MONOPOLISTICALLY COMPETITIVE SEARCH EQUILIBRIUM

FEBRUARY 13, 2019

LABOR MARKET INTERMEDIATION

- **Recruiting Sector**
  - **aka "Labor Market Intermediaries"**
  - **aka "Headhunters"**
  - **aka "Middlemen"**

- **Develop Monopolistically Competitive Recruiting Model**
  - **Moen (1997 *JPE*), Shimer (1996)**
  - **Bilbiie, Ghironi, and Melitz (2012 *JPE*)**
  - **Pissarides (1985 *AER*)**

Based on components of these frameworks

- **Wage model**
- **Implications for aggregate matching**
- **Effects between recruiting-market matches and non-recruiting matches**
- **Implications for general equilibrium**

MAIN QUESTIONS

OUTLINE

- Structure of Recruiting Markets
  - Free entry in recruiting markets
  - Recruiter *ij* profit maximization
  - Cost minimization (directed-search optimization)
- Analytical Results – Part I
  - Surplus sharing condition
  - Aggregate IRS in new job creation
- General Equilibrium Model
  - Physical *k*
  - Directed search labor supply and labor demand conditions
  - Aggregate resource frontier
- Analytical Results – Part II
- Quantitative Results
  - Calibration
  - Steady state and IRFs
- Conclusion

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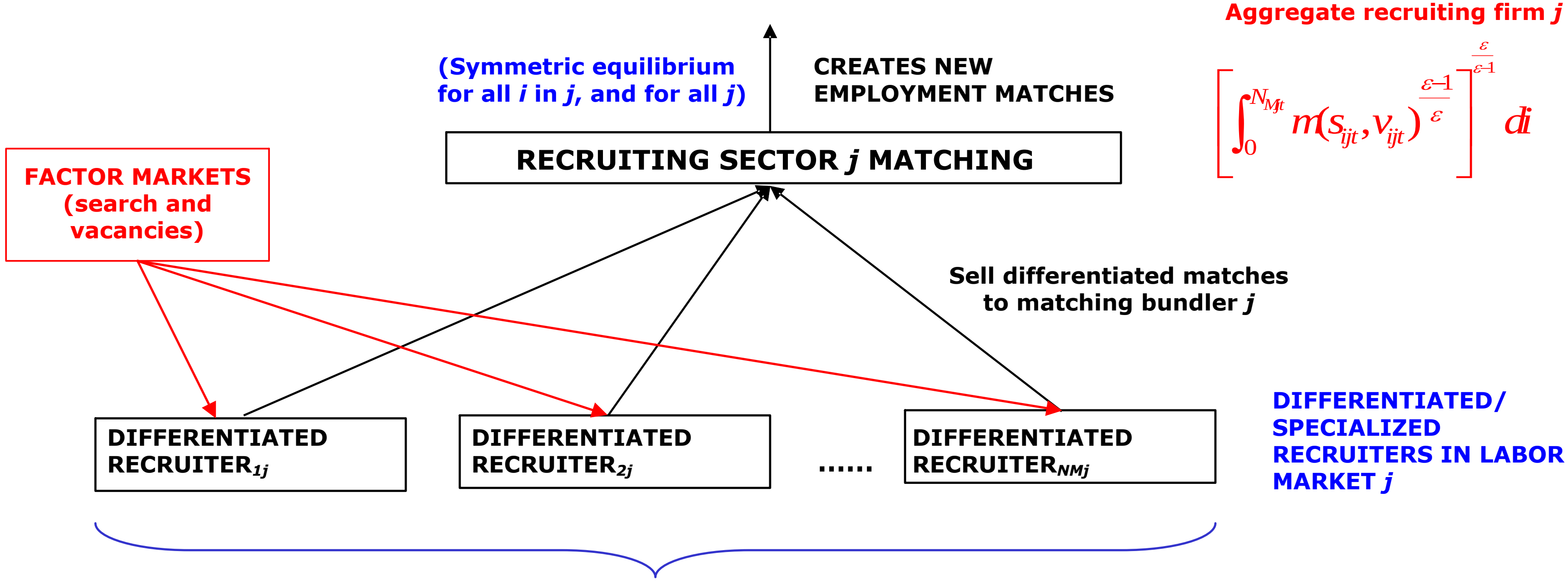
Monopolistic Recruiting Market

- Measure [0, 1] of recruiting markets
  - Perfectly-competitive – index by j
- Measure [0, N_{Mj}] of monopolistic submarkets in recruiting market j
  - Index by ij
  - N_{Mj} endogenously determined via free entry



$$\left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di$$

Measure N_M of monopolistic recruiters, each of which produces a differentiated match

Endogenous Entry in Recruiting Market

- Measure [0, 1] of recruiting markets
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- Free Entry in Recruiting Markets
- Representative Recruiter *j*

Cost of creating new differentiated *m*(.) and entering market

$$\max_{\{N_{Mjt}, N_{MEjt}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[\left(\int_0^{N_{Mjt}} (\rho_{ijt} - mc_{jt}) \cdot m(s_{ijt}, v_{ijt})\, di \right) - \Gamma_{Mt} N_{MEjt} \right]$$

$$N_{Mjt} = (1-\omega) N_{Mjt-1} + N_{MEjt}$$

- Cost of entry Γ_{Mt}
  - Technological
  - R&D
  - Regulatory

$$\Gamma_{Mt} = \Gamma_{Mt}^{TECH} + \Gamma_{Mt}^{R\&D} + \Gamma_{Mt}^{REG} + \ldots$$

Endogenous Entry in Recruiting Market

- **Measure [0, 1] of recruiting markets**
  - **Perfectly-competitive – index by *j***
- **Measure [0, N_{Mj}] of monopolistic submarkets in recruiting market *j***
  - **Index by *ij***
  - **N_{Mj} endogenously determined via free entry**

- **Free Entry in Recruiting Markets**
- **Representative Recruiter *j***

Cost of creating new differentiated *m*(.) and entering market

$$\max_{\{N_{Mjt}, N_{MEjt}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[\left(\int_0^{N_{Mjt}} (\rho_{ijt} - mc_{jt}) \cdot m(s_{ijt}, v_{ijt})\, di \right) - \Gamma_{Mt} N_{MEjt} \right]$$

$$N_{Mjt} = (1-\omega) N_{Mjt-1} + N_{MEjt}$$

- **Free-entry condition – determines new recruiting agencies N_{MEjt}**

$$\Gamma_{Mt} = (\rho_{ijt} - mc_{jt}) \cdot m(s_{ijt}, v_{ijt}) + (1-\omega) E_t \left\{ \Xi_{t+1|t} \Gamma_{Mt+1} \right\} \qquad \text{w/ } i = N_{Mjt}$$

MONOPOLISTIC RECRUITING – MARKET *j*

- Measure [0, 1] of recruiting markets
  - Perfectly-competitive – index by j
- Measure $[0, N_{Mj}]$ of monopolistic submarkets in recruiting market j
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- Matching Aggregator
  - Dixit-Stiglitz
  - ("Benassy")
  - Translog

Incentive for Entry vs. Welfare Benefit of Increasing Returns to Scale

Dixit-Stiglitz Technology Efficiently Balances Tradeoff

Translog and Benassy Technologies
***Inefficiently* Balance Tradeoff**

Monopolistic Recruiting – Market *j*

- Measure [0, 1] of recruiting markets
  - Perfectly-competitive – index by *j*
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- Matching Aggregator
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- Dixit-Stiglitz technology

$$m_{jt} = \left[\int_0^{N_{Mjt}} m_{ijt}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall j$$

labor-market *j* aggregator

dmd_fct

MONOPOLISTIC RECRUITING – MARKET *j*

- **Measure [0, 1] of recruiting markets**
  - **Perfectly-competitive – index by *j***
- **Measure [0, N_{Mj}] of monopolistic submarkets in recruiting market *j***
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  - **N_{Mj} endogenously determined via free entry**

- **Matching Aggregator**
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  - **("Benassy")**
  - **Translog**

- **Demand function for recruiter *ij* (Dixit-Stiglitz)**

$$\rho_{ijt} = m_{ijt}^{-\frac{1}{\varepsilon}} \cdot m_{jt}^{\frac{1}{\varepsilon}} \quad \forall i \in j$$

RECRUITER *ij* – PROFIT-MAXIMIZATION

$$\rho_{ijt} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \cdot mc_{jt}$$

Gross matching-market markup

PERFECT **CSE: ε = infinity (recovers Moen 1997)**

marginal cost of creating new job match

Generally (symmetric equilibrium)

$$\rho(N_{Mt}) = \mu(N_{Mt}) \cdot mc(N_{Mt})$$

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **Profit-maximizing (ρ^*_{ijt}, $m^*(s_{ijt}, v_{ijt})$) chosen**
- **Monopolistic recruiter *ij*'s recruiting problem**
- **Recruiting firm *ij* must attract firms to post vacancies in submarket *ij***
- **Recruiting firm *ij* must attract active job searchers to send résumés to (i.e., search in) submarket *ij***

