
TAX SMOOTHING IN FRICTIONAL LABOR MARKETS

APRIL 13, 2017

OVERVIEW OF MODEL

- ❑ **Infinitely-lived representative household, measure one of members**
 - ❑ Employed members
 - ❑ Unemployed members
 - ❑ Members outside the labor force (“leisure”)
- ❑ **Exogenous stochastic government spending**
 - ❑ Financed via labor income taxation and one-period real state-contingent debt
 - ❑ **Government provides unemployment benefits**
 - ❑ Government provides vacancy subsidies
 - ❑ For completeness of tax instruments (Ramsey issue)
- ❑ **Labor market with matching frictions and wage-setting frictions**
- ❑ **Only an extensive labor margin, no intensive labor margin**
- ❑ **Timing: “instantaneous production”**

Full consumption insurance – standard in DSGE labor search models

Incompleteness of government debt markets NOT driving our results (Aiyagari et al (2002 *JPE*))



HOUSEHOLD OPTIMIZATION

- Maximize expected lifetime utility

$$\max_{\{c_t, n_t, s_t, b_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \underbrace{h((1-p_t)s_t + n_t)}_{\text{disutility of employment + unsuccessful search}} \right]$$

s.t.

$$c_t + b_t = \underbrace{n_t(1-\tau_t^n)w_t}_{\text{measure } n \text{ earn after-tax wage income}} + \underbrace{(1-p_t)s_t\chi}_{\text{measure } ue = (1-p)s \text{ receive } ue \text{ benefit } \chi \text{ (government financed)}} + \underbrace{Rb_{t-1} + (1-\tau^d)d_t}_{\text{Flow budget constraint}} \quad \text{Flow budget constraint}$$

Baseline analysis: set $\tau^d = 1 \rightarrow$ no **profit-taxation** issues driving results

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$$n_t = \underbrace{(1-\rho)n_{t-1}}_{\text{(exogenous) measure of pre-existing employment relationships terminate}} + \underbrace{s_t p_t}_{\text{flow of new employment relationships = measure of searchers } s_t \times \text{probability a searcher successfully lands a job}}$$

Perceived LOM for employment ("instantaneous production")

↓
FOCs with respect c_t, n_t, s_t, b_t

HOUSEHOLDS

- Household LFP condition (the labor supply condition!)

$$\frac{h'(lfp_t)}{u'(c_t)} = p_t \left[(1 - \tau_t^n) w_t + (1 - \rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{1 - p_{t+1}}{p_{t+1}} \right) \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right] + (1 - p_t) \chi$$

- MRS between lfp_t and c_t = expected payoff of searching
 - Unemployment benefit (with probability $1 - p_t$)
 - After-tax wage + continuation value (with probability p_t)

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To recover standard labor supply function (e.g., RBC)

1. $\rho = 1$ (all employment relationships terminate at end of every period)
2. $p = 1$ (probability a searcher finds a job)
3. $\chi = 0$ (no ue benefit because no notion of "ue")

$$\frac{h'(lfp_t)}{u'(c_t)} = (1 - \tau_t^n) w_t$$

FIRMS

- ❑ **Production**
 - ❑ Requires a matched job-worker pair: posting cost γ per vacancy
 - ❑ Individual job i produces $y_{it} = z_t$
 - ❑ Aggregate output $y_t = n_t z_t$ (symmetry across jobs)

- ❑ **Dynamic profit-maximization problem**

$$\max_{\{n_t, v_t\}} \sum_{t=0}^{\infty} \mathbb{E}_{t|0} \left[z_t n_t - w_t n_t - (1 - \tau_t^s) \gamma v_t \right]$$

Ensures completeness of tax instruments

$$n_t = \underbrace{(1 - \rho)n_{t-1}}_{\text{(exogenous) measure of pre-existing employment relationships terminate}} + \underbrace{v_t q_t}_{\text{flow of new employment relationships = \# job-openings x probability an opening attracts a searching individual}}$$

