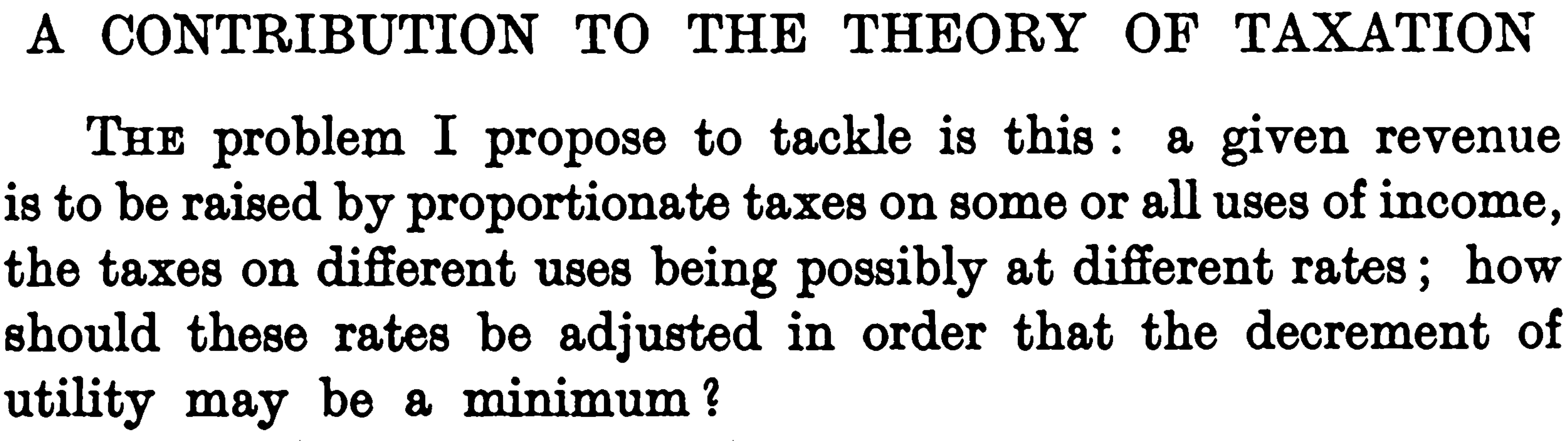
OPTIMAL FISCAL POLICY

APRIL 13, 2017

BASICS OF RAMSEY ANALYSIS

- **Ramsey (1927 *Economic Journal*)**



A CONTRIBUTION TO THE THEORY OF TAXATION

THE problem I propose to tackle is this: a given revenue is to be raised by proportionate taxes on some or all uses of income, the taxes on different uses being possibly at different rates; how should these rates be adjusted in order that the decrement of utility may be a minimum?

- **Stiglitz (2014 NBER working paper)**
  - **Pros and cons of Ramsey taxation framework**
  - **“In Praise of Frank Ramsey’s Contribution to the Theory of Taxation”**

BASICS OF RAMSEY ANALYSIS

- **Maintained assumptions**
  - **Lack of lump-sum taxes (the starting point of Ramsey 1927)**
  - **Completeness of set of proportional tax instruments**
  - **Completeness of government debt markets**
    - **Fully state-contingent set of government bonds issued in *t*, only one yields return depending on realized state in *t*+1**

BASICS OF RAMSEY ANALYSIS

- **Maintained assumptions**
  - **Lack of lump-sum taxes (the starting point of Ramsey 1927)**
  - **Completeness of set of proportional tax instruments**
  - **Completeness of government debt markets**
    - **Fully state-contingent set of government bonds issued in *t*, only one yields return depending on realized state in *t*+1**

- **Completeness of tax instruments?**
  - **Suppose three distinct goods, each with proportional tax rate**
  - **Household optimality conditions**

$$\frac{u_1(c_1,c_2,c_3)}{u_2(c_1,c_2,c_3)} = \frac{(1-\tau_1)\cdot p_1}{(1-\tau_2)\cdot p_2} \qquad \frac{u_2(c_1,c_2,c_3)}{u_3(c_1,c_2,c_3)} = \frac{(1-\tau_2)\cdot p_2}{(1-\tau_3)\cdot p_3}$$