Definitions

$J(w_{ijt})$ **value to goods-producing firm of successfully hiring worker in submarket *ij***

$W(w_{ijt})$ **value to worker of successfully finding a job in submarket *ij***

U **outside option of worker if unsuccessful in finding a job in submarket *ij***

Recruiter *ij* – Cost-Minimization (Dual)

- **Recruiter *ij* total profit function**

$$\mathbf{V}_{M_{ijt}}(s_{ijt}, v_{ijt}) \equiv \left(\rho_{ijt} - mc_{jt}\right) \cdot m(s_{ijt}, v_{ijt})$$

Marginal profit conditions

$$\boxed{\frac{\partial \mathbf{V}_{M_{ijt}}}{\partial s_{ijt}}} = \left(\rho_{ijt} - mc_{jt}\right) \cdot m_{s_{ijt}} \qquad \boxed{\frac{\partial \mathbf{V}_{M_{ijt}}}{\partial v_{ijt}}} = \left(\rho_{ijt} - mc_{jt}\right) \cdot m_{v_{ijt}}$$

$$\equiv \mathbf{V}_{Mijt}^{s} \qquad \equiv \mathbf{V}_{Mijt}^{v}$$

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **Recruiter *ij* marginal profit function**

$$\left(\rho_{ijt} - mc_{jt}\right) \cdot m_{v_{ijt}}$$

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt} \right) \cdot m_{v_{ijt}}$$

subject to

$$\gamma - p_{v_{jt}} - k^{f}(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^{F} = 0$$

$$p_{s_{jt}} + k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1 - k^{h}(\theta_{ijt}))\mathbf{U}_{t} - \mathbf{X}^{H} = 0$$

Recruiter *ij* – Cost-Minimization (Dual)

- **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right)\cdot(1-\xi)\cdot k^{f}(\theta_{ijt})$$

subject to

$$\gamma - p_{v_{jt}} - k^{f}(\theta_{ijt})\cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^{F} = 0$$

$$p_{s_{jt}} + k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^{h}(\theta_{ijt}))\mathbf{U}_{t} - \mathbf{X}^{H} = 0$$

Suppose

$m(s,v) = s^{\xi}v^{1-\xi}$

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right)\cdot(1-\xi)\cdot k^{f}(\theta_{ijt})$$

Suppose

$m(s,v) = s^{\xi} v^{1-\xi}$

subject to

multipliers

$$\gamma - p_{v_{jt}} - k^{f}(\theta_{ijt})\cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^{F} = 0$$ **1**

$$p_{s_{jt}} + k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^{h}(\theta_{ijt}))\mathbf{U}_{t} - \mathbf{X}^{H} = 0$$ $\boldsymbol{\kappa_{ijt}}$

(given CRS m(.), only one multiplier needed)

FOCs wrt w_{ijt} and θ_{ijt}

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1) $$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^H\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \;\longrightarrow\; \boxed{\kappa_{ijt}^H=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

b/c zero proportional taxation on wage

2) $$\left(\rho_{ijt}-mc_{jt}\right)\cdot(1-\xi)\cdot\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}-\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\kappa_{ijt}^H\cdot\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1) $$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^H\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \quad\rightarrow\quad \boxed{\kappa_{ijt}^H=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

b/c zero proportional taxation on wage

2) $$\underbrace{\left(\rho_{ijt}-mc_{jt}\right)}_{\neq 0}\cdot(1-\xi)\cdot\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}-\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\kappa_{ijt}^H\cdot\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

≠ 0

MONOPOLISTICALLY
competitive
recruiting sector

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^H\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \quad\longrightarrow\quad \kappa_{ijt}^H=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}$$

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$$\underbrace{\left(\rho_{ijt}-mc_{jt}\right)}_{\neq 0}\cdot(1-\xi)\cdot\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}-\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\kappa_{ijt}^H\cdot\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

≠ 0

MONOPOLISTICALLY **competitive recruiting sector**

Crucial for *EFFICIENT* matching in decentralized economy

elast

Recruiter *ij* – Cost-Minimization (Dual)

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1) $$-k^{f}(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^{H}\cdot k^{h}(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \;\longrightarrow\; \boxed{\kappa_{ijt}^{H}=-\frac{k^{f}(\theta_{ijt})}{k^{h}(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

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Cobb-Douglas matching

$$m(s,v)=s^{\xi}v^{1-\xi}$$

Combine and rearrange

$$k^{h}(\theta)=\frac{m(s,v)}{s}=m(1,\theta)=\theta^{1-\xi}$$

$$k^{f}(\theta)=\frac{m(s,v)}{v}=m(\theta^{-1},1)=\theta^{-\xi}$$

AND

$$\frac{\partial k^{h}(\theta)}{\partial\theta}=(1-\xi)\theta^{-\xi}$$

$$\frac{\partial k^{f}(\theta)}{\partial\theta}=-\xi\theta^{-\xi-1}$$

Surplus Sharing

❒ **Proposition 1. Monopolistically Competitive Surplus Sharing Condition**

ξ is elasticity of $m_{ij}(.)$ wrt s_{ij}

Wage Model

$$\underbrace{\xi \cdot (1-\xi) \cdot \left(\rho_{ijt} - mc_{jt}\right)}_{\text{payoff accruing to monopolistic recruiter } ij} + \underbrace{(1-\xi) \cdot \left(\mathbf{W}(w_{ijt}) - \mathbf{U}\right)}_{\text{surplus accruing to new employee}} = \underbrace{\xi \cdot \mathbf{J}(w_{ijt})}_{\text{surplus accruing to new employer}}$$

SURPLUS SHARING

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↓ **substitute** $mc = \rho/\mu$

↓ **symmetric equilibrium**

↓ **functional dependence of** $\rho(.)$ **and** $\mu(.)$ **on** N_M

(see Bilbiie, Ghironi, Melitz 2008 NBER WP, 2016 NBER WP)

Surplus Sharing

- **Proposition 1. Monopolistically Competitive Surplus Sharing Condition**

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Wage Model

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payoff accruing to monopolistic recruiter

surplus accruing to new employee

surplus accruing to new employer

- **Observations**
  - **As** $\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\rightarrow 0$ **surplus sharing condition → *PERFECTLY* competitive**

SURPLUS SHARING

- **Proposition 1. Perfectly Competitive Surplus Sharing Condition**

$$(1-\xi)\cdot\left(\mathbf{W}(w_t)-\mathbf{U}\right)=\xi\cdot\mathbf{J}(w_t)$$

surplus accruing to new employee and new employer are
***EFFICIENT* contributions to *m*(.) in decentralized CSE**

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  - **As** $\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\to 0$ **surplus sharing condition → *PERFECTLY* competitive**

Surplus Sharing

- **Proposition 1. Perfectly Competitive Surplus Sharing Condition**

$$(1-\xi)\cdot\left(\mathbf{W}(w_t)-\mathbf{U}\right)=\xi\cdot\mathbf{J}(w_t)$$

Suppose

$m(s,v)=s^{\xi}v^{1-\xi}$

surplus accruing to new employee and new employer are *EFFICIENT* contributions to *m*(.) in decentralized CSE

- **Observations**
  - **As** $\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\to 0$ **surplus sharing condition → *PERFECTLY* competitive**
  - **Depends on matching process *m*(*s*, *v*)**

Marginals vs. Elasticities

- **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right) \cdot m_{s_{ijt}}$$

Marginal profit conditions

$$\boxed{\frac{\partial \mathbf{V}_{M_{ijt}}}{\partial s_{ijt}}} = \left(\rho_{ijt} - mc_{jt}\right) \cdot m_{s_{ijt}}$$

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MARGINALS VS. ELASTICITIES

- **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right) \cdot \xi \cdot k^{h}(\theta_{ijt})$$

Suppose

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$$\frac{\partial \mathbf{V}_{M_{ijt}}}{\partial s_{ijt}} = \left(\rho_{ijt} - mc_{jt}\right) \cdot m_{s_{ijt}}$$

