Department of Applied Economics Johns Hopkins University

Economics 602

Macroeconomic Theory and Policy

Final Exam Suggested Solutions

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NAME: ______________________________

The Exam has a total of four (4) problems and pages numbered one (1) through twelve (12). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use two pages (double-sided) of notes. You may **not** use a calculator.

Problem 1	**/ 25**
Problem 2	**/ 20**
Problem 3	**/ 15**
Problem 4	**/ 40**
TOTAL	**/ 100**

Problem 1: Consumption and Savings in the Two-Period Economy (25 points). Consider a two-period economy in which the government collects only lump-sum taxes from the representative consumer, and in which the representative consumer has no control over his pre-tax income. The lifetime utility function of the representative consumer is $u(c_1, c_2) = \ln c_1 + \ln c_2$, where, as usual, $\ln(.)$ stands for the natural logarithm. We will work here in purely real terms: suppose the consumer's **present discounted value of ALL lifetime REAL pre-tax income is 26, and the present discounted value of ALL lifetime tax payments is 6.** Suppose that the real interest rate between period 1 and period 2 is zero (i.e., r = 0), and also suppose the consumer begins period 1 with zero net assets.

a. **(17 points)** Set up the lifetime Lagrangian formulation of the consumer's problem, in order to answer the following: i) is it possible to numerically compute the consumer's optimal choice of consumption in period 1? If so, compute it; if not, explain why not. ii) is it possible to numerically compute the consumer's optimal choice of consumption in period 2? If so, compute it; if not, explain why not. iii) is it possible to numerically compute the consumer's real asset position at the end of period 1? If so, compute it; if not, explain why not.

Solution: We know that with zero initial assets, the LBC of the consumer is

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2}{1+r} - \frac{t_2}{1+r},$$

in which the right hand side is the present value of lifetime **disposable** (i.e., after-tax) income (the notation is standard from Chapter 7). The Lagrangian is thus

$$u(c_1, c_2) + \lambda\left[y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r} - t_1 - \frac{t_2}{1+r}\right],$$

where λ of course is the Lagrange multiplier (note there's only one multiplier since this is the lifetime formulation of the problem, not the sequential formulation of the problem). The first-order conditions with respect to c_1 and c_2 (which are the objects of choice) are, as usual:

$$u_1(c_1, c_2) - \lambda = 0$$

$$u_2(c_1, c_2) - \frac{\lambda}{1+r} = 0$$

(And of course the FOC with respect to the multiplier just gives back the LBC.) Also as usual, these FOCs can be combined to give the consumption-savings optimality condition, $\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} = 1 + r$. With the given utility function, the marginal utility functions are $u_1 = 1/c_1$ and $u_2 = 1/c_2$, so the consumption-savings optimality condition in this case becomes $c_2 / c_1 = 1 + r$. This can be rearranged to give $c_2 = (1+r)c_1$, which we can then insert in the LBC to give $c_1 + c_1 = y_1 - t_1 + \frac{y_2}{1+r_1} - \frac{t_2}{1+r_1}$ (no, that's not a typo, it's $c_1 + c_1$ after the substitution…).

In this problem, you are given neither y_1 nor y_2. Instead, what you are given is $y_1 + \frac{y_2}{1+r_1} = 26$, **and** $t_1 + \frac{t_2}{1+r} = 6$. Thus, we have that the optimal quantity of period-1 consumption is $c_1^* = 10$ (which solves part i). We can **not** compute c_2^*, however, because we are not given the interest

Problem 1 continued

rate r (which you would need in order to use the expression $c_2 = (1+r)c_1$ computed above. (This solves part ii). To compute the asset position at the end of period 1, we would need to compute $y_1 - c_1^* - t_1$, but because we know neither y_1 nor t_1, we cannot compute this either (which solves part iii).

b. **(8 points)** To demonstrate how important the concept of the real interest rate is in macroeconomics, an interpretation of it (in addition to the several different interpretations we have already discussed in class) is that it reflects the rate of consumption growth between two consecutive periods. **Using only the consumption-savings optimality condition for the given utility function, briefly describe/discuss whether the real interest rate is positively related to, negatively related to, or not at all related to the rate of consumption growth between period one and period two.** For your reference, the definition of the rate of consumption growth rate between period 1 and period 2 is $\frac{c_2}{c_1} - 1$ (completely analogous to, say, the definition of the rate of growth of prices between period 1 and period 2). (**Note:** No mathematics are especially required for this problem; also note this part can be fully completed even if you were unable to get all the way through part a).

