Department of Applied Economics Johns Hopkins University

Economics 602

Macroeconomic Theory and Policy

Midterm Exam – Suggested Solutions

Professor Sanjay Chugh

Fall 2010

NAME: ____

The Exam has a total of four (4) problems and pages numbered one (1) through six (6). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

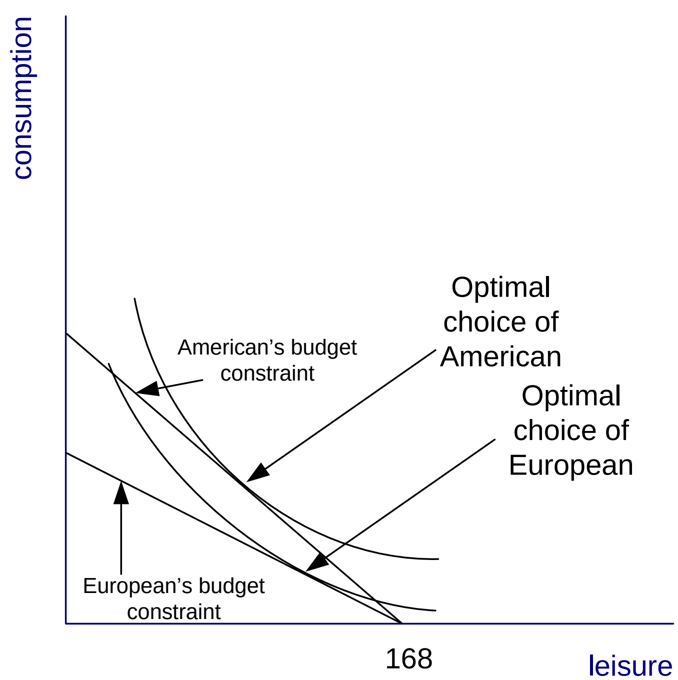
You may use one page (double-sided) of notes. You may **not** use a calculator.

Problem 1	**/ 20**
Problem 2	**/ 30**
Problem 3	**/ 25**
Problem 4	**/ 25**
TOTAL	**/ 100**

Problem 1: European and U.S. Consumption-Leisure Choices (20 points). Europeans work fewer hours than Americans. There are likely very many possible reasons for this, and indeed in reality this fact arises from a combination of many reasons. In this question, you will consider two reasons using the simple (one-period) consumption-leisure model.

a. **(10 points)** Suppose that both the utility functions and pre-tax real wages W/P of American and European individuals are identical. However, the labor income tax rate in Europe is higher than in America. In a **single** carefully-labeled indifference-curve/budget constraint diagram (with consumption on the vertical axis and leisure on the horizontal axis), show how it can be the case that Europeans work fewer hours than Americans. Provide any explanation of your diagram that is needed.