Firm's perceived LOM for total employment ("instantaneous hiring")

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- ❑ **Vacancy-creation condition**

$$\underbrace{\frac{\gamma(1 - \tau_t^s)}{q_t}}_{\text{cost of posting vacancy (inclusive of subsidy or tax)}} = \underbrace{z_t - w_t + (1 - \rho) E_t \left[\mathbb{E}_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right]}_{\text{benefit of posting vacancy}}$$

cost of posting vacancy (inclusive of subsidy or tax)

benefit of posting vacancy

LABOR MARKET

- ❑ Labor-market tightness $\theta_t = v_t/u_t$
 - ❑ Important aggregate variable in matching-based models
 - ❑ Matching probabilities p and q depend only on θ given CRTS matching
 - ❑ **Key statistic for matching efficiency**

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- ❑ Labor-market tightness $\theta_t = v_t/u_t$
 - ❑ Important aggregate variable in matching-based models
 - ❑ Matching probabilities p and q depend only on θ given CRTS matching
 - ❑ **Key statistic for matching efficiency**
- ❑ Matching function $m(s_t, v_t) = \psi s_t^\xi v_t^{1-\xi}$
- ❑ LOM for aggregate employment $n_t = (1 - \rho)n_{t-1} + m(s_t, v_t)$
- ❑ Nash bargaining over wage payment solves

$$\max_{w_t} \underbrace{\left(W_t - U_t \right)^\eta}_{\text{Gain to household of successfully forming another employment relationship}} \underbrace{J_t^{1-\eta}}_{\text{Value to firm of hiring another worker}} \longrightarrow \frac{W_t - U_t}{1 - \tau_t^n} = \frac{\eta}{1 - \eta} J_t$$

$$\longrightarrow w_t = \eta z_t + (1 - \eta) \frac{\chi}{1 - \tau_t^n} + \eta(1 - \rho) E_t \left\{ \Xi_{t+1|t} \left[1 - (1 - p_{t+1}) \frac{1 - \tau_{t+1}^n}{1 - \tau_t^n} \right] \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\}$$

GOVERNMENT AND RESOURCE FRONTIER

- Exogenous government spending financed via
 - Labor income tax
 - One-period state contingent real debt

$$\tau_t^n w_t n_t + b_t + \tau^d d_t = g_t + R_t b_{t-1} + (1 - p_t) s_t \chi + \tau_t^s \gamma v_t$$

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 - **Rather than assuming χ is "home production"**

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- Resource constraint

$$c_t + g_t + \gamma v_t = z_t n_t$$

- = govt budget constraint + hh budget constraint
- Assuming χ is govt-financed allows it to drop out of resource constraint
 - **Makes model more comparable to existing Ramsey models**

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- Precise nature of χ (ue benefit? home production? value of leisure?) not typically specified in DSGE matching models

- Our model articulates both ue benefit and value of leisure

PRIVATE-SECTOR EQUILIBRIUM

- Stochastic processes $\{c_t, n_t, s_t, w_t, \theta_t, R_t, b_t\}_{t=0}^{\infty}$ that satisfy
 - □ Household's bond Euler equation
 - Vacancy-creation condition
 - Labor force participation condition
 - Nash wage outcome
 - Law of motion for employment $n_t = (1 - \rho)n_{t-1} + m(s_t, v_t)$
 - □ Government budget constraint (key condition in Ramsey models)
 - □ Resource constraint $c_t + g_t + \gamma v_t = z_t n_t$
 - Given processes $\{g_t, z_t, \tau_t^n, \tau_t^s\}_{t=0}^{\infty}$

Standard conditions in basic Ramsey models

CALIBRATION

- ❑ **Baseline calibration**
 - ❑ So that exogenous policy (non-Ramsey) equilibrium broadly matches U.S. labor market fluctuations

- ❑ **Preferences and key parameters**

$$u(c_t) - h(lfp_t) = \ln c_t - \frac{\kappa}{1 + 1/\iota} lfp_t^{1+1/\iota}$$

- ❑ Participation (labor supply) elasticity ($\iota = 0.18$)
- ❑ Low worker bargaining power ($\eta = 0.05$)
- ❑ High unemployment benefit (98% of real wage)

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- **Rest of parameters, matching-related and otherwise, standard**
 - $\beta = 0.99$
 - $\rho = 0.10$
 - $\xi = 0.40$
 - AR(1) parameters for LOMs for TFP and government spending
 - Etc.

