Economics 8823
Advanced Macroeconomics
Project 1
Professor Sanjay Chugh
Spring 2018

Objective

As a building block to working with some set(s) of general equilibrium and/or partial equilibrium models (both in this class and, should your research interests eventually take you in that direction, your own continued research), you will compute a **first-order approximation** to the decisions rules of a DSGE/RBC economy. You will use the Schmitt-Grohe and Uribe (2004 *Journal of Economic Dynamics and Control*) algorithm.

Because the primary methodological objective here is **to learn how to implement** such solutions yourself, you are **not** permitted to use off-the-shelf programs provided by Schmitt-Grohe and Uribe, Uhlig, or others or packaged programs such as Dynare.

The Economy

The representative household maximizes lifetime utility

$$\max_{c_t, n_t, k_{t+1}, x_t} E_0 \sum_{t=0}^{\infty} \beta^t \ln\left[c_t - x_t \cdot \frac{\kappa}{1+1/\psi} n_t^{1+1/\psi} \right]$$

subject to its budget constraint

$$c_t + k_{t+1} - (1-\delta)k_t + T_t = w_t n_t + r_t k_t$$

(T_t is a flow lump-sum tax that fully and exactly finances government spending $g_t = \bar{g} \quad \forall t$) and

$$x_t = c_t^{\omega} \cdot x_{t-1}^{1-\omega}.$$

The representative firm maximizes profits

$$\max_{n_t, k_t} \left[z_t f(k_t, n_t) - w_t n_t - r_t k_t \right]$$

by choosing factor inputs n_t and k_t.

The aggregate goods resource constraint is

$$c_t + k_{t+1} - (1-\delta)k_t + g_t = z_t f(k_t, n_t),$$

in which g_t denotes the government's period-t flow of expenditures. The stationary-state (aka, balanced-growth) production technology is

$$f(k_t, n_t) = z_t k_t^{a} n_t^{1-a}$$

and the steady-state level of TFP is $\bar{z} = 1$.

The only exogenous process in the economy is exogenous TFP, and its evolution is characterized by the AR(1) process

$$\ln z_{t+1} = \rho_z \ln z_t + \varepsilon_{t+1}^{z},$$

in which ε_{t+1}^{z} is distributed as i.i.d. $N(0, \sigma_z^2)$. The persistence and standard deviation parameters are, respectively $\rho_z = 0.95$ and $\sigma_z = 0.007$.

Parameter Sets

Numerically compute the deterministic steady-state values of *c*, *n*, *k*, *r*, *w*, and *x* (i.e., consumption, labor, the capital stock, the real interest rate, the real wage, and the term that arises from the Jaimovich-Rebelo preference specification) for this economy using the four parameter sets in Table 1**.**

	Parameter Set A	Parameter Set B	Parameter Set C	Parameter Set D
β	0.99	0.99	0.99	0.99
δ	0.02	0.02	0.02	0.02
ψ	1	1	1	1
α	0.36	0.36	0.36	0.36
ω	1	0.7	0.3	10e-5
κ	**To be determined**	**To be determined**	**To be determined**	**To be determined**
$\bar{g}$	**To be determined**	**To be determined**	**To be determined**	**To be determined**

Table 1. Parameter sets.

For each parameter set, compute (i.e., calibrate) the value of κ so that n^{SS} = 0.30 and the value of $\bar{g}$ so that the long-run share of government expenditures in GDP is 20 percent.

Simulations

Having computed the matrices g_x and h_x for a given parameter set, the next step is to conduct simulations of your model(s). In order to generate simulations, recall that the first-order approximations are given by

$$
\begin{aligned}
y_t &= g(x_t,\sigma) \approx g(\bar{x},0) + g_x \cdot (x_t - \bar{x}) \\
x_{t+1} &= h(x_t,\sigma) \approx h(\bar{x},0) + h_x \cdot (x_t - \bar{x}) + \eta\sigma\varepsilon_{t+1}
\end{aligned}
$$

in which it is easiest to set the perturbation parameter $\sigma = 1$, in which case the matrix η must contain the standard deviations of the model's exogenous state variables. You will be provided with sequences of shocks for the process z_t which is the forcing process for your time-series simulations. Specifically, you will be provided with 200 sequences each of length 200 periods (quarters). These shocks are drawn from an *iid* $N(0, 1)$ distribution, which, when pre-multiplied with the appropriate row of the matrix η yields an *iid* $N(0, \sigma_z^2)$.[1]

Using **both the non-filtered AND the HP-filtered cyclical components of your simulated time series** (specifically, the net percentage deviation (aka log deviation) of each simulated series from its respective trend), calculate, for each time series of interest in a given simulation, standard deviations, first-order serial correlations, and contemporaneous correlation of each variable with GDP.[2,3] Then, compute and report the means and standard deviations of these means, the means and standard deviations of these standard deviations, and the means and standard deviations of these correlations across all simulations. These sets of second-moment statistics (along with the steady state values of the endogenous variables you decide are interesting/relevant to analyze) are what you should report as your simulation-based results (in some appropriate and informative combination of tables and/or graphs and/or text).

