Department of Economics The Ohio State Univeristy

Economics 8823
Advanced Macroeconomics
Project 1 – (Partial/Sketch of) Suggested Solutions
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Objective

As a building block to working with some set(s) of general equilibrium and/or partial equilibrium models (both in this class and, should your research interests eventually take you in that direction, your own continued research), you will compute a **first-order approximation** to the decisions rules of a DSGE/RBC economy. You will use the Schmitt-Grohe and Uribe (2004 *Journal of Economic Dynamics and Control*) algorithm.

Because the primary methodological objective here is **to learn how to implement** such solutions yourself, you are **not** permitted to use off-the-shelf programs provided by Schmitt-Grohe and Uribe, Uhlig, or others or packaged programs such as Dynare.

The Economy

The representative household maximizes lifetime utility

$$\max_{c_t, n_t, k_{t+1}, x_t} E_0 \sum_{t=0}^{\infty} \beta^t \ln\left[c_t - x_t \cdot \frac{\kappa}{1+1/\psi} n_t^{1+1/\psi} \right]$$

subject to its budget constraint

$$c_t + k_{t+1} - (1-\delta)k_t + T_t = w_t n_t + r_t k_t$$

(T_t is a flow lump-sum tax that fully and exactly finances government spending $g_t = \bar{g} \quad \forall t$) and

$$x_t = c_t^{\omega} \cdot x_{t-1}^{1-\omega}.$$

The representative firm maximizes profits

$$\max_{n_t, k_t} \left[z_t f(k_t, n_t) - w_t n_t - r_t k_t \right]$$

by choosing factor inputs n_t and k_t.

The aggregate goods resource constraint is

$$c_t + k_{t+1} - (1-\delta)k_t + g_t = z_t f(k_t, n_t),$$

in which g_t denotes the government's period-t flow of expenditures. The stationary-state (aka, balanced-growth) production technology is

$$f(k_t, n_t) = z_t k_t^{a} n_t^{1-a}$$

and the steady-state level of TFP is $\bar{z} = 1$.

The only exogenous process in the economy is exogenous TFP, and its evolution is characterized by the AR(1) process

$$\ln z_{t+1} = \rho_z \ln z_t + \varepsilon_{t+1}^{z},$$

in which ε_{t+1}^{z} is distributed as i.i.d. $N(0, \sigma_z^2)$. The persistence and standard deviation parameters are, respectively $\rho_z = 0.95$ and $\sigma_z = 0.007$.

Parameter Sets

Numerically compute the deterministic steady-state values of c, n, k, r, w, and x (i.e., consumption, labor, the capital stock, the real interest rate, the real wage, and the term that arises from the Jaimovich-Rebelo preference specification) for this economy using the four parameter sets in Table 1**.**

	Parameter Set A	Parameter Set B	Parameter Set C	Parameter Set D
β	0.99	0.99	0.99	0.99
δ	0.02	0.02	0.02	0.02
ψ	1	1	1	1
α	0.36	0.36	0.36	0.36
ω	1	0.7	0.3	10e-5
κ	**To be determined**	**To be determined**	**To be determined**	**To be determined**
$\bar{g}$	**To be determined**	**To be determined**	**To be determined**	**To be determined**

Table 1. Parameter sets.

For each parameter set, compute (i.e., calibrate) the value of κ so that n^{SS} = 0.30 and the value of $\bar{g}$ so that the long-run share of government expenditures in GDP is 20 percent.

What To Submit

- A clear, concise definition of the (dynamic stochastic) private-sector equilibrium.

- The state vector x_t and the co-state vector y_t.

- The g_x and h_x matrices that correspond to each of the four parameter sets A, B, C, and D (which include, respectively, the Jaimovich and Rebelo (2009) terms $\omega = 1$, $\omega = 0.7$, $\omega = 0.3$, and $\omega = 10e-5$).

- For each of the four parameter sets, impulse response functions to z_t shocks plotted as IRF figures. The IRF figures must include TFP, GDP, C, labor, gross investment, the stock of physical capital, the real interest rate, and the real wage. It is left up to you to determine how many IRF figures to include that take into account these variables – each IRF figure should be **informatively captioned in way that a reader could simply look at the figure and caption and understand the take-away messages(s) without necessarily needing to refer to the main text.**

- One clearly-organized, easy-to-read, and informatively captioned Table that displays simulation-based HP-filtered business cycle statistics for the same set of variables contained in the IRFs.

- One clearly-organized, easy-to-read, and informatively captioned Table that displays simulation-based non-HP-filtered business cycle statistics for the same set of variables contained in the IRFs.

- Brief summary (no more than two pages in length) of the results found, including comparison of the results with those presented in King and Rebelo's (1999) Table 1 and Table 3.

- Include your code (i.e., all the relevant files that one would need to replicate your results).