BASICS OF RAMSEY ANALYSIS

- **Maintained assumptions**
  - **Lack of lump-sum taxes (the starting point of Ramsey 1927)**
  - **Completeness of set of proportional tax instruments**
  - **Completeness of government debt markets**
    - **Fully state-contingent set of government bonds issued in *t*, only one yields return depending on realized state in *t*+1**

- **Completeness of tax instruments?**
  - **Suppose three distinct goods, each with proportional tax rate**
  - **Household optimality conditions**

$$\frac{u_1(c_1,c_2,c_3)}{u_2(c_1,c_2,c_3)}=\frac{(1-\tau_1)\cdot p_1}{(1-\tau_2)\cdot p_2} \qquad \frac{u_2(c_1,c_2,c_3)}{u_3(c_1,c_2,c_3)}=\frac{(1-\tau_2)\cdot p_2}{(1-\tau_3)\cdot p_3}$$

  - **Completeness of tax instruments exists if, given a Ramsey allocation**
    - **There is ≥ one tax rate on each MRS = price ratio condition and…**
    - **…there is a unique mapping from the Ramsey allocation to a set of tax rates**

BASICS OF RAMSEY MACRO FISCAL POLICY

- ☐ **Ramsey problem**

$$\max_{\{c_t,n_t,k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(n_t) \right]$$ **s.t.**

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f\left(k_t, n_t\right)$$

<u>Sequence</u> of Lagrange multipliers $\beta^t \lambda_t$

BASICS OF RAMSEY MACRO FISCAL POLICY

☐ **Ramsey problem**

$$\max_{\{c_t,n_t,k_{t+1}\}} E_0\sum_{t=0}^{\infty}\beta^t\left[u(c_t)-h(n_t)\right]$$ **s.t.**

$$c_t+g_t+k_{t+1}-(1-\delta)k_t=z_t f\left(k_t,n_t\right)$$

Sequence of Lagrange multipliers $\beta^t\lambda_t$

Present-value implementability constraint (PVIC): the PV GBC

$$E_0\sum_{t=0}^{\infty}\beta^t\underbrace{\left[u'(c_t)\cdot c_t-h'(n_t)\cdot n_t\right]}_{\text{Define as } W(c_t,\, n_t)}=A_0$$

Single Lagrange multiplier μ

BASICS OF RAMSEY MACRO FISCAL POLICY

- ☐ **Ramsey problem**

$$\max_{\{c_t, n_t, k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(n_t) \right] \quad \textbf{s.t.}$$

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f\left(k_t, n_t\right)$$

Sequence of Lagrange multipliers $\beta^t\lambda_t$

$$E_0 \sum_{t=0}^{\infty} \beta^t \underbrace{\left[u'(c_t) \cdot c_t - h'(n_t) \cdot n_t \right]}_{\text{Define as } W(c_t,\, n_t)} = A_0$$

Present-value implementability constraint (PVIC): the PV GBC

Single Lagrange multiplier μ

- ☐ **Ramsey FOCs (for $t > 0$, which sidesteps issue of taxation of $t = 0$ initial capital stock and other assets, of which A_0 is a function)**
  - ☐ **Commitment by Ramsey government to its $t > 0$ policies at $t = 0$**
  - ☐ **Discretionary Ramsey government does *not* commit to its $t > 0$ policies at $t = 0$**

BASICS OF RAMSEY MACRO FISCAL POLICY

- ❑ **Ramsey problem**

$$\max_{\{c_t,n_t,k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(n_t)\right] \quad \textbf{s.t.}$$

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f\left(k_t, n_t\right)$$

Sequence of Lagrange multipliers $\beta^t \lambda_t$

Present-value implementability constraint (PVIC): the PV GBC

$$E_0 \sum_{t=0}^{\infty} \beta^t \underbrace{\left[u'(c_t)\cdot c_t - h'(n_t)\cdot n_t\right]}_{\text{Define as } W(c_t,\, n_t)} = A_0$$

