Department of Applied Economics Johns Hopkins University

Economics 602

**Macroeconomic Theory**

**Midterm Exam – Suggested Solutions**

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**NAME:**

The Exam has a total of five (5) problems and pages numbered one (1) through fourteen (14). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use one page (double-sided) of notes. You may **not** use a calculator.

| | |
|---|---|
| **Problem 1** | **/ 20** |
| **Problem 2** | **/ 33** |
| **Problem 3** | **/ 22** |
| **Problem 4** | **/ 13** |
| **Problem 5** | **/ 12** |
| **TOTAL** | **/ 100** |

**Problem 1: Steady State Analysis (20 points).** Suppose that in the infinite-period consumer economy (with no government and hence no taxes at all), a steady state is achieved. Suppose that the beginning of planning horizon assets equals zero, the real interest rate is $r > 0$, the impatience factor is $\beta \in (0,1)$, and the asset that is being studied is something very general, that has (gross) interest rate $1+r$.

a. **(4 points)** In the steady state, the rate of consumption growth between two adjacent periods is defined as $\frac{c_{t+1}}{c_t} - 1$ (completely analogous to how we have defined, say, the rate of growth of prices between period 1 and period 2). **In the steady state, briefly** describe/discuss **(two sentences maximum – rambling essays will not be rewarded)** whether the real interest rate is **positively related to, negatively related to, or not at all related to the rate of consumption growth between period t and period t+1.**

**Solution:** The familiar consumption-savings optimality condition is $\frac{u'(c_t)}{\beta u'(c_{t+1})} = 1 + r_t$. In the steady-state in which $c_t = c_{t+1} = \ldots c$ and $r_t = r_{t+1} = \ldots = r$, this condition simply says that $\frac{1}{\beta} = 1 + r$. **If** we take the $r$ interest rate as exogenous, then if $r$ rises, savings would rise. **But** if we take the steady state as below, this cannot occur.

b. **(4 points)** Define qualitatively an economic steady state, distinguishing between how nominal variables behave and how real variables behave. **And (a related question),** in the economic steady state, what conclusions can we draw about the steady state relationship between something about $\beta$ and something about $r$? **Be as precise as you can be, but make your response no longer than three sentences.**

**Solution:** The answer is implicit in the above solution: a steady state is a condition in which real variables stop moving over time, whereas nominal variables may still move. (If nominal variables do move, there has to be certain conditions that they satisfy.) Again looking above, the steady state relationship between $\beta$ and $r$ is $\frac{1}{\beta} = 1 + r$.

**Problem 1b continued (if you need more space)**

**Regardless of what you found above, from here onwards suppose that** $\frac{1}{\beta} = 1 + r$ **in the steady state.**

c. **(4 points)** Adopting the standard view in the macroeconomics profession that $\beta \in (0,1)$ is the most basic view that economics has of people, provide a **brief** discussion/description **(two sentences maximum)** about how the steady state relationship above came about. In this explanation, you should **not** rely on anything about the banking system, etc; discuss/describe things in terms of primitives.

**Solution:** Under the standard view, if $\beta$ is the basic (i.e., the view of impatience), then $r$ is the reaction of "markets" (not even defined as the banking system – it could be, for example, cavemen) to induce people to wait. Said in a simple (and often intuitive) way, the real interest rate is "the price of time."

**Problem 1 continued**

d. **(4 points)** What if instead $\beta \in (0,1)$ is a type of "shortcut" to capture the idea that, even though people actually have **finite** lives, the model captures it with an infinite-life model. But then, to actually make the people in the **model** "care" about time, there is a $\beta \in (0,1)$ "shortcut" to make the model work. With this "shortcut" view of $\beta$, provide a **brief** discussion/description **(two sentences maximum)** about how the steady state relationship above would be interpreted. In this explanation, you should **not** rely on anything about the banking system, etc; discuss/describe things in terms of primitives.

**Solution:** Under this view (and the auxiliary assumption that people do not care about their children, etc, as much as they do about themselves – some people mentioned this, and some people did not), the $\beta$ interpretation is a stand-in in the **infinite-horizon model** for people's **finite-horizon lives.** The idea of the "price of time" still holds (i.e., we have not changed the model in any way), but the interpretation is, from the point of view of the pure theory, less clean (although that is a subjective view).

e. **(4 points)** Of the two explanations in part c and part d about the basics of $\beta$ (and ignoring the "fact" that macroeconomists typically think of things in terms of part c), which one sounds "better?" Again limit your response to **two sentences maximum, *<u>and</u>* base your answer on the responses you provided above.**

**Solution:** Based on the above two answers, in principle arguments could be made either way. As long as your responses were short and to the point, you received most or full credit.

**Problem 2: Consumption, Labor, and Unemployment (33 points).** In this question, you will take the Chapter 2 model (static consumption-labor) and modify it to see how **search and matching theory** works. There are two basic ideas that underlie search and matching theory: i) it incorporates into basic supply-and-demand analysis the fact that when an individual wants to work (i.e., "supplies labor"), there is a **chance** that employment may not be found; ii) it builds in the idea that "search" takes some effort.

