Labor Search and Matching: General Equilibrium Wedges

April 13, 2017

MATCHING EFFICIENCY

- **Social Planner**

$$\max_{\{c,n_t,s_t,v_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(lfp_t\right) \right]$$

$lfp_t \equiv (1-p_t)s_t + n_{t-1}$

s.t.

$$c_t + g_t + \gamma v_t = z_t n_t$$

Resource constraint

$$n_t = (1-\rho) n_{t-1} + m(s_t, v_t)$$

Aggregate LOM for total employment

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Aggregate LOM for total employment

FOCs

(consider deterministic case)

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{\gamma m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

$$= \gamma \theta_t \frac{\xi}{1-\xi}$$

Static Efficiency Condition.

"Efficient Participation Condition"

Can instead derive directly off transformation frontier of model.

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$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{\gamma\, m_s(s_t,v_t)}{m_v(s_t,v_t)} = \gamma\theta_t \frac{\xi}{1-\xi}$$

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(\dfrac{\gamma}{m_v(s_{t+1},v_{t+1})}\right)(1-m_s(s_{t+1},v_{t+1}))}{\dfrac{\gamma}{m_v(s_t,v_t)} - z_t}$$

Static Efficiency Condition.

"Efficient Participation Condition"

Can instead derive directly off transformation frontier of model.

Intertemporal Efficiency Condition.

"Efficient Vacancies Condition"

Can instead derive directly off transformation frontier of model.

TRANSFORMATION FRONTIER

❑ Construct model-consistent transformation function

"The production set is taken as a primitive datum of the theory…If [the transformation function] *F*(·) is differentiable, and if the production vector *y* satisfies *F*(*y*) = 0, then for any commodities *l* and *k*, the ratio

$$MRT_{l,k}(y) = \frac{\partial F(y)/\partial y_l}{\partial F(y)/\partial y_k}$$

is called the *marginal rate of transformation (MRT) of good l for good k at vector y*…

…A single-output technology is commonly described by means of a *production function f*(*z*)…Holding the level of output fixed, we can define the *marginal rate of technical substitution* ($MRTS_{l,k}$) … Note that $MRTS_{l,k}$ is simply a renaming of the marginal rate of transformation…in the special case of a single-output technology."

***Microeconomic Theory,* Mas-Colell, Whinston, and Green (p. 128 – 130)**

TRANSFORMATION FRONTIER

- Ceteris paribus…
- Does one-unit decrease in $(1 - lfp_t)$ affect c_t?
  - If so, how?
- Does one-unit decrease in c_t affect c_{t+1}?
  - If so, how?

TRANSFORMATION FRONTIER

- Transformation function

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho) n_{t-1} + m(s_t, v_t)$$

TRANSFORMATION FRONTIER

- **Transformation function**

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

- → $v_t = \dfrac{z_t n_t - c_t}{\gamma}$
- **Insert into LOM for n_t to construct** $n_t - (1-\rho)n_{t-1} - m\left(s_t, \dfrac{z_t n_t - c_t}{\gamma}\right) = 0$
- **Use $lfp_t = (1-\rho)n_{t-1} + s_t$ to construct within-period transformation frontier**

$$\Gamma\left(c_t, lfp_t, n_t; \cdot\right) \equiv n_t - (1-\rho)n_{t-1} - m\left(lfp_t - (1-\rho)n_{t-1}, \frac{z_t n_t - c_t}{\gamma}\right) = 0$$

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- **Use IFT to obtain static MRT (participation margin)**

$$MRT_{c_t, lfp_t} = -\frac{\Gamma_{lfp_t}}{\Gamma_{c_t}} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

STATIC **MRT between LFP and Walrasian good**

Transformation Frontier – Intuition

- **One-unit decrease in (1 – lfp_t) …**
- **→ increases s_t by one unit …**
- **→ increases n_t by $m_s(s_t, v_t)$ units …**