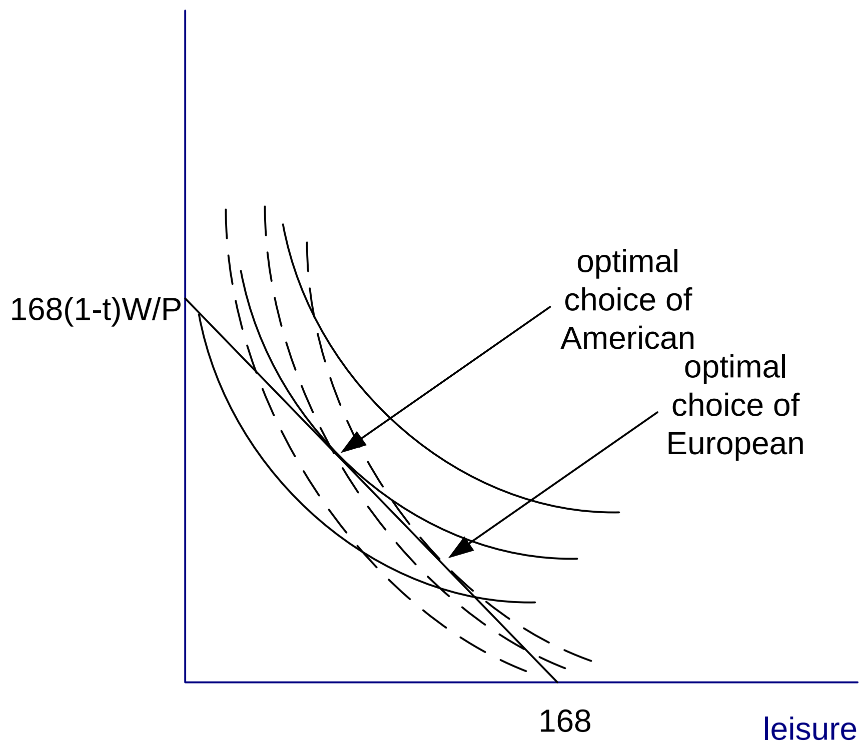
Solution: If Europeans work fewer hours than Americans, then Europeans have more leisure time than Americans, simply because (in our weekly framework) $n+l=168$. Europeans and Americans have identical utility functions, which means that their indifference maps are identical. This means that the difference in hours worked must arise completely from differences in their budget constraints. With a higher labor income tax in Europe, the budget constraint of the European consumer is less steep than the budget constraint of the American, as the diagram below shows (because the slope of the budget constraint is $(1-t)W/P$, and you are given that W/P is the same in the two countries). The diagram shows that the European optimally chooses more leisure (hence less labor) and less consumption than the American. Here, the difference between Europeans and Americans is solely in the relative prices (embodied by the slope of the budget constraint) they face. (For full credit here, you had to somehow make clear that the indifference maps of the representative European and the representative American are identical.)



Problem 1 continued.

b. **(10 points)** Suppose that both the pre-tax real wages W/P and the labor tax rates imposed on American and European individuals are identical. However, the utility function $u^{AMER}(c,l)$ of Americans differs from that of Europeans $u^{EUR}(c,l)$. In a **single** carefully-labeled indifference-curve/budget constraint diagram (with consumption on the vertical axis and leisure on the horizontal axis), show how it can be the case that Europeans work fewer hours than Americans. Provide any explanation of your diagram that is needed.

Solution: In this case, the budget constraints of the European consumer and American consumer are identical, so the difference in hours worked must arise completely from differences in their utility functions. Graphically, this means that the two types of consumers have different indifference maps (i.e., a different set of indifference curves). In the diagram below, the budget line is the common budget line of the European and the American. The solid indifference curves are the American's, while the dashed indifference curves are the European's. With steeper indifference curves, the European's optimal choice along the same budget line must occur at a point that features more leisure (hence less labor) and less consumption than the American's optimal choice. Here, the difference between Europeans and Americans is solely in their preferences.



umption

Problem 2: Patience and the Dynamics of Stock Prices and Consumption (30 points). Suppose the economy is in a steady state at the start of the year 2040. The steady-state level of consumption prior to the start of the year 2040 is c^{SS}, and suppose that the economy has been in this steady state for four years. Thus, $c^{SS} = c_{2039} = c_{2038} = c_{2037} = c_{2036}$. Furthermore, suppose that the steady-state **real** interest rate in the four years prior to the start of the year 2040 is $r^{SS} > 0$.

Perhaps due to several years of economic tranquility, suppose that at the start of the year 2040, the representative consumer **becomes more patient than he used to be before 2040. Furthermore, it is not until the start of the year 2040 that the representative consumer understands that he has become more patient** (thus, the consumer never "anticipated" anytime prior to 2040 that he would "become more patient" in the year 2040).

Denote the representative consumer's subjective discount factor from the year 2040 onwards as β, which, as just described, is a different value than it used to be before 2040; denote the subjective discount factor in the pre-2040 period as β^{PRE}. Despite the change in the representative consumer's patience, both β and β^{PRE} are numbers strictly between zero and one.