In terms of formal notation, let $p^{FIND}$ be the **probability** that an individual searching for a job finds suitable employment. By the definitions of probabilities, $p^{FIND} \in [0,1]$ (that is, the probability is a number between zero and one). **Hence, the probability of not finding a job is 1-$p^{FIND}$.** (**Note:** $p$ does **not** denote a "price.")

And let $s$ be the "search cost," measured in real units (that is, in units of consumption goods) that an individual incurs for **each hour that he/she would like to work.** For example, if the individual desires $n$ = 10 hours of work during the week, the total search cost is 10$s$; if the individual desires $n$ = 20 hours of work during the week, the total search cost is 20$s$; and so on.[1] The search cost is $s \geq 0$.

In quantitative and policy applications that use this framework, a commonly-used utility function is

$$u(c,l) = \ln c - \frac{\theta}{1+1/\psi}(168-l)^{1+1/\psi},$$

in which $\psi$ and $\theta$ (the Greek letters "psi" and "theta," respectively) are **constants** (even though we will not assign any numerical value to them) in the utility function. The consumer has no control over either $\psi$ or $\theta$, and both $\psi > 0$ and $\theta > 0$. **You are to use this utility function throughout the analysis.**

The budget constraint, expressed in **real units** (that is, in units of consumption goods), is

$$c + s\cdot(168-l) = w\cdot(168-l)$$

in which $w$ denotes the **real wage** and $s$ the search cost, both of which are taken as given by the consumer.

**(OVER)**

[1] The way to interpret this is that it is more costly (in a search sense) to find a job the closer it is to a "full time" job because one has to send out more applications, go through more interviews, etc.

**Problem 2 continued**

The Lagrangian for this problem is

$$\ln c - \frac{\theta}{1+1/\psi}(168-l)^{1+1/\psi} + \lambda\left[w\cdot(168-l) - c - s(168-l)\right]$$

or, written even more compactly,

$$\ln c - \frac{\theta}{1+1/\psi}(168-l)^{1+1/\psi} + \lambda\left[(w-s)\cdot(168-l) - c\right]$$

with the term in square brackets the budget constraint. **Finally, for use below, note that the derivative of** $\frac{\theta}{1+1/\psi}(168-l)^{1+1/\psi}$ **with respect to *l* (leisure) is:** $-\theta\cdot(168-l)^{1/\psi}$ **(note the minus sign).**

a. **(4 points)** Provide a qualitative **(two sentences maximum)** interpretation of the budget constraint above (i.e., refer to the term in square brackets in the Lagrangian).

**Solution:** Apart from the presence of the "*s*" term, the budget constraint is interpreted in the usual way: the total wages earned (minus the total search costs) must equal total consumption. There is no savings in the model, so any concept of the "consumption-savings optimality condition" simply does not exist from the point of view of the problem.

b. **(5 points)** In a diagram with consumption on the vertical axis and leisure on the horizontal axis, carefully show the value of the horizontal intercept (i.e., on the leisure axis), briefly describing how you determine this. (Note: there is no need for any optimality condition here; you're simply being asked to first think further about the budget constraint.)

**Solution:** Plotting this relationship also works the same way as in Chapter 2, but you did have to make some **formal** argument (since this model is technically different from the Chapter 2 model): setting $c = 0$ in the budget line, and solving for $l$ gives that $l = 168$.

**Problem 2 continued**

c. **(6 points)** Based on the Lagrangian above, compute the first-order conditions with respect to both $c$ and $l$. **Then,** combine the two first-order conditions to generate the consumption-"leisure" optimality condition. **(Note: your analysis is to be based on the utility function given above.)**

**Solution:** The two FOCs are

$$\frac{1}{c} - \lambda = 0$$

$$\theta \cdot (168 - l)^{1/\psi} - \lambda(w - s) = 0$$

Solving the first line for the multiplier gives $\lambda = \frac{1}{c}$; substituting this into the second line and rearranging gives $\frac{\theta \cdot (168 - l)^{1/\psi}}{1/c} = w - s$. (You didn't have to write it this way, but this is as similar as the condition gets to the standard consumption-leisure optimality condition.)

d. **(6 points)** Not many people are finding jobs successfully. Suppose the Obama administration wants to lower $s$ (the search cost) as a way of helping people. **Given the problem literally as written above,** would this help, would this hurt, or can the issue not be analyzed based on the above? **Limit your answer to three sentences.**

**Solution:** Looking at the "consumption-leisure" condition derived above, lowering $s$ will clearly **raise** the slope of the budget line (by pivoting around the point 168 on the horizontal intercept). With normal (as opposed to "inferior") preferences for consumption and leisure (examine the utility function), this will lead to more options being available for the consumer, and people will choose both higher leisure and higher consumption.