TRANSFORMATION FRONTIER – INTUITION

- ❑ **One-unit decrease in (1 – lfp_t) …**
- ❑ **→ increases s_t by one unit …**
- ❑ **→ increases n_t by** $m_s(s_t, v_t)$ **units …**
- ❑ **→ increases $z_t n_t$ by** $z_t m_s(s_t, v_t)$ **units …**

TRANSFORMATION FRONTIER – INTUITION

- **One-unit decrease in (1 – lfp_t) …**
- **→ increases s_t by one unit …**
- **→ increases n_t by $m_s(s_t, v_t)$ units …**
- **→ increases $z_t n_t$ by $z_t m_s(s_t, v_t)$ units …**
- **To hold n_t constant, v_t must *decrease* by $m_v(s_t, v_t)$ …**
- **… which decreases $z_t n_t$ by $\dfrac{z_t m_v(s_t, v_t)}{\gamma}$ units**

$$\Rightarrow MRT_{c_t, lfp_t} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

TRANSFORMATION FRONTIER

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- Does one-unit decrease in (1 – lfp_t) affect c_t?

$$MRT_{c_t, lfp_t} = -\frac{\Gamma_{lfp_t}}{\Gamma_{c_t}} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

- Does one-unit decrease in c_t affect c_{t+1}?
  - If so, how?

TRANSFORMATION FRONTIER

- Transformation function

$$c_t + \gamma v_t = z_t n_t \qquad n_t = (1-\rho)n_{t-1} + m(s_t, v_t)$$

- → $v_t = \dfrac{z_t n_t - c_t}{\gamma}$, **then insert into LOM for n_t**
- → $n_t - (1-\rho)n_{t-1} - m\left(s_t, \dfrac{z_t n_t - c_t}{\gamma}\right) = 0$
- **Use $lfp_t = (1-\rho)n_{t-1} + s_t$ to construct within-period transformation frontier**

$$\Gamma\left(c_t, lfp_t, n_t; \cdot\right) \equiv n_t - (1-\rho)n_{t-1} - m\left(lfp_t - (1-\rho)n_{t-1}, \frac{z_t n_t - c_t}{\gamma}\right) = 0$$

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STATIC **MRT between LFP and Walrasian good**

$$\frac{\partial n_t}{\partial c_t} = -\frac{\Gamma_{c_t}}{\Gamma_{n_t}} = -\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$$

Marginal effect on n_t of a change in c_t ...which has *intertemporal* consequences

TRANSFORMATION FRONTIER

- **Transformation function across periods**

$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho) n_t - m\left(lfp_{t+1} - (1-\rho) n_t, \frac{z_{t+1} n_{t+1} - c_{t+1}}{\gamma} \right) = 0$$

- **Use IFT to obtain intertemporal MRT**

$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}}$$

TRANSFORMATION FRONTIER

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- Use IFT to obtain intertemporal MRT

$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}} \qquad G_{c_{t+1}} = \frac{m_v(s_{t+1}, v_{t+1})}{\gamma} \qquad G_{c_t} = -(1-\rho)\frac{\partial n_t}{\partial c_t} + (1-\rho)m_s(s_{t+1}, v_{t+1})\frac{\partial n_t}{\partial c_t}$$

Transformation Frontier

- **Transformation function across periods**

$$G\left(c_{t+1}, n_{t+1}, c_t, n_t; \cdot\right) \equiv n_{t+1} - (1-\rho) n(c_t) - m\left(lfp_{t+1} - (1-\rho) n(c_t), \frac{z_{t+1} n_{t+1} - c_{t+1}}{\gamma}\right) = 0$$

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$$\frac{\partial n_t}{\partial c_t} = -\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$$

And convert back into consumption units …

$$IMRT_{c_t, c_{t+1}} = -\frac{G_{c_t}}{G_{c_{t+1}}} = \frac{(1-\rho)\left(\frac{\gamma}{m_v(s_{t+1}, v_{t+1})}\right)\left(1 - m_s(s_{t+1}, v_{t+1})\right)}{\frac{\gamma}{m_v(s_t, v_t)} - z_t}$$