Solution:

The familiar consumption-savings optimality condition is $\frac{u_1}{u_2} = 1 + r$. As we just saw above, for the given utility function, this becomes $\frac{1/c_1}{1/c_2} = 1 + r$, or, rewriting,

$$\frac{c_2}{c_1} = 1 + r .$$

The left-hand-side of this expression obviously measures the consumption growth rate between period 1 and period 2. That is, if c_1 = 100 and c_2 = 103, clearly the consumption growth rate is 3 percent between period 1 and period 2. Which would mean that r = 0.03. If the real interest rate were instead larger, clearly the left-hand-side, c_2/c_1, would be larger as well. Thus, **the higher is the real interest rate, the higher is the consumption growth rate between periods – the real interest rate and the consumption growth rate are positively related to each other.**

Note that simply arguing/explaining here that a rise in the real interest rate leads to a fall in period-1 consumption does not address the question – the question is about the **rate of change of consumption between period 1 and period 2,** not about the **level** of consumption in period 1 by itself.

2. Monetary Policy in the MIU Framework (20 points). In this question, you will analyze, using indifference curve/budget constraint diagrams, the implications of alternative nominal interest rates on the representative consumer's choices of consumption and **real** money balances.

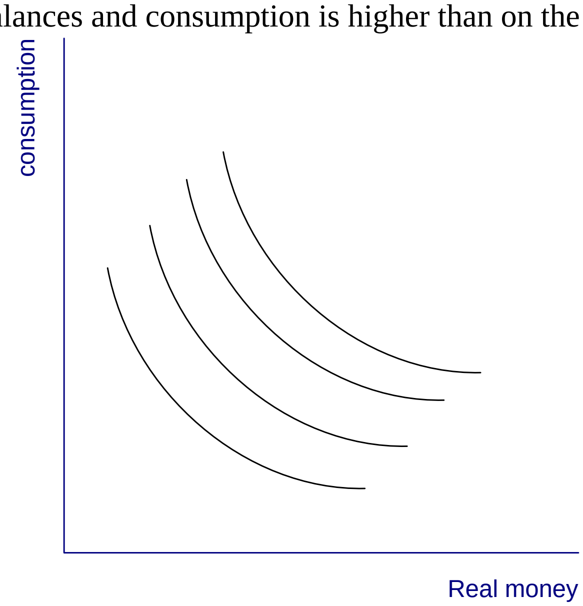
Recall that, with the period-*t* utility function $u(c_t, M_t / P_t)$ (where, as usual, c_t denotes consumption and M_t / P_t denotes **real** money balances), the consumption-money optimality condition (which was derived in Chapter 14) can be expressed as

$$\frac{u_{m_t}(c_t, M_t / P_t)}{u_{c_t}(c_t, M_t / P_t)} = \frac{i_t}{1+i_t},$$

where, again as usual, i_t is the nominal interest rate, $u_c(.)$ denotes the marginal utility of consumption, and $u_m(.)$ denotes the marginal utility of **real** money balances.

a. **(6 points)** Suppose the central bank is considering setting one of two (and only two) **positive** nominal interest rates: i_t^1 and i_t^2, with $i_t^2 > i_t^1$. On the indifference map below, qualitatively (and clearly) sketch relevant budget lines and show the consumer's optimal choices of consumption and real money under the two alternative policies. **On the diagram below, note the point on the vertical axis marked "FIXED" – this denotes a point that must lie on EVERY budget constraint. Clearly label your diagram, including the slopes of the budget lines.**

Solution: Examining the right-hand-side of the above, it is clear that the smaller is i, the flatter is the budget line. Starting from the FIXED point, draw two budget lines, with the budget line with slope *i1* flatter than the budget line with slope *i2*. On the flatter budget line, the consumer's optimal choice of money balances and consumption is higher than on the steeper budget line.



Problem 2 continued

b. **(7 points)** You are a policy adviser to the central bank, and any advice you give is based only on the goal of maximizing the utility of the representative consumer. The central bank asks you to help it choose between the two nominal interest rates i_t^1 and i_t^2 (and only these two). Referring to your work in the diagram above, which nominal interest rate would you recommend implementing? **Briefly** explain.

