Department of Economics The Ohio State University

Economics 8823
Advanced Macroeconomics
Project 2 – (Partial/Sketch of) Suggested Solutions
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Objective

As a building block to working with some set(s) of general equilibrium and/or partial equilibrium models (both in this class and, should your research interests eventually take you in that direction, your own continued research), you will compute a **first-order approximation** to the decisions rules of a DSGE/RBC economy. You will use the Schmitt-Grohe and Uribe (2004 *Journal of Economic Dynamics and Control*) algorithm.

Because the primary methodological objective here is **to learn how to implement** such solutions yourself, you are **not** permitted to use off-the-shelf programs provided by Schmitt-Grohe and Uribe, Uhlig, or others or packaged programs such as Dynare.

The Economy

The representative household maximizes lifetime utility

$$\max_{c_t, n_t^h, s_{ijt}, k_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(c_t) - \frac{\kappa}{1+1/\psi} \cdot \left(n_t^h + \int_0^1 \left(\int_0^1 \left(1 - k_{ijt}^h\right) \cdot s_{ijt} \; di \right) dj \right)^{1+1/\psi} \right)$$

subject to its budget constraint

$$\begin{aligned} c_t + k_{t+1} - (1-\delta)k_t + T_t &= r_t k_t + w_t (1-\rho) n_{t-1}^h \\ &\quad + \int_0^1 \int_0^1 w_{ijt} \cdot k_{ijt}^h \cdot s_{ijt} \; di \; dj + \int_0^1 \int_0^1 \left(1 - k_{ijt}^h\right) \cdot s_{ijt} \cdot \chi \; di \; dj \end{aligned}$$

(T_t is a flow lump-sum tax that fully and exactly finances government spending $g_t + \chi \;\; \forall t$, with χ denoting the fixed government-provided unemployment benefit to each unsuccessful unit of search)

and its perceived law of motion for employment

$$n_t^h = (1-\rho) n_{t-1}^h + \int_0^1 \int_0^1 k_{ijt}^h \cdot s_{ijt} \; di \; dj \, .$$

Using the symmetric equilibrium (that is, $\forall ij$) definition of aggregate labor-force participation (LFP), $lfp_t = n_t^h + (1 - k_t^h) \cdot s_t$, the labor-force participation condition for any submarket *ij* is

$$\frac{h'(lfp_t)}{u'(c_t)} = (1 - k_{ijt}^h) \cdot \chi + k_{ijt}^h \left[w_{ijt} + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1 - k_{jt+1}^h}{k_{jt+1}^h} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right] \;\; \forall ij$$

The representative firm maximizes lifetime discounted profits

$$\max_{n_t^f, v_{ijt}, k_t} \left[\frac{\beta u'(c_t)}{u'(c_0)} \left(z_t f(k_t, n_t^f) - w_t (1-\rho) n_{t-1}^f - r_t k_t - \int_0^1 \int_0^1 \gamma \cdot v_{ijt} \; di \; dj - \int_0^1 \int_0^1 w_{ijt} \cdot k_{ijt}^f \cdot v_{ijt} \; di \; dj \right) \right]$$

by choosing factor inputs n_t^f and k_t along with costly vacancy postings v_{ijt} in each recruiting submarket *ij*. The firm is constrained by its perceived law of motion for employment

$$n_t^f = (1-\rho) n_{t-1}^f + \int_0^1 \int_0^1 k_{ijt}^f \cdot v_{ijt} \; di \; dj \, .$$

The profit-maximizing vacancy posting condition for any submarket *ij* is

$$\gamma = k_{ijt}^{f}\left[z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho)E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \frac{\gamma}{k_{jt+1}^{f}} \right\} \right] \quad \forall ij .$$

A perfectly-competitive recruiting sector intermediates labor demand and labor supply. The (symmetric equilibrium) surplus-sharing condition is

$$\theta_t^{-\epsilon} \cdot \left(\mathbf{W}(w_t) - \mathbf{U} \right) = \mathbf{J}(w_t)$$

in which the relevant value expressions are

$$\mathbf{J}(w_t) = z_t f_n(k_t, n_t) - w_t + (1-\rho)E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \frac{\gamma}{k^f(\theta_{t+1})} \right\},$$

and

$$\mathbf{W}(w_t) - \mathbf{U} = w_t - \chi + (1-\rho)E_t\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \left(\frac{1-k_{t+1}^{h}}{k_{t+1}^{h}} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\}.$$