Compare and contrast the business-cycle moments you find with both Table 1and Table 3 provided in King and Rebelo (1999 *Handbook of Macroeconomics*).

Impulse Responses

Plot impulse response functions for variables of economic interest. (Hint: it would likely be more informative for the reader if several clear IRFs could be contained in one diagram – see `help subplot` in Matlab for more.)

---

[1] The shocks were generated using Matlab's built-in `randn` function. For this project, use the provided sequence of shocks for your simulations (for the sake of some comparability). In subsequent projects of your own, you can use the `randn` function to generate your own random numbers.
[2] Note that some series (such as GDP) may have to be constructed residually if you do not include them as part of your state or costate vectors.
[3] You will be provided with Matlab files that implement the HP filter.

What To Submit

- A clear, concise definition of the (dynamic stochastic) private-sector equilibrium.

- The state vector x_t and the co-state vector y_t.

- The g_x and h_x matrices that correspond to each of the four parameter sets A, B, C, and D (which include, respectively, the Jaimovich and Rebelo (2009) terms $\omega = 1$, $\omega = 0.7$, $\omega = 0.3$, and $\omega = 10e - 5$).

- For each of the four parameter sets, impulse response functions to z_t shocks plotted as IRF figures. The IRF figures must include TFP, GDP, C, labor, gross investment, the stock of physical capital, the real interest rate, and the real wage. It is left up to you to determine how many IRF figures to include that take into account these variables – each IRF figure should be **informatively captioned in way that a reader could simply look at the figure and caption and understand the take-away messages(s) without necessarily needing to refer to the main text.**

- One clearly-organized, easy-to-read, and informatively captioned Table that displays simulation-based HP-filtered business cycle statistics for the same set of variables contained in the IRFs.

- One clearly-organized, easy-to-read, and informatively captioned Table that displays simulation-based **non-**HP-filtered business cycle statistics for the same set of variables contained in the IRFs.

- **Brief, concise description** (no more than two pages in length) of the results found, including comparison of the results with those presented in King and Rebelo's (1999) Table 1 and Table 3.

- Include your code (i.e., all the relevant files that one would need to replicate your results).

(Some) Computational/Programming Guidance

Using Matlab's `fsolve` function to solve for the matrices g_x and h_x is once again the key computational step, as in Project 0.

In order to conduct simulations using the sequences of shocks with which you will be provided, you must essentially proceed "iteratively" through each simulation. To do so, begin with k_0 (which is simply the deterministic steady state value $\bar{k}$) and the "first realization" of the shock to z (that is, the first (period-zero) shock to log z) and compute the period-zero equilibrium outcome using

$$y_0 = g(\bar{x},0) + g_x \cdot (x_0 - \bar{x})$$
$$x_1 = h(\bar{x},0) + h_x \cdot (x_0 - \bar{x}) + \eta\sigma\varepsilon_1$$

Once you have the period-zero equilibrium outcome of the model in hand, compute the period-one equilibrium outcome of the model using

$$y_1 = g(\bar{x},0) + g_x \cdot (x_1 - \bar{x})$$
$$x_2 = h(\bar{x},0) + h_x \cdot (x_1 - \bar{x}) + \eta\sigma\varepsilon_2$$

Continue this way through all periods of the simulation, and then repeat this for each of the simulations. In conducting these simulations, you can and should try to cleverly arrange matrices and vectors in a way that takes advantage of Matlab's comparative advantage (compared to other software programs) in performing matrix manipulations. Be careful about issues such as matrix conformability, in particular with your g_x and h_x matrices.

A "sensibility check" you may want to try on your programs is to check the convergence (to the deterministic steady state) implied by your computed g_x and h_x matrices. To check this, begin with some arbitrary k_0 (say, perhaps 1% or 0.5% above or below the steady state $\bar{k}$) and construct a vector of zeros for the sequence of TFP shocks and deterministic productivity shocks. Iteratively apply your approximated decision rules (as described above) to construct a time-series simulation of the model – the difference, of course, is that this will be a *deterministic simulation* because each period the TFP shock is by assumption zero. If you have computed the correct g_x and h_x, your model variables should clearly converge to their deterministic steady state counterparts.

If you do not find convergence to the deterministic steady state (and you are convinced you are conducting the simulations correctly), there likely is an error in your computed g_x and/or h_x matrix. One "simple" error is that you have found the explosive root of the system (i.e., an eigenvalue outside the unit circle). You can check this using the command `eig(hx)`.