DYNAMICS

		Ramsey		Exogenous Policy Benchmark		Data
		Calibration		Calibration		
		HM	0% and Hosios	HM		
Labor Tax Rate	Mean					22%
	Rel SD					1.4
Market tightness (θ)	Rel SD					11.3
Vacancies	Rel SD					6.3
Unemployment	Rel SD					5.2
LFP	Rel SD					0.20
Real wage	Rel SD					0.52
Static wedge	SD (%)					
Intertemporal wedge	SD (%)					

Gertler and Trigari (2009 JPE)

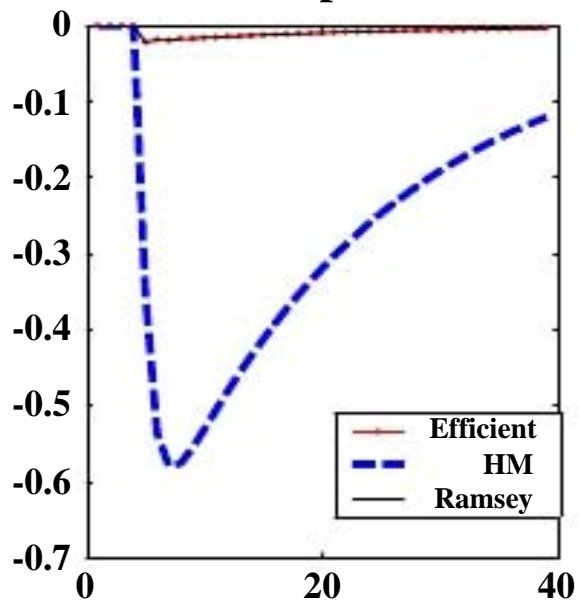
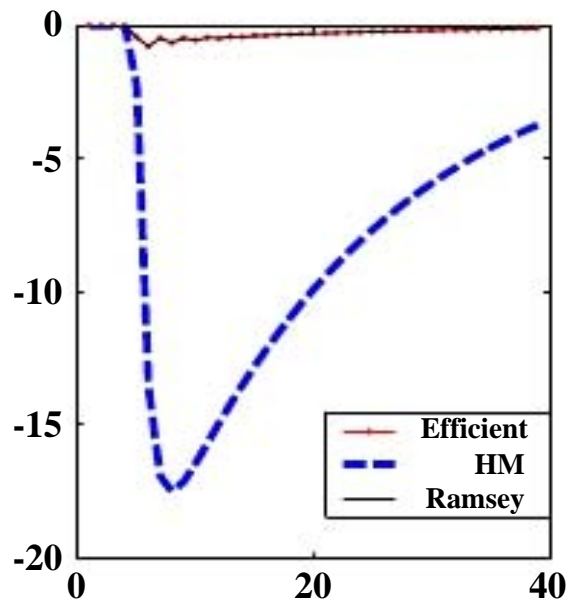
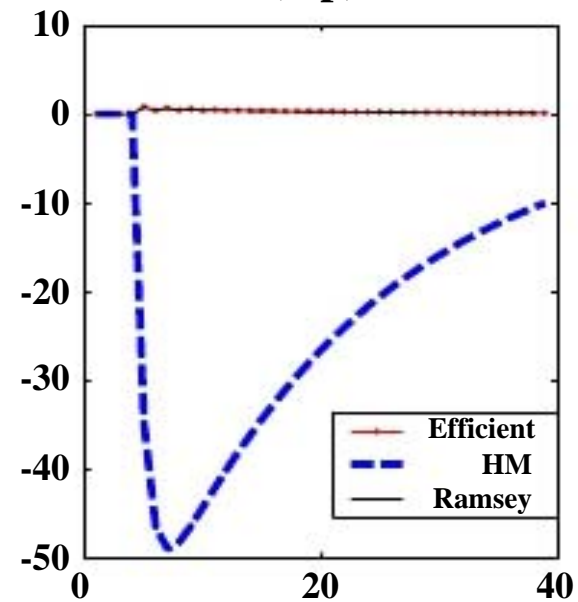