Solutions:

The steady-state equilibrium is defined as a list of variables $\left\{c^{ss}, n^{ss}, k^{ss}, r^{ss}, w^{ss}, x^{ss}\right\}$ that solve: 1) the steady-state household consumption-labor optimality condition; 2) the steady-state household consumption-savings optimality condition (aka, the physical capital Euler expression); 3) the steady-state firm profit-maximizing labor demand function; 4) the steady-state firm profit-maximizing capital demand function; 5) the steady-state evolution of x_t; and 6) the aggregate goods resource constraint.

The endogenous variables (including the values for κ and $\bar{g}$ for each parameter set) are provided in **Error! Reference source not found.** and **Error! Reference source not found.**.

	Parameter Set A	Parameter Set B	Parameter Set C	Parameter Set D
β	0.99	0.99	0.99	0.99
δ	0.02	0.02	0.02	0.02
ψ	1	1	1	1
α	0.36	0.36	0.36	0.36
ω	1	0.7	0.3	10e-5
κ	**12.6800**	**8.0859**	**8.1408**	**12.6728**
$\bar{g}$	**0.2423**	**0.2423**	**0.2423**	**0.2423**

Table 2. Steady-state values for kappa and gbar for each parameter set.

	Parameter Set A	Parameter Set B	Parameter Set C	Parameter Set D
c	0.6794	0.6794	0.6794	0.6794
n	0.3000	0.3000	0.3000	0.3000
k	14.4898	14.4898	14.4898	14.4898
r	0.0301	0.0301	0.0301	0.0301
w	2.5846	2.5846	2.5846	2.5846
x	0.6794	0.6794	0.6794	0.6794

Table 3. Endogenous steady-state equilibrium variables for each parameter set.

Next, regarding dynamics, denote by μ_t the period-t Lagrange multiplier on the evolution expression $x_{JR,t} = c_t^{\omega} x_{JR,t-1}^{1-\omega}$ that the representative household takes as a constraint. The private-sector equilibrium is a set of state-contingent endogenous functions $\{c_t, n_t, inv_t, k_{t+1}, r_t, w_t, x_{JR,t}, \mu_t\}_{t=0}^{\infty}$ that satisfy 1) the first-order condition with respect to consumption; 2) the first-order condition with respect to labor; 3) the consumption-savings optimality condition (aka, the physical capital Euler expression); 4) the firm's profit-maximizing labor demand function; 5) the firm's profit-maximizing capital demand function; 6) the first-order condition with respect to $x_{JR,t}$; 6) the evolution expression for $x_{JR,t}$; 7) the definition of gross investment, $inv_t = k_{t+1} - (1-\delta)k_t$; and 8) the aggregate goods resource constraint. **(Note that this is but one acceptable definition of equilibrium.)** The initial conditions k_{-1} and $x_{JR,0}$ are taken as given as is the exogenous process for (log) TFP $\ln z_{t+1} = \rho_z \ln z_t + \varepsilon_{t+1}^z$, in which ε_{t+1}^z is distributed as i.i.d. $N(0,\sigma_z^2)$.

Define the state vector as $x_t = [k_t, x_{JR,t-1}, z_t]'$ and the co-state vector as $y_t = [c_t, n_t, gdp_t, inv_t, x_{JR,t}, \lambda_t, \mu_t, r_t, w_t]'$, in which λ_t denotes the period-t Lagrange multiplier on the household's budget constraint. **(Note that the state vector *must* be defined in this way, but the co-state vector can be defined in *many* different ways.)**

Given the state and co-state vectors defined above, the first-order approximated (in level-linear terms) g_x and h_x matrices for the several different values of ω are:

***ω* = 1**

$$g_x = \begin{pmatrix} 0.0498 & 0 & 0.2030 \\ -0.0005 & 0 & 0.0651 \\ 0.0288 & 0 & 1.3798 \\ -0.0210 & 0 & 1.1768 \\ 0.0498 & 0 & 0.2030 \\ -0.1080 & 0 & -0.4398 \\ 0.1586 & 0 & -1.3921 \\ -0.0014 & 0 & 0.0343 \\ 0.0658 & 0 & 2.3828 \end{pmatrix} \quad \text{and} \quad h_x = \begin{pmatrix} 0.9590 & 0 & 1.1768 \\ 0.0498 & 0 & 0.2030 \\ 0 & 0 & 0.95 \end{pmatrix}.$$