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$$m(s,v) = s^{\xi} v^{1-\xi}$$

$$m_s = \xi \cdot \theta^{1-\xi}$$

$$m_v = (1-\xi) \cdot \theta^{-\xi}$$

$$\frac{\partial k^f(\theta)}{\partial \theta} = -\xi \theta^{-\xi-1}$$

$$\frac{\partial k^h(\theta)}{\partial \theta} = (1-\xi)\theta^{-\xi}$$

$$k^h(\theta) = \frac{m(s,v)}{s} = m(1,\theta) = \theta^{1-\xi}$$

$$k^f(\theta) = \frac{m(s,v)}{v} = m(\theta^{-1},1) = \theta^{-\xi}$$

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$$m(s,v) = s^{\xi} v^{1-\xi}$$

$$m_s = \xi \cdot \theta^{1-\xi}$$

$$m_v = (1-\xi) \cdot \theta^{-\xi}$$

Ratio characterizes *EFFICIENT* matching (more to say soon..)

$$\frac{\partial k^f(\theta)}{\partial \theta} = -\xi \theta^{-\xi-1}$$

$$\frac{\partial k^h(\theta)}{\partial \theta} = (1-\xi)\theta^{-\xi}$$

Ratio crucial for *EFFICIENT* matching in decentralized economy

$$k^h(\theta) = \frac{m(s,v)}{s} = m(1,\theta) = \theta^{1-\xi}$$

$$k^f(\theta) = \frac{m(s,v)}{v} = m(\theta^{-1},1) = \theta^{-\xi}$$

Marginals vs. Elasticities

- **Marginals and elasticities**

$$\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}} = \frac{\dfrac{s \cdot m_s}{m(s,v)}}{\dfrac{v \cdot m_v}{m(s,v)}}$$

MARGINALS VS. ELASTICITIES

❒ **Marginals and elasticities**

$$-\underbrace{\left(\frac{k^h(\theta)}{k^f(\theta)}\right)}_{=\,\theta}\cdot\left(\frac{\partial k^f(\theta)/\partial\theta}{\partial k^h(\theta)/\partial\theta}\right)=\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}}=\frac{\dfrac{s\cdot m_s}{m(s,v)}}{\dfrac{v\cdot m_v}{m(s,v)}}$$

= θ

FOCs

Marginals vs. Elasticities

- **Marginals and elasticities**

$$-\underbrace{\left(\frac{k^h(\theta)}{k^f(\theta)}\right)}_{=\,\theta}\cdot\left(\frac{\partial k^f(\theta)/\partial\theta}{\partial k^h(\theta)/\partial\theta}\right)=\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}}=\frac{\dfrac{s\cdot m_s}{m(s,v)}}{\dfrac{v\cdot m_v}{m(s,v)}}$$

Suppose

$$m(s,v)=s^{\xi}v^{1-\xi}$$

$$\frac{\partial k^f(\theta)}{\partial\theta}=-\xi\theta^{-\xi-1}$$

$$\frac{\partial k^h(\theta)}{\partial\theta}=(1-\xi)\theta^{-\xi}$$

$$m_s=\xi\cdot\theta^{1-\xi}$$

$$m_v=(1-\xi)\cdot\theta^{-\xi}$$

$$\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}}=\frac{\xi}{1-\xi}$$

Marginals vs. Elasticities

- **Explicit-form wage expression**

$$\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}} = \frac{\xi}{1-\xi}$$

Suppose

$$m(s,v) = s^{\xi} v^{1-\xi}$$

$$w_t = \left(\frac{\frac{\xi}{1-\xi}}{1+\frac{\xi}{1-\xi}} \right) \cdot z_t + \cdot \left(\frac{1}{1+\frac{\xi}{1-\xi}} \right) \chi + \left(\frac{\frac{\xi}{1-\xi}}{1+\frac{\xi}{1-\xi}} \right) \cdot \left(\frac{1-\rho}{1+r} \right) \cdot \gamma \cdot E_t \theta_{t+1}$$

MARGINALS VS. ELASTICITIES

- **Explicit-form wage expression**

$$\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}} = \frac{\xi}{1-\xi}$$

Suppose

$$m(s,v) = s^{\xi} v^{1-\xi}$$

↓

$$w_t = \xi \cdot z_t + \cdot (1-\xi)\chi + (1-\xi) \cdot \left(\frac{1-\rho}{1+r}\right) \cdot \gamma \cdot E_t \theta_{t+1}$$

MARGINALS VS. ELASTICITIES

❒ **Explicit-form wage expression**

$$\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}} = \frac{\xi}{1-\xi}$$

Suppose

$$m(s,v) = s^{\xi} v^{1-\xi}$$

$$w_t = \xi \cdot z_t + \cdot (1-\xi)\chi + (1-\xi) \cdot \left(\frac{1-\rho}{1+r}\right) \cdot \gamma \cdot E_t \theta_{t+1}$$

$$\frac{m_s}{m_v} = \left(\frac{\xi}{1-\xi}\right)\theta$$

MARGINALS VS. ELASTICITIES

- **Marginals and elasticities**

$$-\underbrace{\left(\frac{k^h(\theta)}{k^f(\theta)}\right)}_{=\theta}\cdot\left(\frac{\partial k^f(\theta)/\partial\theta}{\partial k^h(\theta)/\partial\theta}\right)=\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}}=\frac{\frac{s\cdot m_s}{m(s,v)}}{\frac{v\cdot m_v}{m(s,v)}}$$

Suppose dHRW m(.)

$$m(s,v)=\frac{s\cdot v}{\left(s^{\epsilon}+v^{\epsilon}\right)^{1/\epsilon}}$$

MARGINALS VS. ELASTICITIES

❒ **Marginals and elasticities**

$$-\left(\frac{k^h(\theta)}{k^f(\theta)}\right)\cdot\left(\frac{\partial k^f(\theta)/\partial\theta}{\partial k^h(\theta)/\partial\theta}\right)=\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}}=\frac{\frac{s\cdot m_s}{m(s,v)}}{\frac{v\cdot m_v}{m(s,v)}}$$

$\underbrace{\left(\frac{k^h(\theta)}{k^f(\theta)}\right)}$ **= θ**

Suppose dHRW m(.)

$$m(s,v)=\frac{s\cdot v}{\left(s^\epsilon+v^\epsilon\right)^{1/\epsilon}}$$

$$k^h(\theta)=\frac{m(s,v)}{s}=m(1,\theta)=\frac{\theta}{\left[1+\theta^\epsilon\right]^{1/\epsilon}}$$

$$k^f(\theta)=\frac{m(s,v)}{v}=m(\theta^{-1},1)=\frac{1}{\left[1+\theta^\epsilon\right]^{1/\epsilon}}$$

$$\frac{\partial k^h(\theta)}{\partial\theta}=\frac{1}{\left(1+\theta^\epsilon\right)^{1/\epsilon}}\cdot\left[1-\frac{\theta^\epsilon}{1+\theta^\epsilon}\right]$$

$$\frac{\partial k^f(\theta)}{\partial\theta}=-\frac{\theta^\epsilon}{\left(1+\theta^\epsilon\right)^{\frac{1+\epsilon}{\epsilon}}\cdot\theta}$$

$$\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}}=\theta^\epsilon$$

MARGINALS VS. ELASTICITIES

❒ **Explicit-form wage expression**

$$\frac{\varepsilon_{m,s}}{\varepsilon_{m,v}} = \theta^{\epsilon}$$

Suppose dHRW m(.)

$$m(s,v) = \frac{s \cdot v}{\left(s^{\epsilon} + v^{\epsilon}\right)^{1/\epsilon}}$$

↓

$$w_t = \left(\frac{\theta^{\epsilon}}{1+\theta^{\epsilon}}\right) \cdot z_t + \cdot \left(\frac{1}{1+\theta^{\epsilon}}\right) \chi + \left(\frac{\theta^{\epsilon}}{1+\theta^{\epsilon}}\right) \cdot \left(\frac{1-\rho}{1+r}\right) \cdot \gamma \cdot E_t \theta_{t+1}$$

Recruiter *ij* – Cost-Minimization (Dual)

- **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right) \cdot m_{s_{ijt}}$$

subject to

$$\gamma - p_{v_{jt}} - k^{f}(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^{F} = 0$$

$$p_{s_{jt}} + k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1 - k^{h}(\theta_{ijt}))\mathbf{U}_{t} - \mathbf{X}^{H} = 0$$

Suppose dHRW m(.)