In both the pre-2040 and post-2040 periods, the representative consumer's utility function in each period is $u(c_t) = \ln c_t$. If we view each time period as being one year, then, starting from the beginning of period 2040 (i.e., the year 2040), the representative consumer's lifetime utility function is

$$\ln c_{2040} + \beta \ln c_{2041} + \beta^2 \ln c_{2042} + \beta^3 \ln c_{2043} + ...$$

For simplicity, suppose that the nominal price of consumption is **always one in every time period** (that is, $... = P_{2037} = P_{2038} = P_{2039} = P_{2040} = P_{2041} = P_{2042} = ... = 1$ **forever**), and the nominal dividend paid on each share of stock is **always zero in every time period** (that is, $... = D_{2037} = D_{2038} = D_{2039} = D_{2040} = D_{2041} = D_{2042} = ... = 0$ **forever**). The budget constraints faced by the representative consumer starting from the year 2040 are thus

$$\begin{aligned}
&c_{2040} + S_{2040}a_{2040} = Y_{2040} + S_{2040}a_{2039} \\
&c_{2041} + S_{2041}a_{2041} = Y_{2041} + S_{2041}a_{2040} \\
&c_{2042} + S_{2042}a_{2042} = Y_{2042} + S_{2042}a_{2041} \\
&...
\end{aligned}$$

and so on in subsequent years. The rest of the notation is as in Chapter 8: a_t denotes the consumer's stock holdings at the end of a given year t, Y_t denotes the consumer's nominal income during a given year t, and S_t denotes the per-share nominal price of stock during a given year t.

(OVER)

Problem 2 continued

a. (2 points) In no more than one sentence/phrase, define/describe an economic steady state.

Solution: An economic steady state is a condition in which all real (though not necessarily nominal) measures are unchanging from one time period to the next.

b. (6 points) Define the **rate of stock price growth between the years 2038 and 2039** as $\frac{S_{2039}}{S_{2038}}-1$. Was the rate of stock price growth between the years 2038 and 2039 positive, negative, zero, or is it impossible to determine? Carefully justify your answer.

Solution: We know the consumption savings optimality condition from the infinite-period framework can be expressed as $\frac{u'(c_t)}{\beta u'(c_{t+1})} = \left(\frac{S_{t+1}+D_{t+1}}{S_t}\right)\frac{P_t}{P_{t+1}}$ between any two consecutive time periods t and t+1. In this problem, we have that nominal dividends are always zero and the nominal price of consumption is always zero. Viewing the two periods t and t+1 as being 2038 and 2039, as you are asked about here, the consumption-savings optimality condition between those two years can be expressed as $\frac{u'(c_{2038})}{\beta^{PRE}u'(c_{2039})} = \frac{S_{2039}}{S_{2038}}$. Consumption is the same in the years 2038 and 2039, so the $u'(.)$ terms cancel, which leaves us with

$$\frac{1}{\beta^{PRE}} = \frac{S_{2039}}{S_{2038}}$$

from which it readily follows that the ratio $\frac{S_{2039}}{S_{2038}} > 1$ because (as you are told above) $\beta^{PRE} < 1$. Hence, the rate of stock price growth between 2038 and 2039 is $\frac{S_{2039}}{S_{2038}} - 1 > 0$,

c. (4 points) As described above, the representative consumer is more patient starting in 2040 (and beyond) than before 2040. In terms of the subjective discount factors β and β^{PRE}, does this mean that $\beta < \beta^{PRE}$, $\beta > \beta^{PRE}$, $\beta = \beta^{PRE}$, or is it impossible to tell how β compares to β^{PRE}?

Solution: By definition (from Chapter 8), the lower is the subjective discount factor, the more patient are consumers. Hence, if consumers all of a sudden become more patient in 2040, the subjective discount factor **increased** in 2040, i.e., $\beta > \beta^{PRE}$.