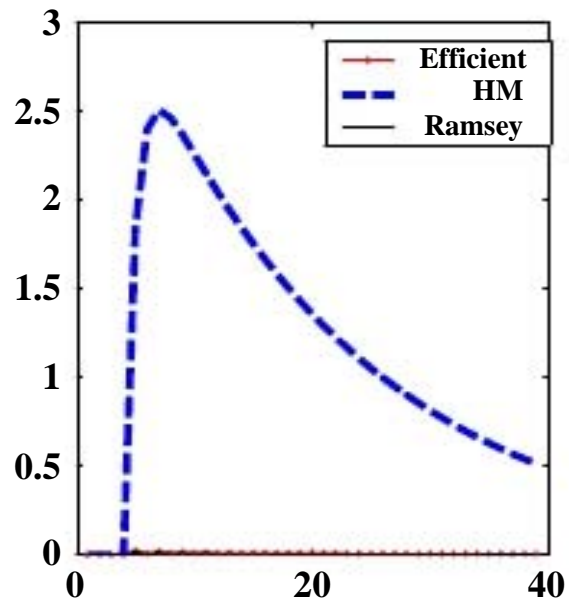
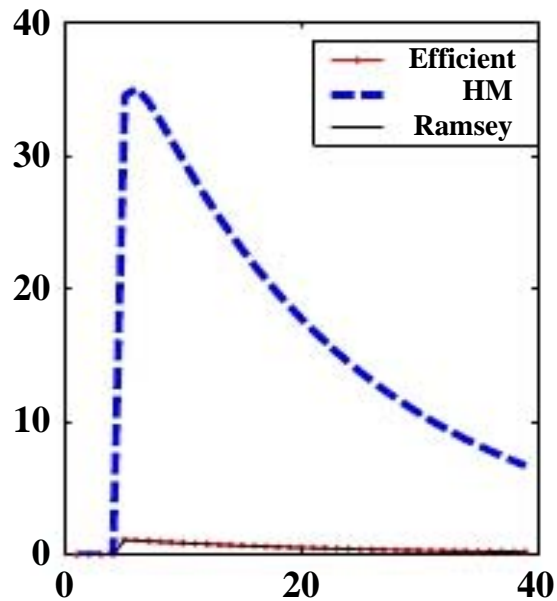
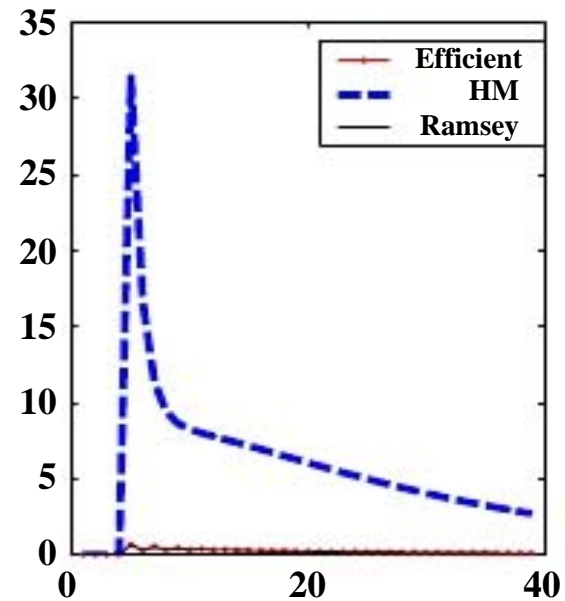
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Labor Tax Rate	Mean	11%		22%		22%
	Rel SD	5.6		1.4		1.4
Market tightness (θ)	Rel SD	1.1		10.9		11.3
Vacancies	Rel SD	1.3		6.9		6.3
Unemployment	Rel SD	1.4		5.4		5.2
LFP	Rel SD	0.13		0.20		0.20
Real wage	Rel SD	0.50		0.28		0.52
Static wedge	SD (%)					
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DYNAMICS