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **Recruiter *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right)\cdot m_{s_{ijt}}$$

subject to

Suppose dHRW m(.)

multipliers

$$\gamma - p_{v_{jt}} - k^f(\theta_{ijt})\cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^F = 0 \qquad \mathbf{1}$$

$$m_{ijt} = \frac{s_{ijt}\cdot\theta_{ijt}}{\left(1+\theta_{ijt}^{\epsilon}\right)^{1/\epsilon}}$$

$$p_{s_{jt}} + k^h(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^h(\theta_{ijt}))\mathbf{U}_t - \mathbf{X}^H = 0 \qquad \kappa_{ijt}$$

$$m_{s_{ijt}} = \theta_{ijt}\cdot\left(1+\theta_{ijt}^{\epsilon}\right)^{-\frac{1}{\epsilon}}$$

$$\frac{\partial m_{s_{ijt}}}{\partial \theta_{ijt}} = \left(1+\theta_{ijt}^{\epsilon}\right)^{-\frac{1}{\epsilon}} - \left(\frac{1}{\epsilon}\right)\theta_{ijt}\cdot\left(1+\theta_{ijt}^{\epsilon}\right)^{-\frac{1}{\epsilon}-1}\cdot\epsilon\cdot\theta_{ijt}^{\epsilon-1}$$

$$= \frac{1}{\left(1+\theta_{ijt}^{\epsilon}\right)^{1/\epsilon}}\cdot\left[1-\frac{\theta_{ijt}^{\epsilon}}{1+\theta_{ijt}^{\epsilon}}\right] = \frac{1}{\left(1+\theta_{ijt}^{\epsilon}\right)^{1+1/\epsilon}}$$

(given CRS m(.), only one multiplier needed)

FOCs wrt w_{ijt} and θ_{ijt}

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **FOCs with respect to w_{ijt} and θ_{ijt}**

$$\frac{\partial \mathbf{V}_{M_{ijt}}}{\partial v_{ijt}} = \left(\rho_{ijt} - mc_{jt}\right)\cdot m_{v_{ijt}} \equiv \mathbf{V}^{v}_{Mijt}$$

$$\frac{\partial \mathbf{V}_{M_{ijt}}}{\partial s_{ijt}} = \left(\rho_{ijt} - mc_{jt}\right)\cdot m_{s_{ijt}} \equiv \mathbf{V}^{s}_{Mijt}$$

1)

$$-k^{f}(\theta_{ijt})\cdot \underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1} + \kappa^{H}_{ijt}\cdot k^{h}(\theta_{ijt})\cdot \underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1} = 0$$

b/c zero proportional taxation on wage

2)

$$\underbrace{\left(\rho_{ijt} - mc_{jt}\right)\cdot \frac{\partial m_{s_{ijt}}}{\partial \theta_{ijt}}}_{} - \frac{\partial k^{f}(\theta_{ijt})}{\partial \theta_{ijt}}\cdot \mathbf{J}(w_{ijt}) + \kappa^{H}_{ijt}\cdot \frac{\partial k^{h}(\theta_{ijt})}{\partial \theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt}) - \mathbf{U}_{t}\right) = 0$$

$$= \frac{\partial \mathbf{V}^{s}_{Mijt}}{\partial \theta_{ijt}} = \left(\rho_{ijt} - mc_{jt}\right)\cdot \frac{\partial m_{s_{ijt}}}{\partial \theta_{ijt}}$$

Recruiter *ij* – Cost-Minimization (Dual)

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^H\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \quad\longrightarrow\quad \kappa_{ijt}^H=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}$$

b/c zero proportional taxation on wage

2)

$$\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\frac{\partial m_{s_{ijt}}}{\partial \theta_{ijt}}}_{=\frac{\partial \mathbf{V}_{Mijt}^s}{\partial \theta_{ijt}}=\left(\rho_{ijt}-mc_{jt}\right)\cdot\frac{\partial m_{s_{ijt}}}{\partial \theta_{ijt}}}-\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}}\cdot \mathbf{J}(w_{ijt})+\kappa_{ijt}^H\cdot\frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

↓ **Divide by $dk^f/d\theta$**

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^H\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \quad\longrightarrow\quad \boxed{\kappa_{ijt}^H=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

b/c zero proportional taxation on wage

2)

$$\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\left(\frac{\partial m_{s_{ijt}}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}\right)}-\mathbf{J}(w_{ijt})+\kappa_{ijt}^H\cdot\left(\frac{\partial k^h(\theta_{ijt})/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}\right)\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

$$=\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\frac{\partial m_{s_{ijt}}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}}$$

$$=\frac{\partial \mathbf{V}_{Mijt}^s/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}$$

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^H\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \quad\longrightarrow\quad \boxed{\kappa_{ijt}^H=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

b/c zero proportional taxation on wage

2)

$$\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\left(\frac{\partial m_{s_{ijt}}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}\right)}-\mathbf{J}(w_{ijt})-\underbrace{\theta_{ijt}^{-1}\cdot\left(\frac{\partial k^h(\theta_{ijt})/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}\right)}_{=\theta^{-\epsilon}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

$$=\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\frac{\partial m_{s_{ijt}}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}}$$

$$=\frac{\partial \mathbf{V}_{Mijt}^s/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}$$

Recruiter *ij* – Cost-Minimization (Dual)

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^H\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \;\longrightarrow\; \kappa_{ijt}^H=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}$$

b/c zero proportional taxation on wage

2)

$$\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\left(\frac{\partial m_{s_{ijt}}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}\right)}-\mathbf{J}(w_{ijt})+\theta_{ijt}^{-\epsilon}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

$$=\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\frac{\partial m_{s_{ijt}}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}}$$

$$=\frac{\partial \mathbf{V}_{Mijt}^s/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}$$

Recruiter *ij* – Cost-Minimization (Dual)

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^{H}\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \quad\longrightarrow\quad \boxed{\kappa_{ijt}^{H}=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

b/c zero proportional taxation on wage

2)

$$\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\left(\frac{\partial m_{s_{ijt}}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}\right)}-\mathbf{J}(w_{ijt})+\theta_{ijt}^{-\epsilon}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

dHRW m(.)

$$=\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\frac{\partial m_{s_{ijt}}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}}=\left(\rho_{ijt}-mc_{jt}\right)\cdot\frac{\partial k^h(\theta_{ijt})/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}$$

$$=\frac{\partial \mathbf{V}_{Mijt}^{s}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}$$

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa^H_{ijt}\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \;\longrightarrow\; \boxed{\kappa^H_{ijt}=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta^{-1}_{ijt}}$$

b/c zero proportional taxation on wage

2)

$$\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\left(\frac{\partial m_{s_{ijt}}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}\right)}-\mathbf{J}(w_{ijt})+\theta^{-\epsilon}_{ijt}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

dHRW m(.)

$$=\underbrace{\left(\rho_{ijt}-mc_{jt}\right)\cdot\frac{\partial m_{s_{ijt}}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}}=\left(\rho_{ijt}-mc_{jt}\right)\cdot\underbrace{\frac{\partial k^h(\theta_{ijt})/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}}_{=-\theta^{1-\epsilon}}$$

$$=\frac{\partial \mathbf{V}^s_{Mijt}/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}$$

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1) $$-k^{f}(\theta_{ijt})\cdot\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}+\kappa_{ijt}^{H}\cdot k^{h}(\theta_{ijt})\cdot\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}=0 \quad\rightarrow\quad \kappa_{ijt}^{H}=-\frac{k^{f}(\theta_{ijt})}{k^{h}(\theta_{ijt})}=-\theta_{ijt}^{-1}$$

$\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}$ =-1; $\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}$ =1

b/c zero proportional taxation on wage

2) $$-\left(\rho_{ijt}-mc_{jt}\right)\cdot\theta_{ijt}^{1-\epsilon}-\mathbf{J}(w_{ijt})+\theta_{ijt}^{-\epsilon}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_{t}\right)=0$$

Recruiter ij – Cost-Minimization (Dual)

❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^{f}(\theta_{ijt})\cdot\underbrace{\frac{\partial\mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^{H}\cdot k^{h}(\theta_{ijt})\cdot\underbrace{\frac{\partial\mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \quad\rightarrow\quad \boxed{\kappa_{ijt}^{H}=-\frac{k^{f}(\theta_{ijt})}{k^{h}(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

b/c zero proportional taxation on wage

2)

$$-\left(\rho_{ijt}-mc_{jt}\right)\cdot\theta_{ijt}^{1-\epsilon}-\mathbf{J}(w_{ijt})+\theta_{ijt}^{-\epsilon}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_{t}\right)=0$$

$$\downarrow$$

$$\theta_{ijt}^{-\epsilon}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_{t}\right)=\mathbf{J}(w_{ijt})+\theta_{ijt}^{1-\epsilon}\cdot\left(\rho_{ijt}-mc_{jt}\right)$$

RECRUITER *ij* – COST-MINIMIZATION (DUAL)

