# MONOPOLISTIC COMPETITION IN A DSGE MODEL

APRIL 4, 2017

# EMPIRICAL AND THEORETICAL CONSIDERATIONS

- **Evidence supports existence of markups in goods markets (i.e., $p > mc$)**
  - **Basu and Fernald (1997 *JPE*) often-cited source**
- **Evidence also supports positive (but small?...) pure economic profits**
- **Are firms always price-takers?**
  - **If not, must endow them with market power**
- **If increasing returns in production exist, a model without market power does not admit an equilibrium with increasing returns**
- **Introduce imperfect competition**
  - **Typically monopolistic competition…**
  - **…a building block of sticky nominal price monetary models**

# WORKHORSE MODEL

- **Dixit-Stiglitz (1977 *AER*) model**
  - **Common specification of imperfect competition in macro models**
  - **Typical building block of sticky price monetary models**
  - **Basic idea: imperfectly-substitutable goods combined yield an aggregate good**

**CES function: $\varepsilon$ the constant elasticity of substitution between any pair of differentiated goods**

$$c_t = \left[ \sum_{i=1}^{N} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

**Discrete number of differentiated goods**

$$c_t = \left[ \int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

**Continuum of differentiated goods**

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Continuum of differentiated goods

- Important properties of aggregator
  - Symmetric in all arguments ← Drives efficiency/optimal policy results
  - Strictly increasing in all arguments
  - Strictly concave in all arguments
  - Homogenous of degree one

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**In some applications, $\varepsilon$ can be time-varying (either endogenously or exogenously)**

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**Discrete number of differentiated goods**

**In some applications, make this endogenous and time-varying, $N_t$**

$$c_t = \left[ \int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

**Continuum of differentiated goods**

- **Important properties of aggregator**
  - **Symmetric in all arguments** ← **Drives efficiency/optimal policy results**
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# TWO EQUIVALENT IMPLEMENTATIONS

- **Consumption aggregator** $c_t = \left[ \int_0^{N_t} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$

“First-stage” problem
- **Consumer chooses $c_t$…** ← Utility-maximization problem

“Second-stage” problem
- **…then chooses each of the $c_{it}$** ← Cost-minimization problem (dual)
- **Each differentiated good *i* produced by a unique producer**
  - **KEY: takes as given the demand function it faces**

# Two Equivalent Implementations

- **Consumption aggregator** $c_t = \left[\int_0^{N_t} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$