Problem 2 continued

d. **(6 points)** Regardless of any events that happen in the year 2040 or the several years following 2040, suppose that **many** years after the year 2040, the economy is once again in a steady state. In this **eventual post-2040 steady state,** is the rate of stock price growth from one year to the next positive, negative, zero, or is it impossible to determine? Carefully justify your answer.

Solution: By exactly the same logic as in part b, we can conclude that $\frac{1}{\beta} = \frac{S_{t+1}}{S_t}$ in the eventual post-2040 steady state. Because $\beta < 1$, we have that $\frac{S_{t+1}}{S_t} > 1$ and hence the rate of stock price growth from any given year to the next in the eventual post-2040 steady state is $\frac{S_{t+1}}{S_t} - 1 > 0$.

e. **(6 points)** Is the rate of stock price growth you analyzed in part d larger than, smaller than, or equal to the rate of stock price growth between the years 2038 and 2039 you analyzed in part b? Or is it impossible to determine? Carefully justify your answer.

Solution: Answering this requires only comparing $\frac{1}{\beta}$ vs. $\frac{1}{\beta^{PRE}}$. By implication of what we concluded in part c, we have $\frac{1}{\beta} < \frac{1}{\beta^{PRE}}$. Putting this conclusion together with the results and analysis of parts b and e, we have that the rate of stock price growth in the eventual post-2040 steady state is **smaller than** the rate of stock price growth in the pre-2040 steady state. This is simply a statement that the real interest rate in the eventual post-2040 steady state is smaller than in the pre-2040 steady state, from the relation that the inverse of the subjective discount factor equals (one plus) the real interest rate; this latter, general, result, has nothing to do with "which" steady state is being examined.

f. **(Harder – 6 points)** In the **eventual post-2040 steady state** (i.e., many years after 2040), is consumption larger than, smaller than, or equal to consumption in the steady state prior to the year 2040? Or is it impossible to determine? Carefully justify your answer.

Solution: With a higher degree of patience (higher β), individuals are (all else equal) more willing to postpone consumption purchases until later, which implies higher savings (a flow). Eventually (i.e., in the eventual post-2040 steady state), the higher **accumulated wealth** (a stock) will indeed allow higher consumption. So, eventually, consumption will rise, even if it may fall during some temporary transition period in the years immediately after 2040.

Problem 2f continued (if you need more space)

Problem 3: Two-Period Economy (25 points). Consider a two-period economy (with no government and hence no taxes), in which the representative consumer has no control over his income. The lifetime utility function of the representative consumer is $u\left(c_1, c_2\right) = \ln c_1 + c_2$, where $\ln$ stands for the natural logarithm (that is not a typo – it is only c_1 that is inside a ln(.) function, c_2 is **not** inside a ln(.) function).

Suppose the following numerical values: the **nominal** interest rate is $i = 0.05$, the nominal price of period-1 consumption is $P_1 = 100$, the nominal price of period-2 consumption is $P_2 = 105$, and the consumer begins period 1 with zero net assets.

a. **(3 points)** Is it possible to numerically compute the **real** interest rate (r) between period one and period two? If so, compute it; if not, explain why not.

Solution: The inflation rate is easily computed as $\pi_2 = \frac{P_2}{P_1} - 1 = \frac{105}{100} - 1 = 0.05$. Then, using the exact Fisher equation, $1 + r = \frac{1+i}{1+\pi_2} = \frac{1.05}{1.05} = 1$, so that $r = 0$.

b. **(14 points)** Set up a **sequential** Lagrangian formulation of the consumer's problem, in order to answer the following: i) is it possible to numerically compute the consumer's optimal choice of consumption in period 1? If so, compute it; if not, explain why not. ii) is it possible to numerically compute the consumer's optimal choice of consumption in period 2? If so, compute it; if not, explain why not.