- **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^H\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \quad\longrightarrow\quad \kappa_{ijt}^H=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}$$

b/c zero proportional taxation on wage

2)

$$-\left(\rho_{ijt}-mc_{jt}\right)\cdot\theta_{ijt}^{1-\epsilon}-\mathbf{J}(w_{ijt})+\theta_{ijt}^{-\epsilon}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

$$\theta_{ijt}^{-\epsilon}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=\mathbf{J}(w_{ijt})+\theta_{ijt}^{1-\epsilon}\cdot\left(\rho_{ijt}-mc_{jt}\right)$$

$$\mathbf{W}(w_{ijt})-\mathbf{U}_t=\theta_{ijt}^{\epsilon}\cdot\mathbf{J}(w_{ijt})+\theta_{ijt}\cdot\left(\rho_{ijt}-mc_{jt}\right)$$

Recruiter *ij* – Cost-Minimization (Dual)

- Consider symmetric equilibrium
- Compare with C-D m(.) monopolistically-competitive surplus sharing

direct relationship between time-varying θ_t and time-varying N_{Mt}

$$\mathbf{W}(w_t) - \mathbf{U}_t = \theta_t^{\epsilon} \cdot \mathbf{J}(w_t) + \theta_t \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right)$$

$$\xi \cdot (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) + (1-\xi) \cdot \left(\mathbf{W}(w_t) - \mathbf{U} \right) = \xi \cdot \mathbf{J}(w_t)$$

Outline

- Structure of Recruiting Markets
  - Free entry in recruiting markets
  - Profit maximization
  - Cost minimization (directed-search optimization) – CRUCIAL
- Analytical Results – Part I
  - Surplus sharing condition
  - Aggregate IRS in new job creation
- General Equilibrium Model
  - Physical *k*
  - Directed search labor supply and labor demand conditions
  - Aggregate resource frontier
- Analytical Results – Part II
- Quantitative Results
  - Calibration
  - Steady state and IRFs
- Conclusion

Surplus Sharing

❒ **Proposition 1. Monopolistic Surplus Sharing Condition**

ξ is elasticity of $m_{ij}(.)$ wrt s_{ij}

Wage Model

$$\underbrace{\xi \cdot (1-\xi) \cdot \left(\rho_{ijt} - mc_{jt}\right)}_{\text{payoff accruing to monopolistic recruiter } ij} + \underbrace{(1-\xi) \cdot \left(\mathbf{W}(w_{ijt}) - \mathbf{U}\right)}_{\text{surplus accruing to new employee}} = \underbrace{\xi \cdot \mathbf{J}(w_{ijt})}_{\text{surplus accruing to new employer}}$$

Surplus Sharing

- **Proposition 1. Monopolistic Surplus Sharing Condition**

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payoff accruing to monopolistic recruiter *ij*

surplus accruing to new employee

surplus accruing to new employer

↓ **substitute $mc = \rho/\mu$**

↓ **symmetric equilibrium**

↓ **functional dependence of $\rho(.)$ and $\mu(.)$ on N_M**

(see Bilbiie, Ghironi, Melitz 2008 NBER WP, 2016 NBER WP)

SURPLUS SHARING

- **Proposition 1. Monopolistic Surplus Sharing Condition**

ξ is elasticity of $m_{ij}(.)$ wrt s_{ij}

Wage Model

$$\underbrace{\xi \cdot (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right)}_{\text{payoff accruing to monopolistic recruiter}} + \underbrace{(1-\xi) \cdot \left(\mathbf{W}(w_t) - \mathbf{U} \right)}_{\text{surplus accruing to new employee}} = \underbrace{\xi \cdot \mathbf{J}(w_t)}_{\text{surplus accruing to new employer}}$$

- **Observations**
  - **As** $\left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \to 0$ **surplus sharing condition → *PERFECTLY* competitive**

Surplus Sharing

- **Proposition 1. Monopolistic Surplus Sharing Condition**

ξ **is elasticity of $m_{ij}(.)$ wrt s_{ij}**

Wage Model

$$\underbrace{\xi\cdot(1-\xi)}_{\text{extra resources?...}}\cdot\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)+\underbrace{(1-\xi)\cdot\left(\mathbf{W}(w_t)-\mathbf{U}\right)}_{\text{surplus accruing to new employee}}=\underbrace{\xi\cdot\mathbf{J}(w_t)}_{\text{surplus accruing to new employer}}$$

extra resources?...

surplus accruing to new employee

surplus accruing to new employer

- **Observations**
  - **As** $\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\rightarrow 0$ **surplus sharing condition → *PERFECTLY* competitive**
  - **Matching elasticity ξ in (0,1)....**
  - **From where do "extra" resources arise?**

AGGREGATE INCREASING RETURNS

- AGGREGATE increasing returns in matching

$$\left[\int_0^{N_{Mjt}} m_{ijt}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} di \quad \forall j$$

Dixit-Stiglitz Aggregation

integrate over *i*

(integrate over *j*)

$$N_{Mt}^{\frac{\varepsilon}{\varepsilon-1}} \cdot m(s_t, v_t)$$

$$\epsilon(N_t) = \frac{\varepsilon}{\varepsilon - 1} - 1$$

IRTS effect (elasticity) – Dixit-Stiglitz

Requires BOTH monopolistic competition AND costs of entry

ala Romer endogenous growth model

AGGREGATE INCREASING RETURNS

- **AGGREGATE increasing returns in matching**

$$\left[\int_0^{N_{Mjt}} m_{ijt}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di \quad \forall j$$

Dixit-Stiglitz Aggregation

↓ **integrate over *i***

(integrate over *j*)

$$N_{Mt}^{\frac{1}{\varepsilon-1}} \cdot N_{Mt} \cdot m(s_t, v_t)$$

$$\epsilon(N_t) = \frac{\varepsilon}{\varepsilon - 1} - 1$$

IRTS effect (elasticity) – Dixit-Stiglitz

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AGGREGATE INCREASING RETURNS

- **AGGREGATE increasing returns in matching**

$$\left[\int_0^{N_{Mjt}} m_{ijt}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di \quad \forall j$$

Dixit-Stiglitz Aggregation

integrate over *i*

(integrate over *j*)

$$N_{Mt}^{\frac{1}{\varepsilon-1}} \cdot N_{Mt} \cdot m(s_t, v_t)$$

$$\epsilon(N_t) = \frac{\varepsilon}{\varepsilon-1} - 1$$

IRTS effect (elasticity) – Dixit-Stiglitz

Requires BOTH monopolistic competition AND costs of entry

ala Romer endogenous growth model

more generally

$$\rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t)$$

Holds for any aggregator

Surplus Sharing – Aggregator-Dependence

- **Dixit-Stiglitz**

$$\xi\cdot(1-\xi)\cdot\underbrace{\left(\frac{1}{\varepsilon}\right)}_{\text{markup effect}}\cdot\underbrace{N_{Mt}^{\frac{1}{\varepsilon-1}}}_{\text{IRTS effect}}+(1-\xi)\cdot\left(\mathbf{W}(w_t)-\mathbf{U}\right)=\xi\cdot\mathbf{J}(w_t)$$

AGGREGATE INCREASING RETURNS

- **AGGREGATE increasing returns in matching**

$$N_{Mt}^{\varphi+1-\frac{\varepsilon}{\varepsilon-1}}\left[\int_0^{N_{Mjt}} m_{ijt}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di \quad \forall j$$

"Benassy" Aggregation

↓ **integrate over *i***

(integrate over *j*)

Requires BOTH monopolistic competition AND costs of entry

ala Romer endogenous growth model

$$\rho(N_{Mt})\cdot N_{Mt}\cdot m(s_t, v_t)$$

Holds for any aggregator

AGGREGATE INCREASING RETURNS

- **AGGREGATE increasing returns in matching**

$$N_{Mt}^{\varphi+1-\frac{\varepsilon}{\varepsilon-1}} \left[\int_0^{N_{Mjt}} m_{ijt}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} di \quad \forall j$$

"Benassy" Aggregation

integrate over *i*

(integrate over *j*)

$$N_{Mt}^{\varphi} \cdot N_{Mt} \cdot m(s_t, v_t)$$

Requires BOTH monopolistic competition AND costs of entry

ala Romer endogenous growth model

$$\rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t)$$

Holds for any aggregator

AGGREGATE INCREASING RETURNS

- AGGREGATE increasing returns in matching

$$N_{Mt}^{\varphi+1-\frac{\varepsilon}{\varepsilon-1}}\left[\int_0^{N_{Mjt}} m_{ijt}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di \quad \forall j$$