Solution: Again as is clear from the diagram, choosing the smaller value of i allows the representative consumer to attain a higher level of utility (a higher indifference curve), so i^1 is preferred to i^2.

c. **(7 points)** Suppose instead the central bank is willing to consider setting any nominal interest rate, not just either i_t^1 or i_t^2. What would your policy recommendation be? **Briefly** justify your recommendation, **and also in the diagram in part (a) sketch and clearly label a new budget line consistent with your policy recommendation.**

Solution: Setting $i = 0$ (or, technically speaking, very very very close to zero) would make the budget line completely flat, and allow the consumer to obtain the highest possible utility. Note that, because indifference curves are downward sloping, if $i < 0$, then there would not be a point of tangency between the budget line and an indifference curve – there would no equilibrium. (Indeed, $i = 0$ is the lowest that nominal interest rates can ever go (something known as the "zero-lower-bound" on interest rates – were they to go lower, a monetary economy (ie, one in which money is used as a medium of exchange) would not exist. A topic for a more advanced course in monetary economics.)

Problem 3: A National Service Program (15 points). Consider the following radical policy proposal: rather than taxes being levied on individuals and the proceeds of those taxes being used by the government to fund various programs, suppose that every individual pays no taxes of any kind, but instead must give ten hours of his time every week to national service. You are to analyze this national service program in the context of the (one-period) consumption-leisure framework. Thus, there are now **three** uses of the individual's time: work, leisure, and national service (the mandatory 10 hours). **Assume the following:**

- Instituting the national service program has no effect on any prices or wages in the economy.
- Any time spent voluntarily performing national service beyond the required 10 hours is considered leisure.

a. **(8 points)** Using the notation developed in Chapter 2 (i.e., c to denote consumption, n to denote hours of work per week, l to denote hours of leisure per week, P to denote the nominal price of consumption, and W to denote the nominal hourly wage), construct the representative agent's (weekly) budget constraint in this model with a national service program. Recall that there are 168 hours in one calendar week. Provide brief economic justification for your work.

Solution: The individual is required to give 10 hours per week to national service. Thus he has (168-10)=158 hours per week left to allocate to either labor or leisure. Proceeding completely analogously as in the standard model, then, we finally arrive at the budget constraint

$$Pc + Wl = 158W .$$

(Refer to Chapter 2 of the Lecture Notes for the full algebraic derivation: the derivation here is identical except that $t = 0$ by assumption and 168 is replaced by 158).

Problem 3 continued

b. **(7 points)** Now recall the standard consumption-leisure framework with no national service program. Suppose that both the consumption tax rate is zero and the labor tax rate is zero. How does the slope of the budget constraint in this economy compare with the slope of the budget constraint in the economy with the national service program in part a? Provide brief economic explanation.

Solution: With no taxes and no national service program, the budget constraint is $Pc + Wl = 168W$. Comparing this with the budget constraint in part a above, we see that the slopes of the two budget constraints are identical – the two constraints only differ in their intercepts. The constraint in part a has intercepts which are smaller than the constraint here in part b. The intuition is that in the consumption-leisure model the individual has **time** to allocate between working and not working. The national service program takes away some of this time but otherwise has no effect on the real (inclusive of taxes) wage W / P.

Problem 4: Financing Constraints and Housing Markets (40 points). Consider an enriched version of the two-period consumption-savings framework from Chapters 3 and 4, in which the representative individual not only makes decisions about consumption and savings, but also housing purchases. For this particular application, it is useful to interpret "period 1" as the "young period" of the individual's life, and interpret "period 2" as the "old period" of the individual's life.

In the young period of an individual's life, utility depends only on period-1 consumption c_1. In the old period of an individual's life, utility depends both on period-2 consumption c_2, as well as his/her "quantity" of housing (denoted h).[1] From the perspective of the beginning of period 1, the individual's lifetime utility function is

$$\ln c_1 + \ln c_2 + \ln h,$$

in which ln(.) stands for the natural log function; the term $\ln h$ indicates that people directly obtain happiness from their housing.

Due to the "time to build" nature of housing (that is, it takes time to build a housing unit), the representative individual has to incur expenses **in his/her young period** to purchase housing **for his/her old period.** The **real** price in period 1 (i.e., measured in terms of period-1 consumption) of a "unit" of housing (again, think of a unit of housing as square footage) is p_1^H, and the **real** price in period 2 (i.e., measured in terms of period-2 consumption) of a unit of housing is p_2^H.

In addition to housing decisions, the representative individual also makes **stock** purchase decisions. The individual begins period 1 with **zero** stock holdings ($a_0 = 0$), and ends period 2 with **zero** stock holdings ($a_2 = 0$). How many shares of stock the individual ends period 1 with, and hence begins period 2 with, is to be optimally chosen. The **real** price in period 1 (i.e., measured in terms of period-1 consumption) of each share of stock is s_1, and the **real** price in period 2 (i.e., measured in terms of period-2 consumption) of each share of stock is s_2. For simplicity, suppose that stock **never** pays any dividends (that is, dividends = 0 always).