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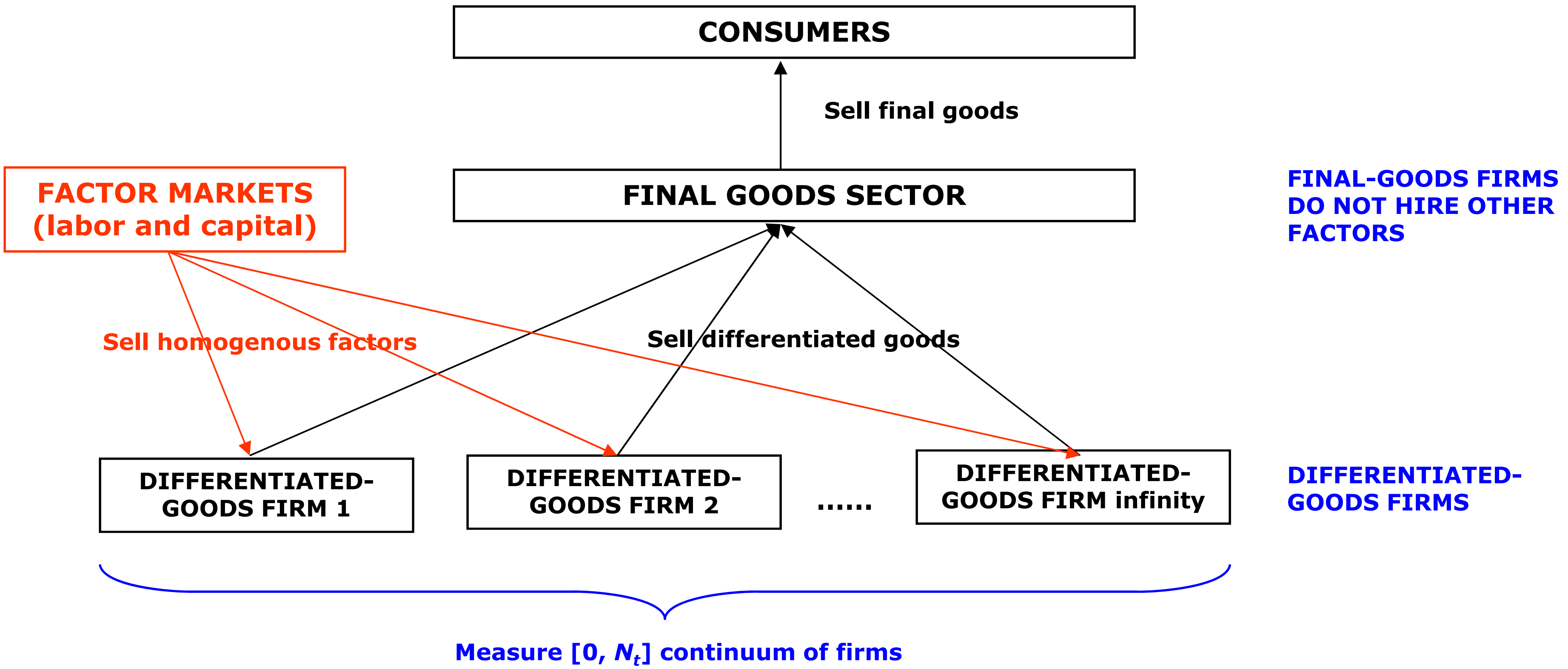
- **Production aggregator** $y_t = \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$
  - **Final-goods producer chooses $y_{it}$…** ← Profit-maximization problem
  - **…to sell a composite final good $y_t$ to consumers** ← Expenditure-minimization problem (dual)
  - **Each differentiated good *i* produced by a unique intermediate-goods producer**
    - **KEY: takes as given the demand function it faces**

- **Isomorphic results**

# MARKET STRUCTURE



# MARKET ORGANIZATION

- **Differentiated producer *i* production technology** $y_{it} = \underbrace{\overbrace{z_t f\left(k_{it}, n_{it}\right)}^{\text{Usual CRS}} - \Phi}$

  **"Net-of-fixed-factor production technology" exhibits IRS (i.e., marginal cost < average cost)**

  **Fixed production factor: a) sometimes useful for calibrating profit share b) often set to zero**

  - **Rotemberg and Woodford (*Frontiers* chapter): "materials cost" foundations**

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- **Differentiated producer *i* hires inputs on perfectly-competitive markets…**

- **…and sells its output on its own *monopolistically-competitive* market**
  - **Sells "directly" to consumers…**
  - **…or to final-goods firms**

- **For starters, suppose $\Phi = 0$ (→ mc = ac assuming CRS)**

# FINAL-GOODS FIRMS

- **Production Model**
  - **(Representative) final goods producer**

$$\max_{\langle y_{it} \rangle_{i=0}^{N_t}} y_t - \int_0^{N_t} p_{it} y_{it} di$$

Substitute in D-S final-goods aggregator

$$\max_{\langle y_{it} \rangle_{i=0}^{N_t}} \left[ \int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^{N_t} p_{it} y_{it} di$$

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  - **FOC with respect to *$y_{it}$***

$$\left(\frac{\varepsilon}{\varepsilon-1}\right)\left[ \int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{1}{\varepsilon-1}} \left(\frac{\varepsilon-1}{\varepsilon}\right) y_{it}^{-1/\varepsilon} - p_{it} = 0$$

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$$\longrightarrow y_{it}^{1/\varepsilon} = p_{it}^{-1} \cdot \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{1}{\varepsilon-1}} \longrightarrow y_{it} = p_{it}^{-\varepsilon} \cdot \left[\int_0^{N_t} y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