"Benassy" Aggregation

integrate over *i*

(integrate over *j*)

$$N_{Mt}^{\varphi}\cdot N_{Mt}\cdot m(s_t, v_t)$$

$$\epsilon(N_t)=\varphi$$

Requires BOTH monopolistic competition AND costs of entry

ala Romer endogenous growth model

IRTS effect *INDEPENDENT* of markup effect

$$\rho(N_{Mt})\cdot N_{Mt}\cdot m(s_t, v_t)$$

Holds for any aggregator

Surplus Sharing – Aggregator-Dependence

- Dixit-Stiglitz

$$\xi \cdot (1-\xi) \cdot \underbrace{\left(\frac{1}{\varepsilon}\right)}_{\text{markup effect}} \cdot \underbrace{N_{Mt}^{\frac{1}{\varepsilon-1}}}_{\text{IRTS effect}} + (1-\xi) \cdot \left(\mathbf{W}(w_t) - \mathbf{U}\right) = \xi \cdot \mathbf{J}(w_t)$$

- "Benassy"

φ measures increasing returns to scale (independent of ε)

$$\xi \cdot (1-\xi) \cdot \underbrace{\left(\frac{1}{\varepsilon}\right)}_{\text{markup effect}} \cdot \underbrace{N_{Mt}^{\varphi}}_{\text{IRTS effect}} + (1-\xi) \cdot \left(\mathbf{W}(w_t) - \mathbf{U}\right) = \xi \cdot \mathbf{J}(w_t)$$

- **Incentive for Entry**
- **Welfare Benefit of Aggregate IRTS**

Surplus Sharing – Aggregator-Dependence

- Dixit-Stiglitz

$$\xi \cdot (1-\xi) \cdot \underbrace{\left(\frac{1}{\varepsilon}\right)}_{\text{markup effect}} \cdot \underbrace{N_{Mt}^{\frac{1}{\varepsilon-1}}}_{\text{IRTS effect}} + (1-\xi) \cdot \left(\mathbf{W}(w_t) - \mathbf{U}\right) = \xi \cdot \mathbf{J}(w_t)$$

- "Benassy"

lim $\varphi \rightarrow 0$

$$\xi \cdot (1-\xi) \cdot \underbrace{\left(\frac{1}{\varepsilon}\right)}_{\text{markup effect}} \cdot 1 + (1-\xi) \cdot \left(\mathbf{W}(w_t) - \mathbf{U}\right) = \xi \cdot \mathbf{J}(w_t)$$

- **Incentive for Entry**
- **Welfare Benefit of Aggregate IRTS**
  - **Declines under Benassy aggregation as $\varphi \rightarrow 0$**

SURPLUS SHARING – AGGREGATOR-DEPENDENCE

- **Dixit-Stiglitz**

$$\xi \cdot (1-\xi) \cdot \underbrace{\left(\frac{1}{\varepsilon}\right)}_{\text{markup effect}} \cdot \underbrace{N_{Mt}^{\frac{1}{\varepsilon-1}}}_{\text{IRTS effect}} + (1-\xi) \cdot \left(\mathbf{W}(w_t) - \mathbf{U}\right) = \xi \cdot \mathbf{J}(w_t)$$

- **"Benassy"**

φ measures increasing returns to scale (independent of ε)

$$\xi \cdot (1-\xi) \cdot \underbrace{\left(\frac{1}{\varepsilon}\right)}_{\text{markup effect}} \cdot \underbrace{N_{Mt}^{\varphi}}_{\text{IRTS effect}} + (1-\xi) \cdot \left(\mathbf{W}(w_t) - \mathbf{U}\right) = \xi \cdot \mathbf{J}(w_t)$$

- **Translog**

$$\xi \cdot (1-\xi) \cdot \underbrace{\left(\frac{(\sigma N_{Mt})^{-1}}{1+(\sigma N_{Mt})^{-1}}\right)}_{\text{markup effect}} \cdot \underbrace{\exp\left(-\frac{1}{2} \cdot \frac{\tilde{N}_M - N_{Mt}}{\sigma \cdot \tilde{N}_M \cdot N_{Mt}}\right)}_{\text{IRTS effect}} + (1-\xi) \cdot \left(\mathbf{W}(w_t) - \mathbf{U}\right) = \xi \cdot \mathbf{J}(w_t)$$

WAGE IN MONOPOLISTIC MARKETS

❒ **Proposition 1. Monopolistic Surplus Sharing Condition**

ξ is elasticity of $m_{ij}(.)$ wrt s_{ij}

$$\underbrace{\xi \cdot (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right)}_{\text{payoff accruing to monopolistic recruiter}} + \underbrace{(1-\xi) \cdot \left(\mathbf{W}(w_t) - \mathbf{U} \right)}_{\text{surplus accruing to new employee}} = \underbrace{\xi \cdot \mathbf{J}(w_t)}_{\text{surplus accruing to new employer}}$$

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payoff accruing to monopolistic recruiter | **surplus accruing to new employee** | **surplus accruing to new employer**

substitute W(.), U, and J(.)

Monopolistic Wage (explicit-form)

$$w_t = \xi \cdot z_t f_n(k_t, n_t) + (1-\xi)\chi + \xi(1-\rho)E_t \left\{ \Xi_{t+1|t} \gamma \cdot \theta_{t+1} \right\}$$
$$-\xi(1-\xi)\left[\left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) - (1-\rho)E_t \left\{ \Xi_{t+1|t} \left(\rho(N_{Mt+1}) - \frac{\rho(N_{Mt+1})}{\mu(N_{Mt+1})} \right) \right\} \right]$$

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substitute W(.), U, and J(.)

Monopolistic Wage (explicit-form)

$$w = \xi \cdot mpn + (1-\xi)\chi + \xi(1-\rho)\beta \cdot \gamma \cdot \theta - \xi(1-\xi)(1-\rho)\beta\left(\rho(N_M) - \frac{\rho(N_M)}{\mu(N_M)} \right)$$

steady state

OUTLINE

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  - Aggregate IRS in new job creation
- **General Equilibrium Model**
  - **Physical *k***
  - **Directed search labor supply and labor demand conditions**
  - **Aggregate resource frontier**
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GENERAL EQUILIBRIUM

- Introduce random-search matching and Nash-bargained wages

General Equilibrium

- Introduce random-search matching and Nash-bargained wages
- Submarket *ij* Labor Supply (directed search)

$$\frac{h'(lfp_t)}{u'(c_t)} = p_{s_{jt}} + k_{ijt}^h \cdot \underbrace{\left[w_{ijt} + (1-\rho)E_t\left\{ \Xi_{t+1|t}\left[\left(\frac{1-k_{jt+1}^h}{k_{jt+1}^h}\right)\left(\frac{h'(lfp_{t+1}) - p_{s_{jt+1}}}{u'(c_{t+1})}\right)\right]\right\}\right]}_{\equiv \mathbf{W}(w_{ijt})} + (1-k_{ijt}^h)\cdot \mathbf{U} \quad \forall ij$$

HH_OPT

General Equilibrium

- Introduce random-search matching and Nash-bargained wages

- Submarket *ij* Labor Supply (directed search)

$$\frac{h'(lfp_t)}{u'(c_t)} = p_{s_{jt}} + k_{ijt}^h \cdot \underbrace{\left[w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left[\left(\frac{1-k_{jt+1}^h}{k_{jt+1}^h} \right) \left(\frac{h'(lfp_{t+1}) - p_{s_{jt+1}}}{u'(c_{t+1})} \right) \right] \right\} \right]}_{\equiv \mathbf{W}(w_{ijt})} + (1-k_{ijt}^h) \cdot \mathbf{U} \quad \forall ij$$

- Submarket *ij* Labor Demand (directed vacancies)

$$\frac{\gamma}{k_{ijt}^f} = p_{v_{jt}} + k_{ijt}^f \cdot \underbrace{\left[z_t \cdot f_n(k_t, n_t) - w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma - p_{v_{jt+1}}}{k_{jt+1}^f} \right\} \right]}_{\equiv \mathbf{J}(w_{ijt})} \quad \forall ij$$

FIRM_OPT

General Equilibrium

- Symmetric equilibrium across *ij*
- Aggregate law of motion for labor

$$n_t = (1-\rho)n_{t-1} + \underbrace{\rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t)}_{\text{new job matches via monopolistic recruiting}} + \underbrace{m(s_{Nt}, v_{Nt})}_{\text{new job matches via random search}}$$

GENERAL EQUILIBRIUM

- **Symmetric equilibrium across *ij***
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$$n_t = (1-\rho)n_{t-1} + \underbrace{\rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t)}_{\text{new job matches via monopolistic recruiting}} + \underbrace{m(s_{Nt}, v_{Nt})}_{\text{new job matches via random search}}$$

- **Aggregate resource frontier**

$$[\text{ Absorption }] = z_t f(k_t, n_t)$$

(Std. procedure for aggregation: sum hh BCs, substitute equil. expressions)