Because housing is a big-ticket item, the representative individual has to accumulate financial assets (stock) while young to overcome the informational asymmetry problem and be able to purchase housing. Suppose the **financing constraint** that governs the purchase of housing is

$$\frac{p_1^H h}{R^H} = s_2 a_1$$

(technically an inequality constraint, but we will assume it always holds with strict equality). In the financing constraint, $R^H > 0$ is a government-controlled **"leverage ratio"** for housing. **Note well the subscripts on variables that appear in the financing constraint.**

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[1] For concreteness, you can think of "quantity" of housing as the square footage and/or the "quality" of the housing space.

Problem 4 continued

Finally, the **real** quantities of income in the young period and the old period are y_1 and y_2, over which the individual has no choice.

The **sequential Lagrangian** for the representative individual's problem lifetime utility maximization problem is:

$$\text{Lagrangian} = \ln c_1 + \ln c_2 + \ln h + \lambda_1[y_1 - c_1 - s_1 a_1 - p_1^H h]$$
$$+ \lambda_2\left[y_2 + s_2 a_1 + p_2^H h - c_2\right] + \mu\left[s_2 a_1 - \frac{p_1^H h}{R^H}\right],$$

in which μ is the Lagrange multiplier on the financing constraint, and λ_1 and λ_2 are, respectively, the Lagrange multipliers on the period-1 and period-2 budget constraints.

a. **(5 points) In no more than two brief sentences/phrases,** qualitatively describe what an **informational asymmetry** is, and why it can be a serious problem in financial transactions.

Problem 4 continued

b. **(5 points) In no more than three brief sentences/phrases,** qualitatively describe the role that the leverage ratio R^H plays in the "housing finance" market. In particular, briefly describe/discuss what higher leverage ratios imply for the individual's ability to finance a house purchase (i.e., "obtain a mortgage").

c. **(5 points)** Based on the sequential Lagrangian presented above, compute the **two** first-order conditions: with respect to a_1 and h. (You can safely ignore any other first-order conditions.)

Solution: The two FOCs are:

$$-\lambda_1 s_1 + \lambda_2 s_2 + \mu s_2 = 0$$

$$\frac{1}{h} - \lambda_1 p_1^H + \lambda_2 p_2^H - \frac{\mu p_1^H}{R^H} = 0$$

Problem 4 continued

d. **(5 points)** Based on the first-order condition with respect to *h* computed in part c, solve for the period-1 real price of housing p_1^H (that is, your final expression should be of the form $p_1^H = ...$ where the term on the right hand side is for you to determine). **(Note: you do NOT have to eliminate Lagrange multipliers from the final expression.)**

Solution: Rearranging the FOC on *h* computed above, we have, after two steps of algebra,

$$p_1^H = \frac{\frac{1}{h} + \lambda_2 p_2^H}{\lambda_1 + \frac{\mu}{R^H}} = \frac{1 + \lambda_2 p_2^H h}{\mu + \lambda_1 R^H} \cdot \frac{R^H}{h}$$

You did not need to write the expression in the second (far right) form, but it is of course mathematically fine if you did, **HOWEVER,** note that if you did write it this way, you had to recognize in part e that if $\mu = 0$, then R^H **does not appear** in the house price expression.

e. **(5 points)** Based on the expression for p_1^H computed in part d, **and assuming that the Lagrange multiplier μ > 0 (recall, furthermore, that R^H > 0),** answer the following: is the period-1 price of housing **larger than or smaller than** what it would be if financing constraints for housing were **not at all an issue?** Or is it impossible to determine? **Carefully** explain the logic of your argument/analysis, and provide brief economic interpretation of your conclusion.