The (symmetric equilibrium) aggregate goods resource constraint is

$$c_t + k_{t+1} - (1-\delta)k_t + g_t + \gamma \cdot v_t = z_t f(k_t, n_t),$$

in which g_t denotes the government's period-t exogenous (non-unemployment benefit) expenditures and the aggregate law of motion of labor is

$$n_t = (1-\rho)n_{t-1} + m(s_t, v_t).$$

The goods production technology is

$$f(k_t, n_t) = z_t k_t^{a} n_t^{1-a}$$

with the steady-state level of TFP of $\bar{z} = 1$, and the aggregate matching technology is

$$m(s_t, v_t) = \frac{s_t \cdot v_t}{\left(s_t^{\epsilon} + v_t^{\epsilon} \right)^{1/\epsilon}} \Leftrightarrow m(.) = \frac{s \cdot \theta}{\left(1 + \theta^{\epsilon} \right)^{1/\epsilon}}$$

(hence the appearance of labor market tightness θ_t in the surplus-sharing expression).

The two exogenous processes in the economy are exogenous TFP and exogenous government spending, and their evolutions are characterized, respectively, by the AR(1) processes

$$\ln z_{t+1} = \rho_z \ln z_t + \varepsilon_{t+1}^z$$

and

$$\ln g_{t+1} = (1-\rho_g)\ln \bar{g} + \rho_g \ln g_t + \varepsilon_{t+1}^g$$

in which ε_{t+1}^z is distributed as i.i.d. $N(0,\sigma_z^2)$ and ε_{t+1}^g is distributed as i.i.d. $N(0,\sigma_g^2)$. The persistence and standard deviation parameters for the two exogenous processes are, respectively, $\rho_z = 0.95$ and $\sigma_z = 0.007$ and $\rho_g = 0.97$ and $\sigma_g = 0.027$.

Definition of Equilibrium

The period-*t* state of the economy is $S_t \equiv [k_t, n_t^{LAG}, z_t, g_t]$. A symmetric private-sector equilibrium contains nine endogenous state-contingent processes $\{c_t, k_{t+1}, n_t, lfp_t, s_t, v_t, \theta_t, w_t, r_t\}$ characterized by the following nine equilibrium conditions:

The aggregate goods resource constraint

$$c_t + k_{t+1} - (1-\delta)k_t + g_t + \gamma \cdot v_t = z_t f(k_t, n_t), \tag{1}$$

the aggregate labor of motion for labor

$$n_t = (1-\rho)n_{t-1} + m(s_t, v_t), \tag{2}$$

the definition of aggregate LFP

$$lfp_t = (1-\rho)n_{t-1} + s_t, \tag{3}$$

the definition of labor market tightness, $\theta_t \equiv v_t / s_t$, which is equilibrium expression (4), the standard capital Euler condition

$$1 = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(1 + z_{t+1} f_k(k_{t+1}, n_{t+1}) - \delta\right)\right\}, \tag{5}$$

the symmetric-equilibrium job creation condition

$$\gamma = k_t^f \left[z_t f_n(k_t, n_t) - w_t + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \frac{\gamma}{k_{t+1}^f} \right\} \right] , \qquad (6)$$

and the symmetric-equilibrium participation conditions by households

$$\frac{h'(lfp_t)}{u'(c_t)} = k_t^h \cdot w_t + (1-k_t^h) \cdot \chi$$
$$+ k_t^h \cdot (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1-k_{t+1}^h}{k_{t+1}^h} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} . \qquad (7)$$

The representative firm's profit-maximizing choice for k_t leads to the equilibrium condition $z_t f_k(k_t, n_t) = r_t \ \forall t$, which is equilibrium condition (8). Finally, we need an explicit-form wage expression that closes the definition of general equilibrium.