Solution: The sequential Lagrangian for this problem (here cast in real terms, but you could have case it in nominal terms as well) is

$$u(c_1, c_2) + \lambda_1[y_1 - c_1 - a_1] + \lambda_2[y_2 + (1+r)a_1 - c_2],$$

where λ_1 and λ_2 are the multipliers on the period-1 and period-2 budget constraints. The first-order condition with respect to c_1 is $u_1(c_1, c_2) - \lambda_1 = 0$, with respect to c_2 is $u_2(c_1, c_2) - \lambda_2 = 0$, and with respect to a_1 is $-\lambda_1 + \lambda_2(1+r) = 0$. The third FOC allows us to conclude $\lambda_1 = \lambda_2(1+r)$. Substituting this into the FOC on c_1 gives $u_1(c_1, c_2) = \lambda_2(1+r)$. Next, the FOC on c_2 allows us to obtain $\lambda_2 = u_2(c_1, c_2)$. Substituting this into the previous expression gives us $u_1(c_1, c_2) = u_2(c_1, c_2)(1+r)$, or $\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} = 1 + r$, which of course is the usual consumption-savings optimality condition. Using the given functional form, the consumption-savings optimality condition for this problem can be expressed as $\frac{1/c_1}{1} = 1 + r$, which immediately allows us to conclude $c_1 = \frac{1}{1+r} = \frac{1}{1} = 1$, which completes part i. However, c_2 **cannot** be computed here because you are given no numerical values regarding income, either in present-value or period-by-period form.

Problem 3continued

c. **(8 points)** The rate of consumption growth between period 1 and period 2 is defined as $\frac{c_2}{c_1}-1$ (completely analogous to how we have defined, say, the rate of growth of prices between period 1 and period 2). Using **only** the consumption-savings optimality condition for the **given** utility function, **briefly** describe/discuss **(rambling essays will not be rewarded)** whether the real interest rate is **positively related to, negatively related to, or not at all related to the rate of consumption growth between period one and period two.** (**Note:** No mathematics are especially required for this problem; also note this part can be fully completed even if you were unable to get all the way through part b).

Solution: The familiar consumption-savings optimality condition is $\frac{u_1(c_1,c_2)}{u_2(c_1,c_2)}=1+r$. As we just saw above, for the given utility function, this becomes $\frac{1/c_1}{1}=1+r$, or, rearranging,

$$c_1=\frac{1}{1+r}.$$

For the consumption-savings optimality condition associated with this particular utility function (which is **quasi-linear** in period-2 consumption), r seems to affect only the period-1 optimal choice of consumption and does **not** affect the growth rate of consumption across periods. Since you were asked to base your analysis on the consumption-savings optimality condition, the conclusion would thus be that r is not at all related to the rate of consumption growth for this utility function, instead affecting only the short-run level of consumption.

However, it is the case that in the full solution to the problem (i.e., using the consumption-savings optimality condition in tandem with the consumer's lifetime budget constraint to solve jointly for both short-run and long-run consumption), c_2 rises when r rises (to see this, substitute the consumption-savings optimality condition into the LBC, and solve for c_2). The fact that c_2 rises when r rises coupled with the result that c_1 falls when r rises means that indeed the consumption growth rate between period 1 and period 2 rises when r rises. You were not required to take the analysis this far since you were asked only to base the analysis on the consumption-savings optimality condition – however (and many answers ran into this difficulty), **if** you decided to take this route you had to take it correctly.