TRANSFORMATION FRONTIER – INTUITION

- **One unit reduction in c_t …**
- **→ increases v_t by $1/\gamma$ units**
- **→ increases n_t by $\frac{m_v(s_t, v_t)}{\gamma}$ units**
- **→ *increases* c_t by $\frac{z_t m_v(s_t, v_t)}{\gamma}$ units**

TRANSFORMATION FRONTIER – INTUITION

- One unit reduction in c_t …
- → increases v_t by $1/\gamma$ units
- → increases n_t by $\frac{m_v(s_t, v_t)}{\gamma}$ units
- → *increases* c_t by $\frac{z_t m_v(s_t, v_t)}{\gamma}$ units
- … so resulting change in c_t is …

Must be netted out…

…in order to hold period-*t* output constant

$$\frac{\gamma - z_t m_v(s_t, v_t)}{\gamma} (< 1)$$

- → increase in v_t by $\frac{1}{\gamma - z_t m_v(s_t, v_t)}$ units for ***ONE-UNIT*** **DECREASE IN** c_t

TRANSFORMATION FRONTIER – INTUITION

- **Increase in v_t by** $\frac{1}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_t, v_t)$ by** $\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_{t+1}, v_{t+1})$ by** $(1-\rho)\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**

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- **→ increase in $m(s_t, v_t)$ by** $\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **→ increase in $m(s_{t+1}, v_{t+1})$ by** $(1-\rho)\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$ **units …**
- **To hold n_{t+1} constant, s_{t+1} must <u>*decrease*</u> by $m_s(s_{t+1}, v_{t+1})$ …**
- **→ increase in v_{t+1} by** $(1-\rho)\left(\dfrac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}\right)\left(1 - m_s(s_{t+1}, v_{t+1})\right)$ **units …**

TRANSFORMATION FRONTIER – INTUITION

- **Increase in v_{t+1} by** $(1-\rho)\left(\frac{m_v(s_t,v_t)}{\gamma - z_t m_v(s_t,v_t)}\right)\left(1-m_s(s_{t+1},v_{t+1})\right)$ **units …**

- **→ increases by c_{t+1}** $\frac{(1-\rho)\left(\frac{\gamma}{m_v(s_t,v_t)}\right)\left(1-m_s(s_{t+1},v_{t+1})\right)}{\frac{\gamma}{m_v(s_t,v_t)} - z_t}$ **units**

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- **Does one-unit decrease in c_t affect c_{t+1}?**

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MATCHING EFFICIENCY

- **Efficiency characterized by**

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{\gamma m_s(s_t, v_t)}{m_v(s_t, v_t)} = \gamma\theta_t \frac{\xi}{1-\xi}$$

= Static MRT$_t$

Static Efficiency Condition.

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho)\left(\frac{\gamma}{m_v(s_{t+1}, v_{t+1})}\right)(1 - m_s(s_{t+1}, v_{t+1}))}{\frac{\gamma}{m_v(s_t, v_t)} - z_t}$$

= Intertemporal MRT$_t$

Intertemporal Efficiency Condition.

MATCHING EFFICIENCY

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= Static MRT$_t$ **= Intertemporal MRT$_t$**

Static Efficiency Condition. **Intertemporal Efficiency Condition.**

- **Ramsey theory: stabilizing THESE wedges is optimal!**
  - **MRTs in DSGE search and matching model: Arseneau and Chugh (2012 *JPE*)**
- **Contribution to understanding efficiency in DGE models with "entry" margins**
  - **MRTs in new monetarist models: Aruoba and Chugh (2010 *JET*)**
  - **MRTs in customer market models: Arseneau, Chahrour, Chugh, and Finkelstein Shapiro (2015 *JMCB*)**
  - **MRTs in endogenous product variety framework: Chugh and Ghironi (2015)**