Begin by substituting the $\mathbf{J}(w_t) = ...$ that appears in the middle of p. 3 and also the $\mathbf{W}(w_t) - \mathbf{U} = ...$ into the surplus-sharing condition above. Doing so gives

$$\theta_t^{-\epsilon} \cdot \left(w_t - \chi + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \left(\frac{1-k_{t+1}^h}{k_{t+1}^h} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right)$$
$$= z_t f_n(k_t, n_t) - w_t + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \frac{\gamma}{k^f(\theta_{t+1})} \right\}$$

Rearranging terms slightly gives

$$w_t + \theta_t^{-\epsilon} \cdot w_t - \theta_t^{-\epsilon} \cdot \chi + \theta_t^{-\epsilon} \cdot (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \left(\frac{1-k_{t+1}^h}{k_{t+1}^h} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\}$$
$$= z_t f_n(k_t, n_t) + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \frac{\gamma}{k^f(\theta_{t+1})} \right\}$$

Next, collecting terms in w_t gives

$$w_t \cdot [1+\theta_t^{-\epsilon}] = z_t f_n(k_t, n_t) + \theta_t^{-\epsilon} \cdot \chi + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \frac{\gamma}{k^f(\theta_{t+1})} \right\}$$
$$- \theta_t^{-\epsilon} \cdot (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \left(\frac{1-k_{t+1}^h}{k_{t+1}^h} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\}.$$

Finally, dividing by $[1+\theta_t^{-\epsilon}]$ yields the ninth equilibrium condition

$$w_t \cdot = \underbrace{\left(\frac{1}{1+\theta_t^{-\epsilon}} \right)}_{=1/(1+\varepsilon_{m,v}^{dHRW}/\varepsilon_{m,s}^{dHRW})} \cdot \left(z_t f_n(k_t, n_t) + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \frac{\gamma}{k^f(\theta_{t+1})} \right\} \right)$$
$$- \underbrace{\left(\frac{\theta_t^{-\epsilon}}{1+\theta_t^{-\epsilon}} \right)}_{=(\varepsilon_{m,v}^{dHRW}/\varepsilon_{m,s}^{dHRW})/(1+\varepsilon_{m,v}^{dHRW}/\varepsilon_{m,s}^{dHRW})} \cdot \left(\chi + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \left(\frac{1-k_{t+1}^h}{k_{t+1}^h} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right)$$

(9)

which is the explicit-form wage expression $w_t \quad \forall t$.

Hence there are nine equilibrium equations that simultaneously solve for the nine state-contingent decision functions.

Parameter Values

Numerically compute the deterministic steady-state values of c, n, v, θ, k, r, w (i.e., consumption, the stock of labor, vacancies, labor-market tightness, the stock of physical capital, the real interest rate, and the real wage) for this economy using the following parameter values in Table 1**.**

	Parameter Values	Description
β	0.99	Quarterly subjective discount factor
δ	0.02	Depreciation rate of physical capital
ψ	0.20	Elasticity of *lfp*
α	0.36	Cobb-Douglas production function elasticity
ϵ	1.25	dHRW matching function elasticity
ρ	0.10	Quarterly separation rate of existing jobs
κ	5.601989740253326 **(please use precise value in code)**	Utility scaling parameter (to target $lfp^{ss} = 0.74$ for U.S. prime-age workers)
$\bar{g}$	0.949262460464700 **(please use precise value in code)**	Steady-state level of government spending; 20% of GDP
χ	1.162438762991125 **(please use precise value in code)**	Government-provided ue benefits
γ	1.457571757458351 **(please use precise value in code)**	Cost of posting one job vacancy (total vacancy costs = γv)

Table 1. Parameter values.

Implementation

You should define the co-state vector as $y_t = [c_t, n_t, v_t, \theta_t, s_t, w_t, r_t, k_t^h, k_t^f]^T$, in which the last two elements of the co-state vector are (based on the particular functional form specified for the matching technology) $k_t^h = \frac{\theta}{\left(1+\theta^\epsilon\right)^{1/\epsilon}}$ and $k_t^f = \frac{1}{\left(1+\theta^\epsilon\right)^{1/\epsilon}}$, and you should define the state vector as $x_t = [k_t, n_t^{LAG}, z_t, g_t]^T$ in order to avoid needing to compute the partials of the equilibrium equations with respect to the state vector x_{t+2}. To avoid computing this partial (which simplifies the code), you are adding an auxiliary variable n_t^{LAG} to the period-t state vector x_t **(NOTE: THIS IS NOT A TYPO)**, which then requires you to add an auxiliary equation to the set of equilibrium equations; the auxiliary equation to add is

$$\text{LINKING\_N} = n_{t+1}^{LAG} = n_t \tag{10}$$

and is named "LINKING_N" for obvious reasons. Note that **you ARE NOT being asked to substitute this expression in any of the model's equilibrium equations.**[1]

