need $\tau^n = \tau^s = 0$ in every period, $\eta = \xi$, $\chi = 0$

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(See eqn. (29) for intertemporal wedge)

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CONCLUSIONS

- **Labor tax smoothing not optimal in DSGE search and matching model**
  - **Calibrated to match key labor market dynamics under exogenous tax policy**
  - **Rigid real wage (delivered through Nash-Hosios bargaining as benchmark) the important feature of the model**

- **But wedge smoothing IS optimal**
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- **Ramsey fluctuations in allocations efficient regardless of calibration**

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- But wedge smoothing IS optimal
  - Basic Ramsey theory
- Ramsey fluctuations in allocations efficient regardless of calibration
- Welfare-relevant notions of wedges
  - Developing matching-model concepts of efficiency and MRTs for use in virtually any matching application
- Could think of "labor wedge" as featuring both static and intertemporal dimensions
  - Use as framework to empirically measure labor wedges