ω = 0.7

$$g_x = \begin{pmatrix} 0.0347 & 0 & 0.3417 \\ -0.0003 & 0 & 0.0900 \\ 0.0294 & 0 & 1.4442 \\ -0.0053 & 0 & 1.1025 \\ 0.0243 & 0 & 0.2392 \\ -0.1080 & 0 & -0.4398 \\ 0.0243 & 0 & 0.2392 \\ -0.0014 & 0 & 0.0343 \\ 0.0658 & 0 & 2.3828 \end{pmatrix} \text{ and } h_x = \begin{pmatrix} 0.9747 & 0 & 1.1025 \\ 0.0243 & 0 & 0.2392 \\ 0 & 0 & 0.95 \end{pmatrix}.$$

ω = 0.3

$$g_x = \begin{pmatrix} 0.0381 & 0 & 0.4425 \\ 0.0036 & 0 & 0.1369 \\ 0.0393 & 0 & 1.5654 \\ 0.0012 & 0 & 1.1228 \\ 0.0114 & 0 & 0.1328 \\ -0.0849 & 0 & -0.8519 \\ 0.1117 & 0 & -1.1051 \\ -0.0014 & 0 & 0.0343 \\ 0.0658 & 0 & 2.3828 \end{pmatrix} \text{ and } h_x = \begin{pmatrix} 0.9812 & 0 & 1.1228 \\ 0.0114 & 0 & 0.1328 \\ 0 & 0 & 0.95 \end{pmatrix}$$

ω = 10e-14

$$g_x = \begin{pmatrix} 0.0243 & 0 & 0.7738 \\ 0.0055 & 0 & 0.2207 \\ 0.0443 & 0 & 1.7819 \\ 0.0020 & 0 & 1.0079 \\ 0.0000 & 0 & 0.0000 \\ -0.1187 & 0 & -2.3921 \\ -0.2546 & 0 & -21.5253 \\ -0.0014 & 0 & 0.0343 \\ 0.0658 & 0 & 2.3828 \end{pmatrix} \text{ and } h_x = \begin{pmatrix} 1 & 0 & 1.0079 \\ 0.0000 & 0 & 0.0000 \\ 0 & 0 & 0.95 \end{pmatrix}$$

The impulse response profiles in Figure 2 and Figure 2 display fluctuations upon a one-time positive exogenous TFP shock for the several values of ω.