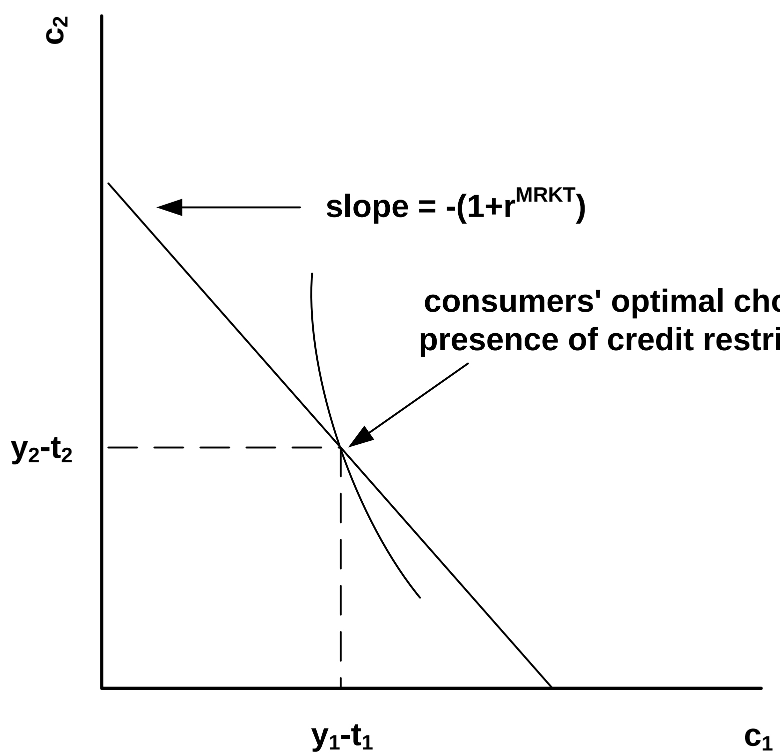
Many answers also simply discussed vaguely the consumption-savings optimality condition to argue something – you were told to base the analysis on the given utility function, so a general analysis did not address the issue.

Finally, note that simply arguing/explaining here that a rise in the real interest rate leads to a fall in period-1 consumption does not address the question – the question is about the **rate of change of consumption between period 1 and period 2,** not about the **level** of consumption in period 1 by itself.

Problem 4: The Credit Crunch and Government Loan Programs (25 points). Consider the two-period framework of fiscal policy from Chapter 7, in which both the representative consumer and the government live for the entire two periods. In real terms, the government spends g_1 and g_2 in periods 1 and 2, and collects from the representative consumer total tax revenues t_1 and t_2 (which are collected lump-sum). The **market** real interest rate is r^{MRKT}, which is the slope of the **consumer's LBC** shown in the diagram below. Note that the diagram below is NOT of the economy-wide resource frontier – for the analysis in this problem, you are to use the consumer's LBC.

There is a credit crunch going on, which **prevents consumers from borrowing from private-market lenders at all** during period 1. (If consumers could borrow from private market lenders, the real interest rate on private-market loans would be r^{MRKT}.) For simplicity, suppose that at the beginning of period 1, the representative consumer has zero net assets – that is, $a_0 = 0$. And, as usual in analysis of the two-period framework, assume that both the government and the representative consumer end period 2 with zero net assets – that is, $a_2 = 0$ and $b_2 = 0$.

Fiscal policy makers are considering various policy options to try to ease the consequences of the credit crunch. Suppose, perhaps for political reasons, that one option that is NOT being considered at all is changing government spending in either or both period 1 or period 2.



(OVER)

Problem 4 continued

a. **(4 points)** One option Congress is considering is to lower lump-sum taxes in period 1. Would this cause taxes in period 2 (t_2) to rise, decline, or remain unchanged? Or is it impossible to determine? Briefly explain (you may refer to the diagram above if necessary).

Solution: Simple examination of the government lifetime budget constraint allows us to conclude that, because neither g_1 nor g_2 is being altered, taxes in period two must rise so that the government's lifetime solvency condition holds.

b. **(6 points)** If Congress does enact the fiscal policy reform described in part a, would the economy's consumption in period 1 (c_1) rise, fall, or remain unchanged? Or is it impossible to determine? Briefly explain (you may refer to the diagram above if necessary).