Single Lagrange multiplier μ

- ❑ **Ramsey FOCs (for $t > 0$, which sidesteps issue of taxation of $t = 0$ initial capital stock and other assets, of which A_0 is a function)**

$$u'(c_t^{RP}) - \lambda_t^{RP} + \mu\cdot W_c(c_t^{RP}, n_t^{RP}) = 0$$

$$-h'(n_t^{RP}) + \lambda_t^{RP} z_t f_n\left(k_t^{RP}, n_t^{RP}\right) + \mu\cdot W_n(c_t^{RP}, n_t^{RP}) = 0$$

$$-\lambda_t^{RP} + \beta E_t\left\{\lambda_{t+1}^{RP}\left[z_{t+1} f_k(k_{t+1}^{RP}, n_{t+1}^{RP}) + 1 - \delta\right]\right\} = 0$$

BASICS OF RAMSEY MACRO FISCAL POLICY

- ❑ **Ramsey FOCs (for *t* > 0, which sidesteps issue of taxation of *t* = 0 initial capital stock and other assets, of which A_0 is a function)**

$$u'(c_t^{RP}) - \lambda_t^{RP} + \mu \cdot W_c(c_t^{RP}, n_t^{RP}) = 0$$

$$-h'(n_t^{RP}) + \lambda_t^{RP} z_t f_n\left(k_t^{RP}, n_t^{RP}\right) + \mu \cdot W_n(c_t^{RP}, n_t^{RP}) = 0$$

$$-\lambda_t^{RP} + \beta E_t\left\{\lambda_{t+1}^{RP}\left[z_{t+1} f_k(k_{t+1}^{RP}, n_{t+1}^{RP}) + 1 - \delta\right]\right\} = 0$$

- ❑ **Social Planner FOCs**

BASICS OF RAMSEY MACRO FISCAL POLICY

- **Ramsey FOCs (for *t* > 0, which sidesteps issue of taxation of *t* = 0 initial capital stock and other assets, of which A_0 is a function)**

$$u'(c_t^{RP}) - \lambda_t^{RP} + \mu \cdot W_c(c_t^{RP}, n_t^{RP}) = 0$$

$$-h'(n_t^{RP}) + \lambda_t^{RP} z_t f_n\left(k_t^{RP}, n_t^{RP}\right) + \mu \cdot W_n(c_t^{RP}, n_t^{RP}) = 0$$

$$-\lambda_t^{RP} + \beta E_t\left\{\lambda_{t+1}^{RP}\left[z_{t+1} f_k(k_{t+1}^{RP}, n_{t+1}^{RP}) + 1 - \delta\right]\right\} = 0$$

- **Social Planner FOCs**

$$u'(c_t^{SP}) - \lambda_t^{SP} = 0$$

$$-h'(n_t^{SP}) + \lambda_t^{SP} z_t f_n\left(k_t^{SP}, n_t^{SP}\right) = 0$$

$$-\lambda_t^{SP} + \beta E_t\left\{\lambda_{t+1}^{SP}\left[z_{t+1} f_k(k_{t+1}^{SP}, n_{t+1}^{SP}) + 1 - \delta\right]\right\} = 0$$

$\downarrow$

Evaluate at deterministic steady states

BASICS OF RAMSEY MACRO FISCAL POLICY

- ☐ **Ramsey FOCs (for $t > 0$) at deterministic steady state**

$$u'(c^{RP}) - \lambda^{RP} + \mu \cdot W_c(c^{RP}, n^{RP}) = 0$$

$$-h'(n^{RP}) + \lambda^{RP} z \cdot f_n\left(k^{RP}, n^{RP}\right) + \mu \cdot W_n(c^{RP}, n^{RP}) = 0$$

$$-\lambda^{RP} + \beta\lambda^{RP}\left[z \cdot f_k(k^{RP}, n^{RP}) + 1 - \delta\right] = 0$$