[1] In the model's set of equilibrium equations described above (which you should implement in your code as stated, please do **not** substitute out any equilibrium condition), note that both n_{t-1} and n_{t+1} appear – the former in the period-t aggregate labor of motion for labor and the latter in the period-t capital Euler condition. Given the canonical form $E_t\left[f(y_{t+1}, y_t, x_{t+1}, x_t)\right] = 0$ used in Schmitt Grohe and Uribe's (2004) perturbation algorithm, the additional "LINKING" equation and the additional auxiliary variable are needed to state the model in this canonical form.

Impulse Responses

Plot impulse response functions for variables of economic interest. More precisely, you are asked to provide two different IRF figures, each containing four panels.

Figure 1 should contain: the IRF for n_t, in panel (1,1), the IRF for c_t, in panel (1,2), the IRF for k_{t+1} in panel (1,3), and the IRF for gdp_t, in panel (1,4). **In EACH of these four panels, plot the impulse response for BOTH a one-time, one-standard deviation positive impulse in TFP and a one-time, one-standard deviation positive impulse in *g*.** (Hint: it would likely be informative for the reader if different colors or different line styles could be used in the IRFs contained in each panel – see `help subplot` in Matlab for more.)

And Figure 2 should contain: the IRF for w_t, in panel (1,1), the IRF for θ_t in panel (1,2), the IRF for k_t^h in panel (1,3), and the IRF for k_t^f in panel (1,4). **In EACH of these four panels, plot the impulse response for BOTH a one-time, one-standard deviation positive impulse in TFP and a one-time, one-standard deviation positive impulse in *g*.** (Hint: it would likely be informative for the reader if different colors or different line styles could be used in the IRFs contained in each panel – see `help subplot` in Matlab for more.)

While the pair of IRF figures is the bare minimum required in the dynamic analysis, you are of course welcome (encouraged) to provide other variables' impulse profiles, change the magnitude of the SDs of the exogenous processes, change the sign of the impulse, and so on.

What To Submit

- The g_x and h_x matrices that correspond to the parameter set provided and based on the **exact** definitions above of the ordering of the elements in the state vector x_t and the ordering of the elements in the co-state vector y_t.

- The deterministic steady state values for all of the variables used in the dynamics.

- Two separate impulse response figures, each with four panels, as detailed above.

- Note whether or not either matching probability ever goes outside the (0, 1) bounds of probabilities.

- Include your code (i.e., all the relevant files that one would need to **EASILY** replicate your results).

Solutions: The steady-state of the model is described in Table 2.

c/gdp	k/gdp	n	v	θ	s	w	r	k^h	k^f	lfp
0.71	11.95	0.68	0.11	0.87	0.13	2.32	0.03	0.53	0.61	0.74

Table 2. Steady-state values.

Defining the state vector as $x_t = [k_t, n_t^{LAG}, z_t, g_t]^T$ and the costate vector as $y_t = [c_t, n_t, v_t, \theta_t, s_t, w_t, r_t, k_t^h, k_t^f, lfp_t]^T$ (note that another element, lfp_t , was added to the originally suggested costate vector), the g_x and h_x matrices are

$$g_x = \begin{pmatrix} 0.0340 & 0 & 0.4467 & -0.5213 \\ 0 & 0 & 0.0948 & 0.0155 \\ 0.0003 & 0 & 0.1912 & 0.0248 \\ 0.0063 & 0 & 0.6358 & -0.0079 \\ -0.005 & 0 & 0.1269 & 0.0297 \\ 0.0244 & 0 & 2.4965 & -0.0257 \\ -0.0004 & 0 & 0.0285 & 0.0004 \\ 0.0021 & 0 & 0.2124 & -0.0026 \\ -0.0020 & 0 & -0.2050 & 0.0025 \\ -0.0005 & & 0.1268 & 0.0297 \end{pmatrix} \text{ and } h_x = \begin{pmatrix} 0.9716 & 0 & 2.5117 & -0.4208 \\ 0 & 0 & 0.0948 & 0.0155 \\ 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0.97 \end{pmatrix}$$

Figure 1 and Figure 2 provide impulse response functions to both a one-time, one-standard shock positive shock to TFP and a one-time, one-standard shock positive shock to government spending.