Solution: The key was to recognize/argue that Ricardian equivalence does **not** hold in this problem, so consumption in period 1 **is** affected. Specifically, observation of the given diagram shows that consumers are **not** at their overall lifetime utility maximizing choice of (c1,c2) since there is no tangency between an indifference curve and the LBC at the point indicated in the diagram. By sketching in a few more indifference curves, it should be apparent that if consumers somehow could purchase more c1, their lifetime utility would be higher.

From the period 1 budget constraint of consumers, which is $c_1 + a_1 = y_1 - t_1$ **and given that a_1 = 0** (which follows from the credit constraint), we can conclude that a decrease in t_1 allows a higher value of c_1, which, from the logical argument of the previous paragraph, consumers would prefer.

An alternative proposal (besides the fiscal reform described in part a) being considered by Congress is to **directly lend to consumers.** Denote by L the quantity of **loans** that Congress would/could make directly to consumers in period 1 **(which are distinct from consumers' assets that are measured in the variables a_0, a_1, and a_2)**, and suppose that the government would be willing to charge a real interest r^{GOV} **lower** than would be available on private markets – that is, $r^{GOV} < r^{MRKT}$. If consumers did borrow from the government in period 1, they would have to repay these loans, inclusive of interest at the rate r^{GOV}, in period 2. The period-1 and period-2 budget constraints of the representative consumer and the government under this direct lending facility would read:

$$c_1 + a_1 = y_1 - t_1 + L$$
$$c_2 + a_2 + (1 + r^{GOV})L = y_2 - t_2 + (1 + r^{MRKT})a_1$$
$$g_1 + b_1 + L = t_1$$
$$g_2 + b_2 = t_2 + (1 + r^{GOV})L + (1 + r^{MRKT})b_1$$

Problem 4 continued

c. **(5 points)** In the diagram in the statement of the problem (above), **clearly and carefully sketch how the consumer's LBC is modified** by the introduction of the government loan program. Provide any (brief) explanation for your sketch that is required, and clearly **label** the element(s) you sketch. (**Hint:** before sketching the modified LBC, think about how the "usual" derivation of the LBC from Chapter 3 and 4 gets modified in this case?).

Solution: Given that $r^{GOV} < r^{MRKT}$, the way the diagram modifies is as follows: starting from the point $(y_1 - t_1, y_2 - t_2)$, the **lower-right segment of the LBC** becomes a straight line with slope $-(1+r^{GOV})$, rather than the shown lower-right segment of the LBC (which has slope $-(1+r^{MRKT})$. That is, there is a kink in the LBC induced by the government loan program: the LBC becomes flatter for values of c1 that imply **borrowing** during period 1 than for values of c1 that imply **saving** during period 1. This is shown in the diagram on the next page.

An intuitive/logical argument along the lines above was sufficient, but if you wanted to pursue this more mathematically, we can see this by deriving a "new LBC" using only the interest rate r^{GOV}. (i.e., redo the derivation of the LBC from Chapter 3 by using r^{GOV} in place of r^{MRKT}, and then recognize/argue that the only portion of THIS LBC that is relevant is IF consumers want to borrow during period 1, which is if the desired value of c_1 is larger than y_1-t_1).