- ☐ **Social Planner FOCs at deterministic steady state**

$$u'(c^{SP}) - \lambda^{SP} = 0$$

$$-h'(n^{SP}) + \lambda^{SP} z \cdot f_n\left(k^{SP}, n^{SP}\right) = 0$$

$$-\lambda^{SP} + \beta \cdot \lambda^{SP}\left[z \cdot f_k(k^{SP}, n^{SP}) + 1 - \delta\right] = 0$$

BASICS OF RAMSEY MACRO FISCAL POLICY

- **Ramsey FOCs (for $t > 0$) at deterministic steady state**

$$u'(c^{RP}) - \lambda^{RP} + \mu \cdot W_c(c^{RP}, n^{RP}) = 0 \tag{1}$$

$$-h'(n^{RP}) + \lambda^{RP} z \cdot f_n\left(k^{RP}, n^{RP}\right) + \mu \cdot W_n(c^{RP}, n^{RP}) = 0 \tag{2}$$

$$-\cancel{\lambda^{RP}} + \beta \cancel{\lambda^{RP}} \left[z \cdot f_k(k^{RP}, n^{RP}) + 1 - \delta \right] = 0 \tag{3}$$

- **Social Planner FOCs at deterministic steady state**

$$u'(c^{SP}) - \lambda^{SP} = 0 \tag{4}$$

$$-h'(n^{SP}) + \lambda^{SP} z \cdot f_n\left(k^{SP}, n^{SP}\right) = 0 \tag{5}$$

$$-\cancel{\lambda^{SP}} + \beta \cdot \cancel{\lambda^{SP}} \left[z \cdot f_k(k^{SP}, n^{SP}) + 1 - \delta \right] = 0 \tag{6}$$

BASICS OF RAMSEY MACRO FISCAL POLICY

- **Ramsey FOCs (for $t > 0$) at deterministic steady state**

$$u'(c^{RP}) - \lambda^{RP} + \mu \cdot W_c(c^{RP}, n^{RP}) = 0 \tag{1}$$

$$-h'(n^{RP}) + \lambda^{RP} z \cdot f_n\left(k^{RP}, n^{RP}\right) + \mu \cdot W_n(c^{RP}, n^{RP}) = 0 \tag{2}$$

$$-\cancel{\lambda^{RP}} + \beta \cancel{\lambda^{RP}}\left[z \cdot f_k(k^{RP}, n^{RP}) + 1 - \delta\right] = 0 \tag{3}$$

- **Social Planner FOCs at deterministic steady state**

$$u'(c^{SP}) - \lambda^{SP} = 0 \tag{4}$$

$$-h'(n^{SP}) + \lambda^{SP} z \cdot f_n\left(k^{SP}, n^{SP}\right) = 0 \tag{5}$$

$$-\cancel{\lambda^{SP}} + \beta \cdot \cancel{\lambda^{SP}}\left[z \cdot f_k(k^{SP}, n^{SP}) + 1 - \delta\right] = 0 \tag{6}$$

- **(3) and (6) imply Ramsey-optimal *k/n* ratio = efficient *k/n* ratio**
  - **(Given maintained assumption of CRS production *f*(.))**
  - **A crucial result!**
  - **Second-best *k/n* ratio = first-best *k/n* ratio**
  - **Chamley (1986 *ECTA*), Judd (1985 *JPub*) seminal references**

ZERO CAPITAL INCOME TAX

- **What does this imply for Ramsey-optimal tax rates?**
- **Recall household optimization**
  - **With labor income tax and capital income tax (and no lump-sum taxes)**

$$\max_{\{c_t,n_t,k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h(n_t) \right] \quad \textbf{s.t.} \quad c_t + k_{t+1} = (1-\tau_t^n) w_t n_t + \left[1 + (1-\tau_t^k)(r_t - \delta) \right] k_t$$

  - **Steady-state consumption-labor optimality (labor supply condition)**

$$\frac{h'(n)}{u'(c)} = (1-\tau^n) z \cdot f_n(k,n)$$

= *w* in equilibrium

  - **Steady-state consumption-savings optimality (capital Euler condition)**

= *r* in equilibrium

$$\cancel{u'(c)} = \beta \cancel{u'(c)} \left(1 + (1-\tau^k)(z \cdot f_k(k,n) - \delta) \right)$$