**Use D-S aggregator** $\longrightarrow$

$$y_{it} = p_{it}^{-\varepsilon} \cdot y_t$$

**Demand Function**

**Dixit-Stiglitz**

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    - **Takes as given all $p_{it}$**

  - **Profit-maximization leads to demand functions for each underlying differentiated good *i***

$$y_{it} = p_{it}^{-\varepsilon} \cdot y_t$$

**TAKEN AS GIVEN BY DIFFERENTIATED FIRM *i***

**Each differentiated firm *i* chooses its $p_i$ to maximize profit**

**Relative price of firm *i*'s output**

**Aggregate output a *shifter* of firm *i*'s demand function**

# DIFFERENTIATED-GOODS FIRMS

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  - **Differentiated goods producer *i***

$$\max_{p_{it}} p_{it} y_{it} - w_t n_{it} - r_t k_{it}$$

**Substitute in demand function**

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- **A "two-stage" optimization problem**
  - **Stage 1: Choose optimal $p_i$**
  - **(Intermediate "stage"): "choose" to produce the $y_i$ corresponding to the optimal choice of $p_i$**
  - **Stage 2: Choose factor inputs to produce $y_i$ at minimum cost**

i.e., production $y_i$ pinned down from downward-sloping demand curve

**GIVEN 1) CRS $f(k, n)$ and 2) $\Phi = 0$**

**→ mc = ac = CONSTANT (with respect to quantity)**

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**STAGE-1 PROBLEM**

$$\max_{p_{it}} p_{it} p_{it}^{-\varepsilon} y_t - mc_t y_{it}$$

Substitute in demand function

$$\max_{p_{it}} p_{it} p_{it}^{-\varepsilon} y_t - mc_t p_{it}^{-\varepsilon} y_t$$

# DIFFERENTIATED-GOODS FIRMS

- **Production Model or Consumption Model**
  - **Differentiated goods producer *i* optimal choice of $p_i$**

$$p_{it} = \underbrace{\frac{\varepsilon}{\varepsilon - 1}} \cdot mc_t$$

**Gross product-market markup**

**Linked *only* to degree of substitutability**

**RBC model: $\varepsilon$ = infinity (perf. comp.)**

**Monopoly model requires $\varepsilon > 1$ and $\varepsilon <$ infinity**

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  - **Given optimal ($p_i$, $y_i$)**

**NOTE: cost-minimization equivalent to profit-maximization *GIVEN* ($p_i$, $y_i$) -- i.e., DUAL PROBLEM**

$$\max_{k_{it}, n_{it}} p_{it} z_t f(k_{it}, n_{it}) - w_t n_{it} - r_t k_{it}$$

**substitute** $p_{it} = \left[z_t f(k_{it}, n_{it})\right]^{-1/\varepsilon} y_t^{1/\varepsilon}$ **from dmd. fct.**

$$\max_{k_{it}, n_{it}} \left[z_t f(k_{it}, n_{it})\right]^{1-1/\varepsilon} y_t^{1/\varepsilon} - w_t n_{it} - r_t k_{it}$$

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  - **Factor demands ($k_i$, $n_i$) solve**

$$\frac{\varepsilon - 1}{\varepsilon} p_{it} z_t f_k(k_{it}, n_{it}) = r_t \qquad \frac{\varepsilon - 1}{\varepsilon} p_{it} z_t f_n(k_{it}, n_{it}) = w_t$$

# BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Symmetric equilibrium across all *i***

$$\frac{\varepsilon-1}{\varepsilon} p_t z_t f_k(k_t, n_t) = r_t \quad \& \quad \frac{\varepsilon-1}{\varepsilon} p_t z_t f_n(k_t, n_t) = w_t \quad \& \quad p_t = \frac{\varepsilon}{\varepsilon-1} \cdot mc_t$$