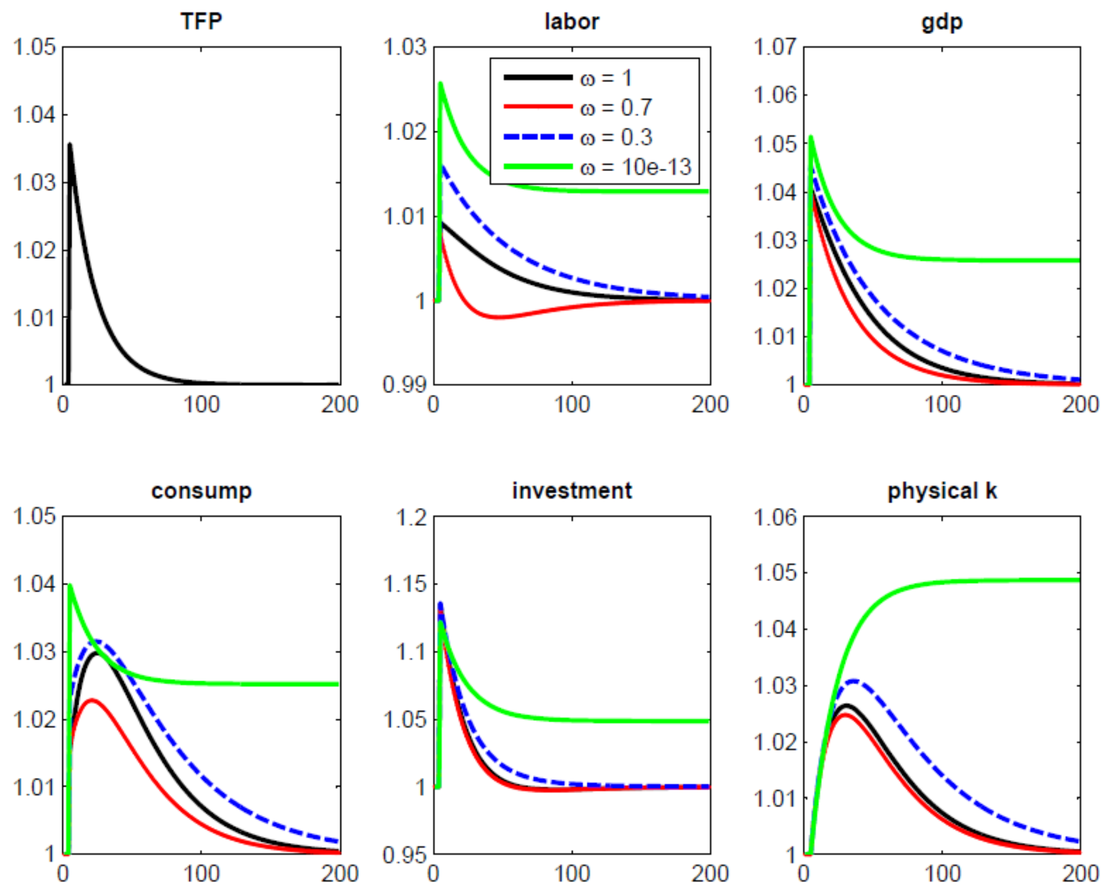

Figure 1. Impulse responses of quantities after one-time TFP shock for several values of ω. The vertical axis plots percentage deviation from the respective parameter set's deterministic steady state. Referring to the top middle panel, it is clear that the smaller is the value of ω, the larger is the impact magnitude of labor and the slower is the return to the initial steady state.

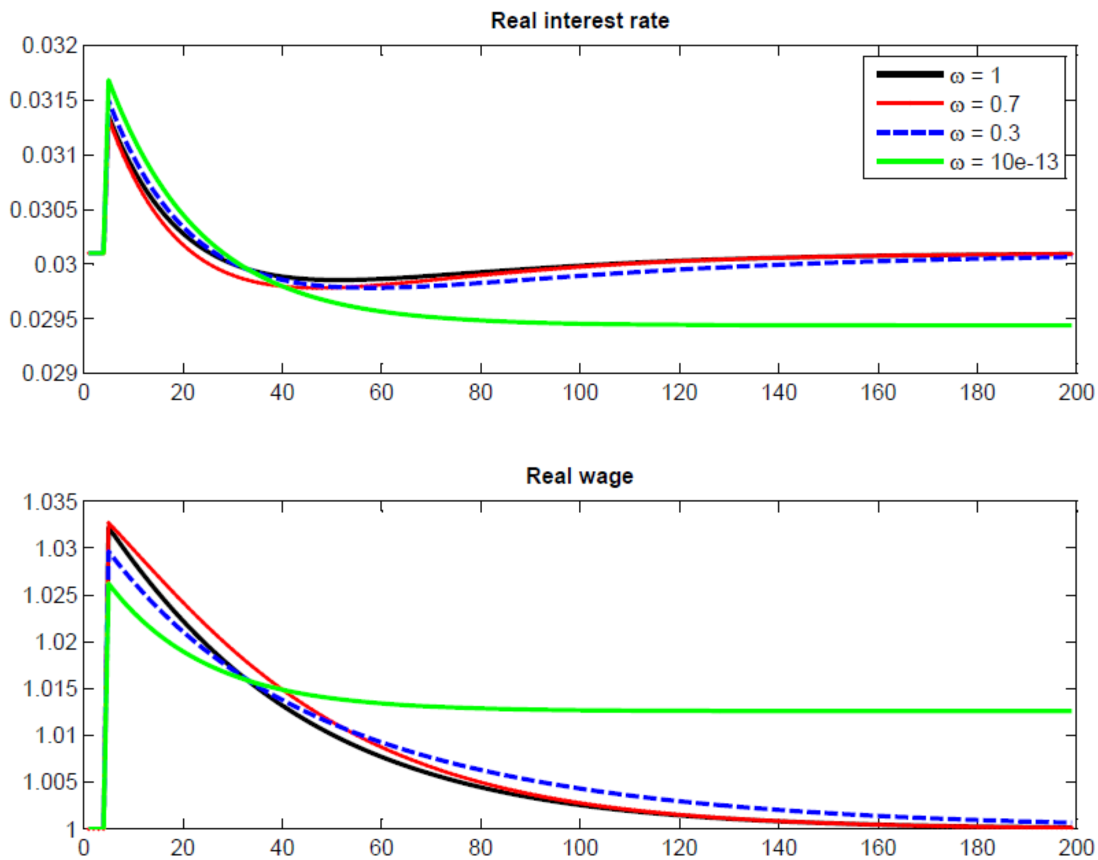

Figure 2. Impulse responses of real interest rate and real wage after one-time TFP shock for several values of ω.

For $\omega = 1$ (KPR preferences) and $\omega = 0$ (i.e., nesting GHH preferences), your code should:

- Set the multiplier associated with the JR expression $x_t = c_t^{\omega} x_{t-1}^{1-\omega}$ to zero in the deterministic steady state
- Set $\bar{x}^{JR} = 1$ in the deterministic steady state
- Set $x_t^{JR} = 1 \ \forall t$

- And, **only for the nested GHH version** $\omega = 0$**,** express the dynamic equilibrium capital-investment condition (aka, Euler equation) so that the real interest rate is **exogenous** – the natural example in context is the deterministic steady state $\bar{r}$, which never changes during business-cycle fluctuations:

$$\lambda_t = \beta \cdot E_t[\lambda_{t+1} \cdot (1 - \delta + \bar{r})]$$