ZERO CAPITAL INCOME TAX

- **What does this imply for Ramsey-optimal tax rates?**
- **Recall household optimization**
  - **With labor income tax and capital income tax (and no lump-sum taxes)**

$$\max_{\{c_t,n_t,k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t)-h(n_t)\right] \quad \textbf{s.t.} \quad c_t + k_{t+1} = (1-\tau_t^n)w_t n_t + \left[1+(1-\tau_t^k)(r_t-\delta)\right]k_t$$

  - **Steady-state consumption-labor optimality (labor supply condition)**

$$\frac{h'(n)}{u'(c)} = (1-\tau^n)z\cdot f_n(k,n)$$

= *w* in equilibrium

  - **Steady-state consumption-savings optimality (capital Euler condition)**

= *r* in equilibrium

$$u'(\cancel{c}) = \beta u'(\cancel{c})\Big(1+(1-\tau^k)(z\cdot f_k(k,n)-\delta)\Big)$$

  - **Ramsey-optimal capital income tax rate = 0!**
  - **Don't tax intertemporal margin at all in the long run…**
  - **…even though Ramsey government has to raise revenue through distortionary taxes**

POSITIVE LABOR INCOME TAX

- **What does this imply for Ramsey-optimal tax rates?**
  - **Steady-state consumption-labor optimality (labor supply condition)**

$$\frac{h'(n)}{u'(c)} = (1-\tau^n)z \cdot f_n(k,n)$$

  - **Steady-state consumption-savings optimality (capital Euler condition)**

$$u'\cancel{(c)} = \beta \cancel{u'(c)}\left(1+(1-\tau^k)(z \cdot f_k(k,n)-\delta)\right)$$

  - **Ramsey-optimal capital income tax rate = 0!**
  - **Don’t tax intertemporal margin at all in the long run…**
  - **…even though Ramsey government has to raise revenue through distortionary taxes**

- **All revenue must be raised through positive labor income tax**

- **Two central Ramsey macro fiscal policy results**

DYNAMICS OF TAX RATES

- ☐ **Outside the steady state?**
- ☐ **Focus on labor income tax rate (simple to consider)**
  - ☐ **Consumption-labor optimality (labor supply condition)**

$$\frac{h'(n_t)}{u'(c_t)} = (1 - \tau_t^n) z_t f_n(k_t, n_t)$$

DYNAMICS OF TAX RATES

- **Outside the steady state?**
- **Focus on labor income tax rate (simple to consider)**
  - **Consumption-labor optimality (labor supply condition)**

$$\underbrace{\frac{h'(n_t)}{u'(c_t)}}_{= \mathbf{MRS}_t} = (1-\tau_t^n)\underbrace{z_t f_n(k_t, n_t)}_{= \mathbf{MPN}_t}$$

$$\rightarrow MRS_t = (1-\tau_t^n) MPN_t$$

- **Labor income tax is a wedge between labor supply and labor demand**
- **Along the business cycle?**
  - **Consider utility form** $u(c_t) - h(n_t) = \ln c_t - \frac{\kappa}{1+1/\iota} n_t^{1+1/\iota}$

ι is labor supply elasticity with respect to real wage

DYNAMICS OF TAX RATES

- **Along the business cycle?**
  - **Consider utility form** $u(c_t) - h(n_t) = \ln c_t - \frac{\kappa}{1+1/\iota} n_t^{1+1/\iota}$

ι is labor supply elasticity with respect to real wage

- **Compute first and second derivatives of *u*(.) and *h*(.)…**
  - **…which are needed to compute W_c(.) and W_n(.)**
- **Do some algebra combining the Ramsey FOCs …**

DYNAMICS OF TAX RATES

- **Along the business cycle?**
  - **Consider utility form** $u(c_t) - h(n_t) = \ln c_t - \frac{\kappa}{1+1/\iota} n_t^{1+1/\iota}$

ι is labor supply elasticity with respect to real wage

- **Compute first and second derivatives of *u*(.) and *h*(.)…**
  - **…which are needed to compute W_c(.) and W_n(.)**