↓ **implies**

$$mc_t = \frac{w_t}{z_t f_n(k_t, n_t)} = \frac{r_t}{z_t f_k(k_t, n_t)}$$

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**Symmetric equilibrium price of good? Substitute demand functions into DS aggregator and compute…**

$$p_t = ?\ldots$$

# BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Price index**

**Final good**

$$y_t = \int_0^{N_t} p_{it} \cdot y_{it} di$$

**Substitute demand functions**

$$y_{it} = p_{it}^{-\varepsilon} \cdot y_t$$

$$y_t = \int_0^{N_t} p_{it} \cdot p_{it}^{-\varepsilon} y_t di$$

**$y_t$ independent of index of integration $i$**

$$y_t = y_t \cdot \int_0^{N_t} p_{it}^{1-\varepsilon} di$$

**Cancel $y_t$ terms**

$$1 = \int_0^{N_t} p_{it}^{1-\varepsilon} di$$

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**$y_t$ independent of index of integration $i$** →

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**Cancel $y_t$ terms** →

$$1 = \int_0^{N_t} p_{it}^{1-\varepsilon} di$$

**Substitute $p_{it} = p_t$ for all $i \in [0, N_t]$** →

$$1 = \int_0^{N_t} p_t^{1-\varepsilon} di$$

**$p_t$ independent of index of integration $i$** →

$$1 = p_t^{1-\varepsilon} \cdot \int_0^{N_t} 1 \cdot di$$

**Rewrite** →

$$1 = p_t^{1-\varepsilon} \cdot N_t$$

**Solve for $p_t$** →

$$p_t = N_t^{\frac{1}{\varepsilon - 1}}$$

- **Symmetric equilibrium price depends on measure [0, $N_t$] of monopolistically-competitive firms**
  - **If $N_t = 1 \rightarrow p_t = 1$**

# BUILDING THE EQUILIBRIUM

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$$\frac{\varepsilon-1}{\varepsilon}p_t z_t f_k(k_t,n_t)=r_t \quad \& \quad \frac{\varepsilon-1}{\varepsilon}p_t z_t f_n(k_t,n_t)=w_t \quad \& \quad p_t=\frac{\varepsilon}{\varepsilon-1}\cdot mc_t$$

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**Symmetric equilibrium D-S price of good**

$$p_t=N_t^{\frac{1}{\varepsilon-1}}$$

**With measure [0, $N_t$] of intermediate firms… what if measure [0, 1] of firms?**

# BUILDING THE EQUILIBRIUM

- **Production Model or Consumption Model**
  - **Symmetric equilibrium across all *i***

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$$mc_t = \frac{w_t}{z_t f_n(k_t, n_t)} = \frac{r_t}{z_t f_k(k_t, n_t)}$$

**Symmetric equilibrium D-S price of good**

$$p_t = 1$$

**With measure [0, 1] of intermediate firms**

$$mc_t = \frac{\varepsilon-1}{\varepsilon}$$

**< 1 with $\varepsilon$ > 1 and $\varepsilon$ < infinity**

**Monopoly power causes factor prices to fall below marginal products…hence inefficiently low equilibrium factor use…hence inefficiently low total output**