DEFINITION – GENERAL EQUILIBRIUM

- **State-contingent stochastic functions**
  $\{ c, n, lfp, k', N_M, N_{ME}, s, v, \theta, s_N, v_N, \theta_N, w, w_N, p_v, p_s \}_{t=0}$ that satisfy
  - **Search directed towards monopolistic submarkets**
  - **Vacancies directed towards monopolistic submarkets**
  - **Monopolistic wage surplus sharing**
  - **Free-entry condition for recruiters**
  - **Aggregate law of motion for recruiters**
  - **Aggregate law of motion for employment**

  - **s_N and v_N in random-search matching channel**

  - **Aggregate LFP (determined by $h'(lfp_t)/u'(c_t)$)**
  - **Capital Euler equation**
  - **Nash wage surplus sharing**

  - **Aggregate goods resource frontier**
  - **Input prices p_{vt} and p_{st} (markdown of respective marginal products)**
  - **Definitions of tightness θ_t and θ_{Nt}**

Given stochastic process $\left\{ z_t \right\}_{t=0}^{\infty}$

and initial conditions k_0, n_{-1}, N_{M-1} (State vector: x_t = [k_t, n_{t-1}, N_{Mt-1}, z_t])

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SPILLOVER EFFECTS

- **Proposition 2 (Static Model). Assume Dixit-Stiglitz matching aggregation.**

$$\frac{\partial N_M^*}{\partial \eta} = \frac{\partial \theta^*}{\partial \eta} = 0 \text{ if } \eta = \xi \text{ (efficient bargaining power)}$$

η is worker's bargaining power

ξ is elasticity of $m_{ij}(.)$ wrt s_{ij}

Spillover Effects

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***η* is worker's bargaining power**

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- **Lemma (Static Model).**

$$\frac{\partial N_M^*}{\partial \eta} > 0 \text{ and } \frac{\partial \theta^*}{\partial \eta} > 0 \text{ iff } \eta < \xi \text{ (low worker bargaining power)}$$

$$\frac{\partial N_M^*}{\partial \eta} < 0 \text{ and } \frac{\partial \theta^*}{\partial \eta} < 0 \text{ iff } \eta > \xi \text{ (high worker bargaining power)}$$

Spillover Effects

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- **Distortion (wage) in random search causes distortion in recruiting sector**
  - **Despite *efficient* Dixit-Stiglitz aggregation**

Quant. Verification

Spillover Effects

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- **Distortion (wage) in random search causes distortion in recruiting sector**
  - **Despite *efficient* Dixit-Stiglitz aggregation**
- **Causality of distortionary spillover does NOT run in opposite direction**
  - **Intuition: insufficient margins of adjustment**

Quant. Verification

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CALIBRATION

- **Utility**

$$u(c_t) - h(lfp_t) = \ln c_t - \frac{\kappa}{1+1/\iota} lfp_t^{1+\frac{1}{\iota}}$$

- **Aggregate LFP**

$$lfp_t \equiv (1-\rho)n_{t-1} + s_t N_{Mt} + s_{Nt}$$

- **Cobb-Douglas matching function**

$$m(s_t, v_t) = m^{EFF} \cdot s_t^{\xi} v_t^{1-\xi}$$

(for both matching functions)

***m*^EFF^ larger in recruiting market**

CALIBRATION

- **Utility**

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(for both matching functions)

***m^{EFF}* larger in recruiting market**

- **β = 0.99**
- **Matching elasticity ξ =0.4**
- **Exogenous job-separation rate ρ = 0.10**
- **Exogenous recruiter exit rate ω = 0.05**

- **Stochastic TFP process**

$$\ln z_{t+1} = \rho_z \ln z_t + \epsilon_t^z$$

- **(Table 4 contains other baseline parameters)**

Spillover Effects

Proposition 2

$$\frac{\partial N_M^*}{\partial \eta} = 0 \text{ and } \frac{\partial \theta^*}{\partial \eta} = 0 \text{ if } \eta = \xi$$

Lemma

$$\frac{\partial N_M^*}{\partial \eta} > 0 \text{ and } \frac{\partial \theta^*}{\partial \eta} > 0 \text{ iff } \eta < \xi$$

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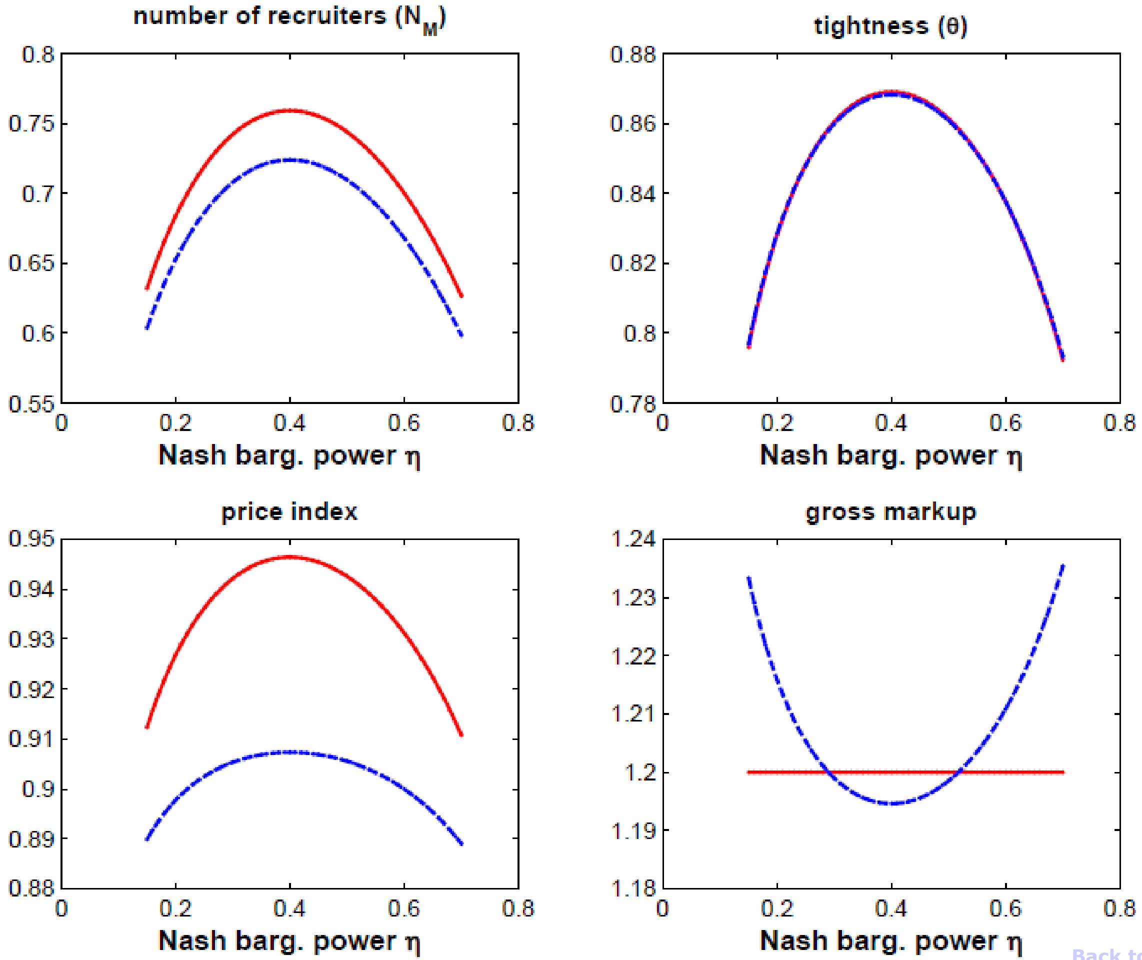

Back to Prop. 2
Outline

Outline

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SUMMARY

- Monopolistically Competitive Recruiting Model
  - Moen (1997 *JPE*), Shimer (1996), Pissarides (1985 *AER*)
  - Bilbiie, Ghironi, and Melitz (2012 *JPE*)

- Tractable Model
- Easy to Extend

- Provides New Competitive Wage Model
- Aggregate Increasing Returns in Intermediated Matching
- Expansion of Aggregate Resource Frontier
- Effects Between Non-Intermediated and Intermediated Matching

MONOPOLISTIC RECRUITING – MARKET *j*

- A continuum of aggregate recruiting agencies
  - Each aggregate recruiting agency is perfectly competitive
  - Easier to deal with mathematically than discrete infinity (tools of calculus can be applied)

- Representative recruiting agency *j*'s profit function

Relative price ρ_{ijt} of submarket recruiter *ij*

$$m(s_{jt}, v_{jt}) - \int_0^{N_{Mjt}} \rho_{ijt} \cdot m_{ijt} \cdot di$$

Substitute aggregate Dixit-Stiglitz matching technology

$$\left[\int_0^{N_{Mjt}} m_{ijt}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^{N_{Mjt}} \rho_{ijt} \cdot m_{ijt} \cdot di$$

MONOPOLISTIC RECRUITING – MARKET *j*

- **Representative recruiter's profit-maximization problem**

$$\max_{\{m_{ijt}\}_{i=0....N_{Mjt}}} \left[\int_0^{N_{Mjt}} m_{ijt}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^{N_{Mjt}} \rho_{ijt} \cdot m_{ijt} \cdot di$$

Chooses profit-maximizing quantity of input of *each* submarket match.