- ❑ **Ramsey fluctuations IDENTICAL to efficient fluctuations for ANY (η, χ) pair**
 - ❑ In terms of fluctuations around a given steady state
 - ❑ Steady-state levels of (τ^n, τ^s) depend on (η, χ) pair

lfp**s****(1-p)s****n****l****v**


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- ❑ **Interpretation: Ramsey government always ensures efficient labor-market fluctuations (v_t, s_t, θ_t)**
 - ❑ By appropriately adjusting (τ^n, τ^s) over the business cycle

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- ❑ **Wedge dynamics**
 - ❑ Ramsey smooths both static wedge....
 - ❑ ...and intertemporal wedge

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LFP	Rel SD	0.13	0.13	0.20		0.20
Real wage	Rel SD	0.50	1.1	0.28		0.52
Static wedge	SD (%)	0.08	0	22.9	0.66	
Intertemporal wedge	SD (%)	0	0	12.3	0.63	

Gertler and Trigari
(2009 JPE)

STATIC AND INTERTEMPORAL CONDITIONS

- Efficiency characterized by

$$\frac{h'(lfp_t)}{u'(c_t)} = \frac{\gamma m_s(s_t, v_t)}{m_v(s_t, v_t)} = \gamma \theta_t \frac{\xi}{1-\xi}$$

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1-\rho) \left(\frac{\gamma}{m_v(s_{t+1}, v_{t+1})} \right) (1 - m_s(s_{t+1}, v_{t+1}))}{\frac{\gamma}{m_v(s_t, v_t)} - z_t}$$

- Decentralized equilibrium conditions characterized by

$$\frac{h'(lfp_t)}{u'(c_t)} = \underbrace{\left[\frac{\chi(1-\xi)}{\gamma \cdot \xi \cdot \theta_t} + (1-\tau_t^n)(1-\tau_t^s) \frac{\eta(1-\xi)}{\xi(1-\eta)} \right]}_{\text{= wedge between static MRS}_t \text{ and static MRT}_t} \gamma \theta_t \frac{\xi}{1-\xi}$$

= wedge between static
MRS_t and static MRT_t

To obtain zero static wedge in every period,
need $\tau^n = \tau^s = 0$ in every period, $\eta = \xi$, $\chi = 0$

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(See eqn. (29) for intertemporal wedge)

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To obtain zero intertemporal wedge in every period, need $\tau^n = \tau^s = 0$ in every period, $\eta = \xi, \chi = 0$

CONCLUSIONS

- ❑ **Labor tax smoothing not optimal in DSGE search and matching model**
 - ❑ **Calibrated to match key labor market dynamics under exogenous tax policy**
 - ❑ **Rigid real wage (delivered through Nash-Hosios bargaining as benchmark) the important feature of the model**

- ❑ **But wedge smoothing IS optimal**
 - ❑ **Basic Ramsey theory**

- ❑ **Ramsey fluctuations in allocations efficient regardless of calibration**

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- ❑ **Welfare-relevant notions of wedges**
 - ❑ **Developing matching-model concepts of efficiency and MRTs for use in virtually any matching application**

- ❑ **Could think of “labor wedge” as featuring both static and intertemporal dimensions**
 - ❑ **Use as framework to empirically measure labor wedges (in progress)**