# MONOPOLISTICALLY-COMPETITIVE EQUILIBRIUM

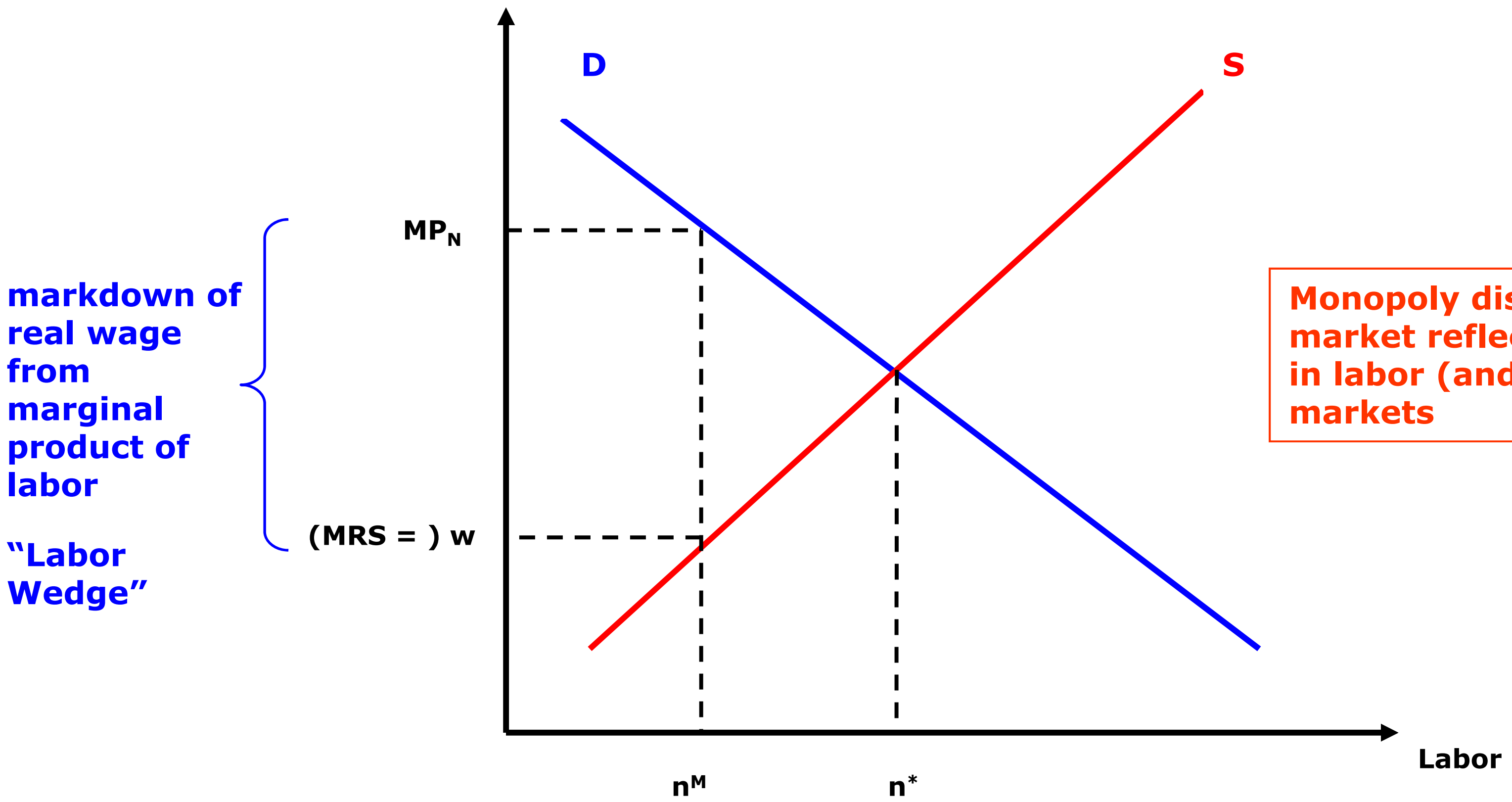
- **Equilibrium Conditions (symmetric across all differentiated goods)**
  - **Consumption-leisure optimality condition**
  - **Consumption-savings optimality condition**
  - **Aggregate resource constraint**

$$c_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t)$$

  - **(Market clearing in labor, capital, and goods markets)**
  - $mc_t = \frac{\varepsilon - 1}{\varepsilon} \quad \forall t$ **( < 1 with $\varepsilon$ > 1)**
  - **Factor prices a mark*down* of marginal products**

$$w_t = \frac{\varepsilon - 1}{\varepsilon} \cdot z_t f_n(k_t, n_t), \;\; r_t = \frac{\varepsilon - 1}{\varepsilon} \cdot z_t f_k(k_t, n_t)$$

# THE LABOR WEDGE



# THE CAPITAL WEDGE