- **Do some algebra combining the Ramsey FOCs …**

$$\underbrace{\kappa \cdot n_t^{1/\iota} \cdot c_t}_{=\ \mathrm{MRS}_t} = \underbrace{\left[1 + \mu\left(\frac{1+\iota}{\iota}\right)\right]^{-1}}_{=\ \text{wedge between } \mathrm{MRS}_t \text{ and } \mathrm{MPN}_t} \cdot \underbrace{z_t f_n(k_t, n_t)}_{=\ \mathrm{MPN}_t}$$

DYNAMICS OF TAX RATES

- **Along the business cycle?**
  - **Consider utility form** $u(c_t)-h(n_t)=\ln c_t-\frac{\kappa}{1+1/\iota}n_t^{1+1/\iota}$

ι is labor supply elasticity with respect to real wage

- **Compute first and second derivatives of *u*(.) and *h*(.)…**
  - **…which are needed to compute W_c(.) and W_n(.)**
- **Do some algebra combining the Ramsey FOCs …**

$$\underbrace{\kappa\cdot n_t^{1/\iota}\cdot c_t}_{=\ \mathrm{MRS}_t}=\underbrace{\left[1+\mu\left(\frac{1+\iota}{\iota}\right)\right]^{-1}}_{=\ \text{wedge between } \mathrm{MRS}_t \text{ and } \mathrm{MPN}_t}\cdot\underbrace{z_t f_n(k_t,n_t)}_{=\ \mathrm{MPN}_t}$$

- **Wedge is a (endogenous…) constant between MRS and MPN in every time period**
  - **$\mu = 0$ (the case of lump-sum taxes) ➔ wedge = 0**
  - **$\mu > 0$ (the Ramsey case) ➔ wedge ≠ 0**

DYNAMICS OF TAX RATES

- **Along the business cycle?**
- **Wedge is a (endogenous…) constant between MRS and MPN in every time period…**

$$\underbrace{\kappa \cdot n_t^{1/\iota} \cdot c_t}_{=\ \text{MRS}_t} = \underbrace{\left[1+\mu\left(\frac{1+\iota}{\iota}\right)\right]^{-1}}_{=\ \text{wedge between MRS}_t\text{ and MPN}_t} \cdot \underbrace{z_t f_n(k_t, n_t)}_{=\ \text{MPN}_t}$$

- **…thus labor income tax rate is constant over time (for this utility form)**
    - **Nearly constant if move to slightly different *h*(*n*) function**

DYNAMICS OF TAX RATES

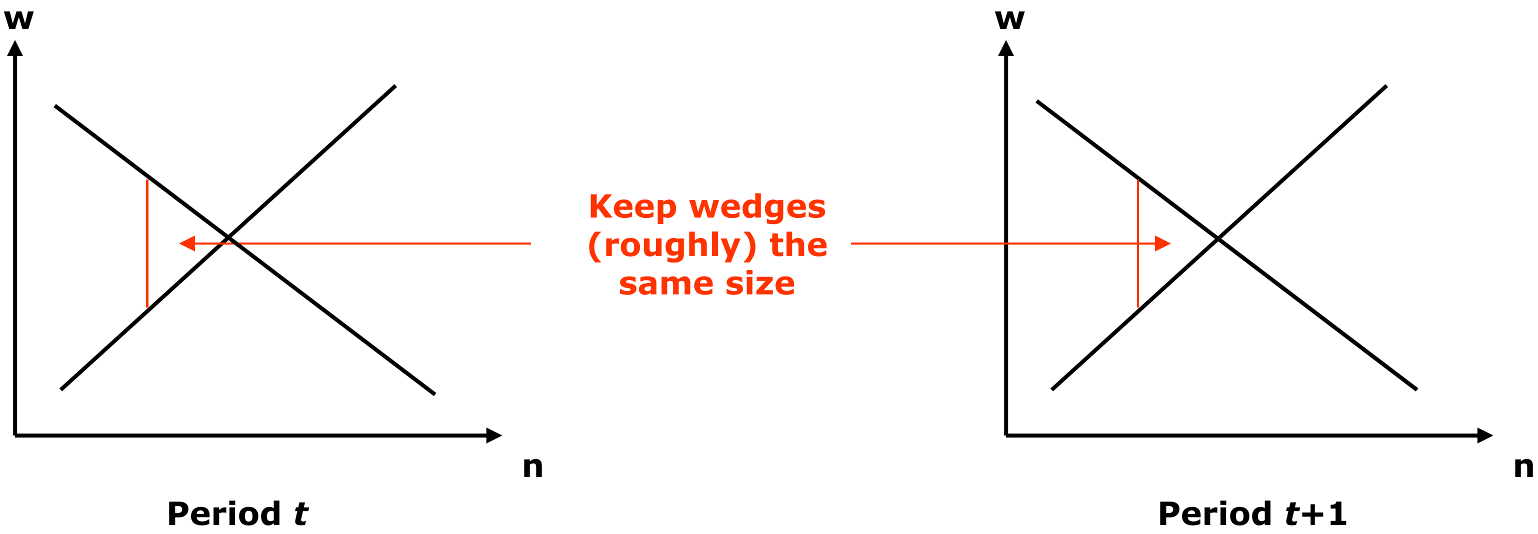
- **Along the business cycle?**
- **Wedge is a (endogenous…) constant between MRS and MPN in every time period…**