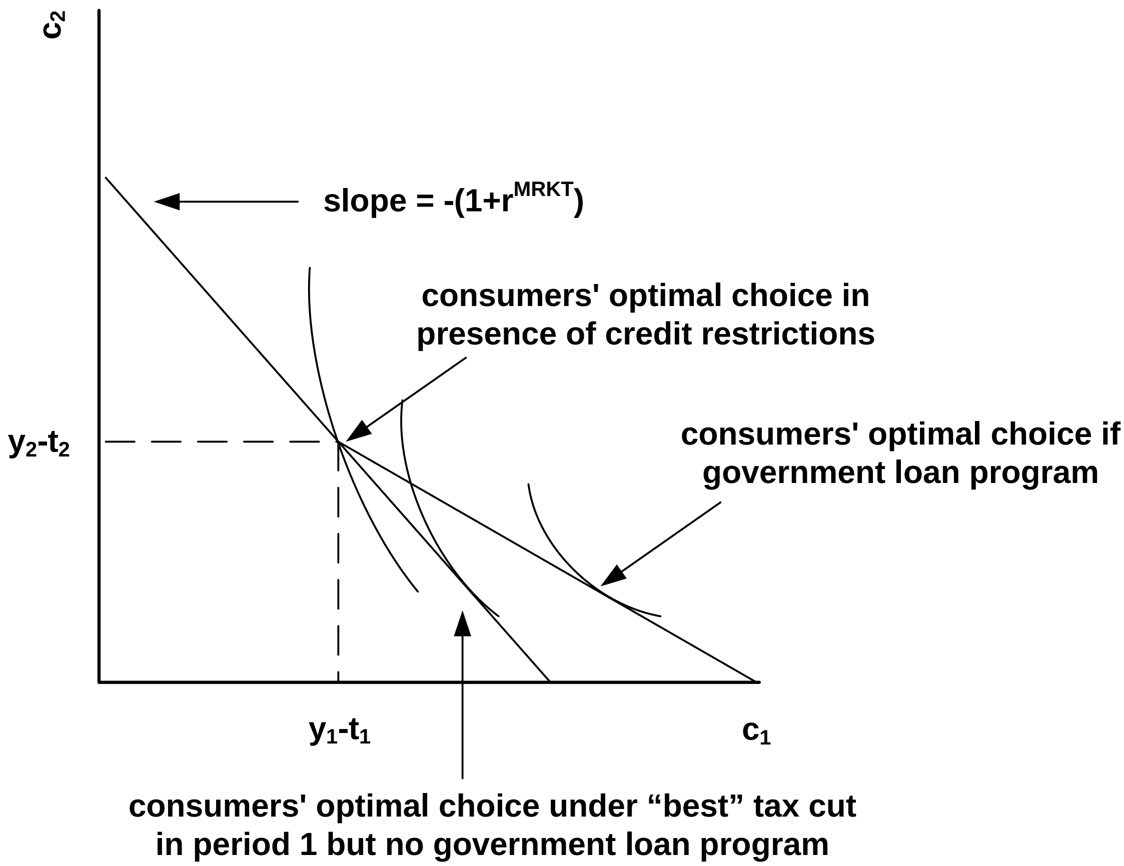
d. **(Harder – 10 points) Based only on your analysis in parts a, b, and c,** which of the two fiscal policy options (the tax reform of part b or the direct lending program of part c) would make the representative consumer better off in a lifetime utility sense (i.e., in terms of welfare)? Carefully describe the logic behind your conclusion, referring, if necessary, to the diagram above.

Solution: No matter whether you analyzed part c intuitively/logically or more mathematically, the main starting point for the analysis of this question was the result/observation that the "kinked" LBC is flatter in the lower-right segment than in the upper-left segment.

From the analysis/result in part b, we concluded that a tax cut in period 1 would increase c1, and hence increase lifetime utility. By how much lifetime utility is increases by a tax cut in period 1 depends on the magnitude of the tax cut. The diagram on the next page illustrates the outcome if the tax cut is sufficiently large that it allows consumers to reach their truly optimal (i.e., not constrained at all by credit restrictions) lifetime pattern of consumption. At this point, consumers have debt at the end of period 1, which they will have to repay in period 2 at the interest rate r^{MRKT}.

Compare this to the case of the government loan program at the **lower** interest rate $r^{GOV} < r^{MRKT}$. Given that we know that consumers would like to be able to borrow in period 1, borrowing at a lower rate is clearly better than borrowing at a higher rate. This is reflected in the fact that along the lower-right segment of the LBC that the government loan program induces, the optimal choice features an even higher level of lifetime utility than is attainable under the "best possible tax cut" but no government loan program. So the government loan program is clearly superior, from the point of view of enhancing consumers' utility, than the tax cut.

Problem 4d continued



END OF EXAM