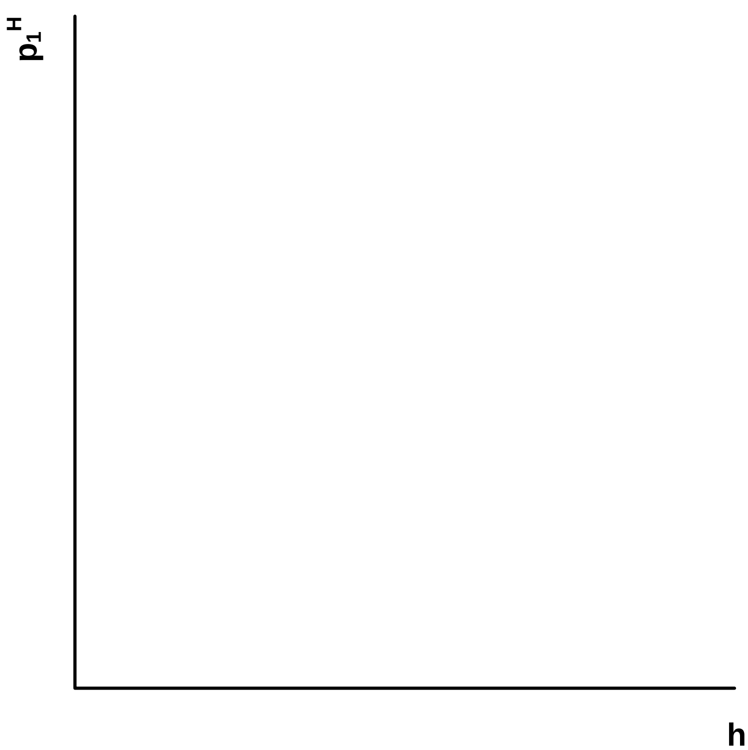
Solution: If financing constraints do not matter for housing purchases, we would have $\mu = 0$. Relative to an economy in which $\mu = 0$, the **first** expression obtained in part d clearly shows (holding all else constant) that if $\mu > 0$, the price of housing p_1^H **is lower** (and, again, if $\mu = 0$, then the leverage ratio R^H simply does not appear in the house price expression at all). The economic intuition is simply the informational asymmetries that impinge on housing purchases (or, more precisely, the loan/mortgage that must be taken out to finance a housing purchase), which limit the quantity that an individual can borrow and hence limit the size/value of housing purchases an individual can make (relative to an economy in which informational asymmetries are not present or do not have any effect whatsoever, which is the $\mu = 0$ case).

Problem 4 continued

For the remainder of this problem (i.e., for parts f, g, and h), suppose that $\lambda_1 = \lambda_2 = 1$**.**

f. **(7 points)** Consider the period-1 housing market, as depicted in the diagram below, which shows the quantity *h* of housing drawn on the horizontal axis and the period-1 price, p_1^H, of housing drawn on the vertical axis. Using the house-price expression computed in part d, qualitatively sketch the relationship between *h* and p_1^H that it implies. Your sketch should make clear whether the relationship is upward-sloping, downward-sloping, perfectly horizontal, or perfectly vertical. Clearly present the algebraic/logical steps that lead to your sketch, and clearly label your sketch.

Solution: With $\lambda_1 = \lambda_2 = 0$, the house price expression computed in part d simplifies a bit, to $p_1^H = \dfrac{\frac{1}{h} + p_2^H}{1 + \frac{\mu}{R^H}}$. In the space of (p_1^H, h) shown below, simple inspection shows that this defines a downward sloping (and convex, but you did not need to take the analysis this far) relationship. This is the **housing demand function.**



Problem 4 continued

g. **(4 points)** In the **same** sketch in part f, clearly show and label what happens if p_2^H rises. (Examples of what could "happen" are that the relationship you sketched rotates, or shifts, or both rotates and shifts, etc.) Explain the logic behind your conclusion, and provide brief economic interpretation of your conclusion.

Solution: Inspection of the house-price expression in part f clearly shows there is a positive relationship between p_1^H and p_2^H. This is true for any given *h*, hence the entire housing demand function shifts to the right if p_2^H rises. The economics is that, because housing is an asset that has market value (in that respect, exactly like stock), if the price of the asset is going to be higher in the future (period 2), that makes it more attractive to purchase in period 1, hence demand for it rises (shifts).

h. **(4 points)** In the **same** sketch in part f, clearly show and label what happens if R^H rises. (Examples of what could "happen" are that the relationship you sketched rotates, or shifts, or both rotates and shifts, etc.) Explain the logic behind your conclusion, and provide brief economic interpretation of your conclusion.

Solution: Inspection of the house-price expression in part f clearly shows there is a positive relationship between p_1^H and R^H. This is true for any given *h*, hence the entire housing demand function shifts to the right if R^H rises. The economics is that a rise in R^H allows a larger housing purchase (in either house size or price or both) for a given market value of financial assets, $S_2 a_1$. Thus, as the allowed "leverage ratio" for housing rises, demand for it rises (shifts).

END OF EXAM