- **FOC with respect to m_{ijt} (for all *ij*)**

.
.
.

- **…after several rearrangements**

$$m_{ijt} = \rho_{ijt}^{-\varepsilon} \cdot m_{jt} \quad \Leftrightarrow \quad \rho_{ijt} = m_{ijt}^{-\frac{1}{\varepsilon}} \cdot m_{jt}^{\frac{1}{\varepsilon}} \quad \forall i \in j$$

DEMAND FUNCTION FOR RECRUITER *ij*

MONOPOLISTIC RECRUITING – SUBMARKET *ij*

- **Focus on profit-maximization of an arbitrary monopolistic recruiter *ij***
- **Assume zero fixed costs of creating a match**
- **Operates a constant-returns-to-scale (CRS) matching technology in order to create its specialized, differentiated match**
  - **CRS: if all inputs are scaled up by the factor *x*, total output is scaled up by the factor *x***
  - **Implementation of theory requires specifying neither the factors of production (i.e., active search *s*, vacancies *v*, etc) nor a matching function (*m*(.))**

$$m(s_{ijt}, v_{ijt}) = s_{ijt}^{\xi} v_{ijt}^{1-\xi}$$

MONOPOLISTIC RECRUITING – SUBMARKET *ij*

- **Focus on profit-maximization of an arbitrary monopolistic recruiter *ij***
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  - **Implementation of theory requires specifying neither the factors of production (i.e., active search *s*, vacancies *v*, etc) nor a matching function (*m*(.))**

Together, these imply a simple description of production

- **Marginal cost of creating a match**
  - **= average cost of creating a match**
  - **is invariant to the quantity of matches created**
    - **i.e., mc is NOT a function mc(quantity of matches)**

$$m(s_{ijt}, v_{ijt}) = s_{ijt}^{\xi} v_{ijt}^{1-\xi}$$

SUBMARKET *ij* – STAGE ONE OPTIMIZATION

- **Monopolistic recruiter *ij*'s price-maximization problem**

Total revenue depends on match creation and its *own* submarket *ij* price.

***mc* is NOT a function of matches created (due to CRS m(.))**

FC = 0 → *mc* = *ac*

$$\max_{\rho_{ijt}} \left[\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m(s_{ijt}, v_{ijt}) \right]$$

SUBMARKET *ij* – STAGE ONE OPTIMIZATION

- **Monopolistic recruiter *ij*'s price-maximization problem**

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$$\max_{\rho_{ijt}} \left[\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m(s_{ijt}, v_{ijt}) \right]$$

Substitute in demand function for recruiter *ij*

$$m_{ijt} = \rho_{ijt}^{-\varepsilon} \cdot m_{jt}$$

Critical point for analysis of monopoly: the recruiter *understands* and *internalizes* the effect of *its* price on the quantity that it creates.

$$\max_{\rho_{ijt}} \left[\rho_{ijt}^{1-\varepsilon} \cdot m(s_{jt}, v_{jt}) - mc_{jt} \cdot \rho_{ijt}^{-\varepsilon} \cdot m(s_{jt}, v_{jt}) \right]$$

- **Profit-maximization ("stage one")**
  - **Compute FOC with respect to relative price ρ_{ij}**

SUBMARKET *ij* – STAGE ONE OPTIMIZATION

- **Monopolistic recruiter *ij*'s price-maximization problem**

$$\max_{\rho_{ijt}}\left[\rho_{ijt}^{1-\varepsilon}\cdot m(s_{jt},v_{jt})-mc_{t}\cdot\rho_{ijt}^{-\varepsilon}\cdot m(s_{jt},v_{jt})\right]$$

- **FOC with respect to ρ_{ijt}**

$$(1-\varepsilon)\cdot\rho_{ijt}^{-\varepsilon}\cdot m(s_{jt},v_{jt})+\varepsilon\cdot\rho_{ijt}^{-\varepsilon-1}\cdot mc_{jt}\cdot m(s_{jt},v_{jt})=0$$

SUBMARKET *ij* – STAGE ONE OPTIMIZATION

- **Monopolistic recruiter *ij*'s price-maximization problem**

$$\max_{\rho_{ijt}} \left[\rho_{ijt}^{1-\varepsilon} \cdot m(s_{jt}, v_{jt}) - mc_t \cdot \rho_{ijt}^{-\varepsilon} \cdot m(s_{jt}, v_{jt}) \right]$$

- **FOC with respect to ρ_{ijt}**

$$(1-\varepsilon) \cdot \rho_{ijt}^{-\varepsilon} \cdot m(s_{jt}, v_{jt}) + \varepsilon \cdot \rho_{ijt}^{-\varepsilon-1} \cdot mc_{jt} \cdot m(s_{jt}, v_{jt}) = 0$$

- **Algebraic rearrangement**

Optimal relative price of recruiter *j* is a markup *ε*/(*ε* – 1) over marginal cost of creating specialized/different match.

KEY PRICING RESULT OF DIXIT-STIGLITZ THEORY.

$$\rho_{ijt} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \cdot mc_{jt}$$

Gross matching-market markup

Linked *only* to degree of substitutability across monopolistic recruiters *i*

PERFECT **CSE: *ε* = infinity**

Monopolistic matching: *ε* > 1 and *ε* < infinity

HOUSEHOLD OPTIMIZATION

- **Household utility**

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h\left(n_t + \underbrace{(1-k_{Nt}^h)\cdot s_{Nt}}_{=ue_t^N} + \int_0^1 \left(\int_0^{N_{Mjt}} \underbrace{(1-k_{ijt}^h)\cdot s_{ijt}}_{=ue_{ijt}} \, di \right) dj \right) \right]$$

flow budget constraint

$$c_t + k_{t+1} + T_t = (1+r_t-\delta)k_t + w_t(1-\rho)n_{t-1} + w_{Nt}\cdot k_{Nt}^h \cdot s_{Nt} + \int_0^1 \int_0^{N_{Mjt}} w_{ijt}\cdot k_{ijt}^h \cdot s_{ijt} \, di \, dj$$

$$+ \int_0^1 \int_0^{N_{Mjt}} p_{s_{jt}} \cdot s_{ijt} \, di \, dj + (1-k_{Nt}^h)\cdot s_{Nt}\chi + \int_0^1 \int_0^{N_{Mjt}} \left(1-k_{ijt}^h\right)\cdot s_{ijt}\chi \, di \, dj + \int_0^1 \Pi_{jt}^M \, dj + \Pi_t^F$$

perceived LOM for labor

$$n_t = (1-\rho)n_{t-1} + k_{Nt}^h \cdot s_{Nt} + \int_0^1 \int_0^{N_{Mjt}} k_{ijt}^h \cdot s_{ijt} \, di \, dj$$

- **FOCs wrt c_t, n_t, k_{t+1}, s_{Nt}, s_{ijt}**

GE

Firm Optimization

- Firm lifetime profit function

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ z_t f(k_t, n_t) - r_t k_t - \gamma_N v_{Nt} - \int_0^1 \int_0^{N_{Mjt}} \gamma v_{ijt} \; di \;\; dj + \int_0^1 \int_0^{N_{Mjt}} p_{v_{jt}} v_{ijt} \; di \;\; dj \right\}$$

$$- E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ w_t \cdot (1-\rho) n_{t-1} + w_{Nt} \cdot k_{Nt}^f \cdot v_{Nt} + \int_0^1 \int_0^{N_{Mjt}} w_{ijt} \cdot k_{ijt}^f \cdot v_{ijt} \; di \; dj \right\}$$

perceived LOM for labor

$$n_t = (1-\rho) n_{t-1} + k_{Nt}^f \cdot v_{Nt} + \int_0^1 \int_0^{N_{Mjt}} k_{ijt}^f \cdot v_{ijt} \; di \; dj$$

- **FOCs wrt k_t, n_t, v_{Nt}, v_{ijt}**

GE