$$\underbrace{\kappa \cdot n_t^{1/\iota} \cdot c_t}_{=\ \mathrm{MRS}_t} = \underbrace{\left[1+\mu\left(\frac{1+\iota}{\iota}\right)\right]^{-1}}_{=\ \text{wedge between } \mathrm{MRS}_t \text{ and } \mathrm{MPN}_t} \cdot \underbrace{z_t f_n(k_t, n_t)}_{=\ \mathrm{MPN}_t}$$

- **…thus labor income tax rate is constant over time (for this utility form)**
  - **Nearly constant if move to slightly different *h*(*n*) function**

- **Labor income tax smoothing**
  - **Key Ramsey macro fiscal policy result**
  - **Keep deadweight losses constant across markets over time**
    - **aka wedges constant**

TAX SMOOTHING VS. WEDGE SMOOTHING



Keep wedges (roughly) the same size

Period *t*

Period *t*+1

- ❑ **Ramsey wants to keep these wedges constant**

$$MRS_t = \mathrm{WEDGE}_t \cdot MPN_t \; \forall t$$

Wedge (Walrasian labor market)

GENERAL EQUILIBRIUM WEDGES

APRIL 13, 2017

TRANSFORMATION FUNCTION

- ☐ **Construct model-consistent transformation function**

"The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

$$MRT_{l,k}(y) = \frac{\partial F(y) / \partial y_l}{\partial F(y) / \partial y_k}$$

is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…

***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

TRANSFORMATION FUNCTION

☐ **Construct model-consistent transformation function**

"The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

$$MRT_{l,k}(y) = \frac{\partial F(y)/\partial y_l}{\partial F(y)/\partial y_k}$$

is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…

…A single-output technology is commonly described by means of a *production function f*(*z*)…Holding the level of output fixed, we can define the *marginal rate of technical substitution (MRTS$_{l,k}$)* … Note that *MRTS$_{l,k}$* is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."

***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

TRANSFORMATION FUNCTION

- ☐ **Construct model-consistent transformation function**

  "The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

  $$MRT_{l,k}(y) = \frac{\partial F(y)/\partial y_l}{\partial F(y)/\partial y_k}$$

  is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…

  …A single-output technology is commonly described by means of a *production function f*(*z*)…Holding the level of output fixed, we can define the *marginal rate of technical substitution (MRTS$_{l,k}$)* … Note that *MRTS$_{l,k}$* is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."