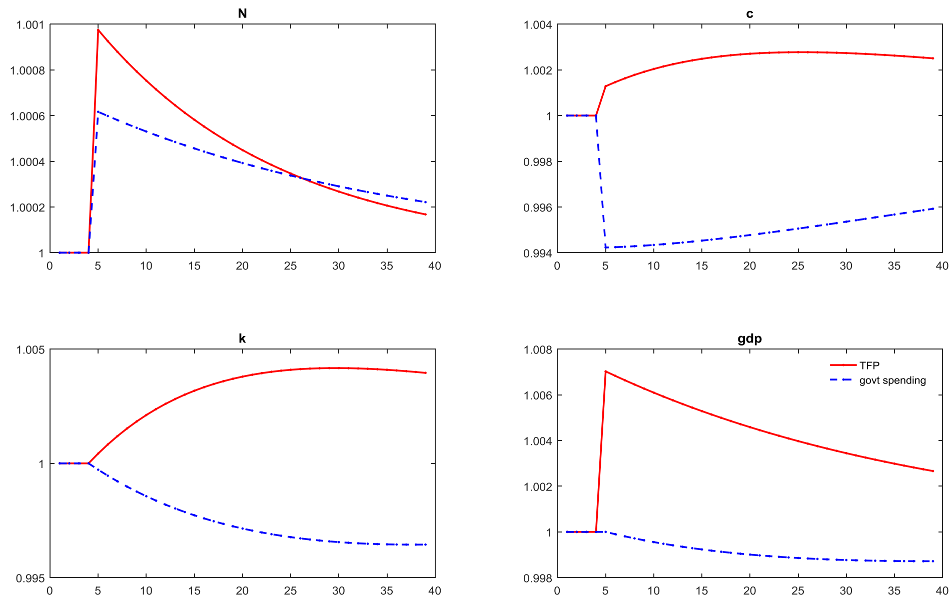

Figure 1. Impulse responses after one-time positive TFP shock (red lines) and one-time positive government spending shock (dashed blue lines).

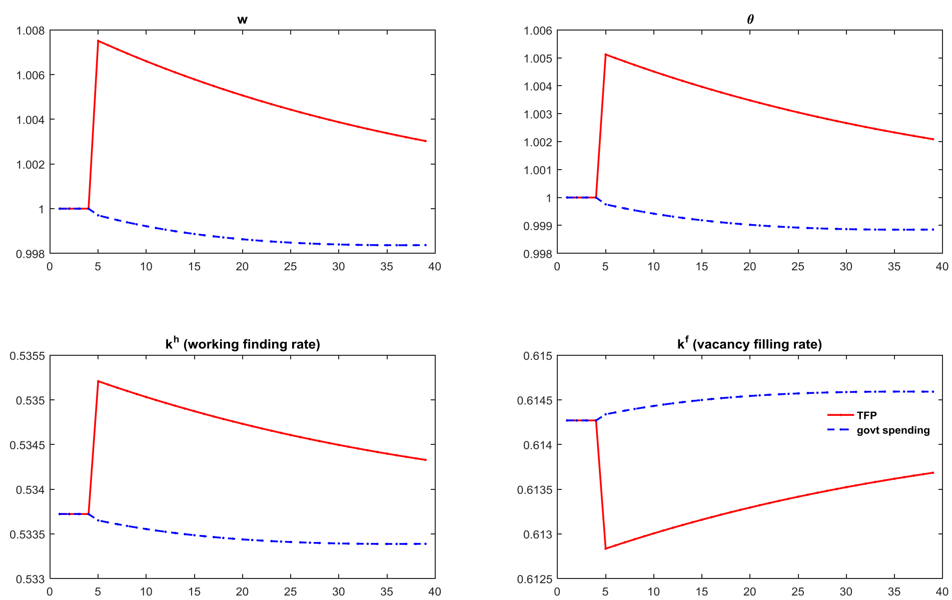

Figure 2. Impulse responses after one-time positive TFP shock (red lines) and one-time positive government spending shock (dashed blue lines).