  ***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

- ☐ **RBC model**

- ☐ **Does one-unit decrease in 1-n_t affect c_t?**
  - ☐ **If so, how?**

- ☐ **Does one-unit decrease in c_t affect c_{t+1}?**
  - ☐ **If so, how?**

TRANSFORMATION FUNCTION

- ❑ **Transformation function of RBC model**

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f\left(k_t, n_t\right)$$

Goods resource constraint

- ❑ **One-unit decrease in 1-n_t → one-unit increase in n_t**
- ❑ **One-unit increase in n_t → output increases by $z_t f_n(k_t, n_t)$ units**

TRANSFORMATION FUNCTION

- ❑ **Transformation function of RBC model**

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f\left(k_t, n_t\right)$$

Goods resource constraint

- ❑ **One-unit decrease in 1-n_t → one-unit increase in n_t**
- ❑ **One-unit increase in n_t → output increases by $z_t f_n(k_t, n_t)$ units**
- ❑ **Increase of output by $z_t f_n(k_t, n_t)$ units → c_t increases by $z_t f_n(k_t, n_t)$ units**

$$\boxed{MRT_{c_t, n_t} \equiv z_t f_n(k_t, n_t)}$$

TRANSFORMATION FUNCTION

- ☐ **Construct model-consistent transformation function**

  > **"The production set is taken as a primitive datum of the theory…If [the transformation function] $F(\cdot)$ is differentiable, and if the production vector y satisfies $F(y) = 0$, then for any commodities l and k, the ratio**
  >
  > $$MRT_{l,k}(y) = \frac{\partial F(y)/\partial y_l}{\partial F(y)/\partial y_k}$$
  >
  > **is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…**
  >
  > **…A single-output technology is commonly described by means of a *production function* $f(z)$…Holding the level of output fixed, we can define the *marginal rate of technical substitution* ($MRTS_{l,k}$) … Note that $MRTS_{l,k}$ is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."**
  >
  > ***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

- ☐ **RBC model**

- ☐ **Does one-unit decrease in 1-n_t affect c_t?**
  - ☐ $MRT_{c_t,n_t} \equiv z_t f_n(k_t, n_t)$

- ☐ **Does one-unit decrease in c_t affect c_{t+1}?**
  - ☐ **If so, how?**

TRANSFORMATION FUNCTION

- ❑ **Transformation function of RBC model (between *t* and *t* + 1)**

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t) \qquad c_{t+1} + g_{t+1} + k_{t+2} - (1-\delta)k_{t+1} = z_{t+1} f(k_{t+1}, n_{t+1})$$

- ❑ **One-unit decrease in c_t → one-unit increase in k_{t+1}**

TRANSFORMATION FUNCTION

- ☐ **Transformation function of RBC model (between *t* and *t* + 1)**

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t) \qquad c_{t+1} + g_{t+1} + k_{t+2} - (1-\delta)k_{t+1} = z_{t+1} f(k_{t+1}, n_{t+1})$$

- ☐ **One-unit decrease in c_t → one-unit increase in k_{t+1}**
- ☐ **One-unit increase in k_{t+1} → output increases by $z_{t+1}f_k(k_{t+1},n_{t+1})$ units**
- ☐ **Increase of output by $z_{t+1}f_k(k_{t+1},n_{t+1})$ units**
- ☐ **→ c_{t+1} increases by**

$$\boxed{MRT_{c_t,c_{t+1}} \equiv 1 + z_{t+1} f_k(k_{t+1}, n_{t+1}) - \delta}$$

TRANSFORMATION FUNCTION

- **Construct model-consistent transformation function**

> **"The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio**
>
> $$MRT_{l,k}(y) = \frac{\partial F(y) / \partial y_l}{\partial F(y) / \partial y_k}$$
>
> **is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…**
>
> **…A single-output technology is commonly described by means of a *production function f*(*z*)…Holding the level of output fixed, we can define the *marginal rate of technical substitution (MRTS$_{l,k}$)* … Note that *MRTS$_{l,k}$* is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."**
>
> ***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

- **RBC model**

- **Does one-unit decrease in 1-n_t affect c_t?**
  - $MRT_{c_t,n_t} \equiv z_t f_n(k_t, n_t)$

- **Does one-unit decrease in c_t affect c_{t+1}?**
  - $MRT_{c_t,c_{t+1}} \equiv 1 + z_{t+1} f_k(k_{t+1}, n_{t+1}) - \delta$