**Problem 2 continued**

e. **(6 points)** Not many people are finding jobs successfully. Suppose the Obama administration wants to raise $p^{FIND}$ (the probability of successfully finding a job) as a way of helping people. **Given the problem literally as written above,** would this help, would this hurt, or can the issue not be analyzed based on the above? **Limit your answer to three sentences.**

**Solution:** This variable is harder to analyze given the way the problem is setup above; in particular, the fact that $p^{FIND}$ does not even show up in the problem makes it seemingly impossible. So if we only had the above representation, the main argument to make was that the variable $p^{FIND}$ "does not even matter."

f. **(6 points)** If you concluded in part d and/or part e that $s$ and/or $p^{FIND}$ does **not** work in the problem as written above, **qualitatively** describe what one would need to do to make it/them work? **Alternatively,** if they both **did** work, why is there nothing further to do?

**Solution:** However, we could recast the problem from part e. Given the general nature of the question, we would have to write the problem in such a way as to have $p^{FIND}$ explicitly appear, and then we could do comparative statics on it. A big-picture point here is that there are **more** than two uses of time (i.e., not just labor and leisure). (Note that there is no need to recast the problem from part d.)

**Problem 3: Two-Period Economy (22 points).** Consider a two-period economy (with no government and hence no taxes at all), in which the representative consumer has no control over his income $y_1$ and $y_2$. The lifetime utility function of the representative consumer is $u(c_1, c_2) = \ln c_1 + c_2$, where ln(.) stands for the natural logarithm. (Note: that is not a typo – it is **only** $c_1$ that is inside a ln(.) function, $c_2$ is **not** inside a ln(.) function).

There is only a single asset the consumer trades; and the consumer begins period one with zero assets.

On the asset that consumers trade, the real interest rate is initially $r > 0$. As a mathematical proposition, this is fine, but think of this $r$ as **very much** larger than zero. In particular, think about the "credit crisis" in the fall of 2008, when certain values of $r$ went to **historically** large values (and some of them are still very large).

For some concreteness, let's think of "period 1" as the fall of 2008 – 2013, and "period 2" to be 2014 – the end of time.

a. **(5 points)** Does the lifetime utility function display diminishing marginal utility in $c_1$? **And,** does it display diminishing marginal utility in $c_2$? Briefly explain, **in no more than three sentences.** (Note the **two** separate questions here.)

**Solution:** Given the utility function, there is curvature (and strict concavity) in $c_1$, so lifetime utility does display diminishing marginal utility in $c_1$. The utility function does **not** display curvature (i.e., it is actually completely linear) in $c_2$, so lifetime utility does **not** display diminishing marginal utility in $c_2$.

**Problem 3 continued**

b. **(5 points)** The "credit crisis" of the fall of 2008 – 2013 begins, and $r$ shoots way up. From a **marginal utility perspective (note this phrase),** does the optimal choice of $c_1$ rise or fall? **And, related,** does the individual **care** about this rise or fall **from a pure (i.e., per-unit) marginal utility perspective?** (Note: your analysis is to be conducted from the perspective of the very beginning of period 1.)

**Solution:** With the rise in $r$ (and if we wanted to think about things graphically), the LBC would pivot around the point ($y_1$, $y_2$) and become steeper. The optimal choice of $c_1$ falls (and, for part c below, the optimal choice of $c_2$ rises). Given the **diminishing marginal utility** for $c_1$, if $c_1$ declines, then the **marginal utility of $c_1$ rises.** (There was a fair amount of confusion amongst responses between **utility** and **marginal utility**, which are two **different** concepts.)

c. **(5 points)** The "credit crisis" of the fall of 2008 – 2013 begins, and $r$ shoots way up. From a **marginal utility perspective (note this phrase),** does the optimal choice of $c_2$ rise or fall? **And, related,** does the individual **care** about this rise or fall **from a pure (i.e., per-unit) marginal utility perspective?** (Note: your analysis is to be conducted from the perspective of the very beginning of period 1.)

**Solution:** With the rise in $r$, and considering just the marginal utility function of $c_2$, there is clearly no change in this marginal utility. Thus, on a pure (i.e., per-unit) marginal utility perspective, the individual does not care about the change in $c_2$. Even though, as noted in part b, the optimal choice of $c_2$ does rise. (Just as there was a fair amount of confusion between **utility** and **marginal utility** (as in part b), solutions to this question also showed a fair amount of confusion between the **per-unit ("pure") marginal utility** and the **total marginal utility** of consumption of $c_2$.)

**Problem 3 continued**

d. **(7 points)** The Federal Reserve notices what's happening. Supposing that the Fed can control both real interest rates and nominal interest rates, it dramatically reduces $r$. Does the Fed's actions do anything to offset the impact on the **pure (i.e., per-unit) marginal utility of $c_1$? And,** does the Fed's actions do anything to offset the impact on the **pure (i.e., per-unit) marginal utility of $c_2$? Explain** your answers precisely. (Note: your analysis is to be conducted from the perspective of the very beginning of period 1.)

**Solution:** The decline in $r$ induced by the Fed's policy actions simply reverse the effects that were described in parts b and c. (So you can simply think of reversing, from the perspective of the theory, the effects described above.) In terms of the real-world effects of the Fed's (various and many!) policies – the jury is still out.

**Problem 4: A Contraction in Credit Availability (13 points).** The graph below (on the next page) shows our usual two-period indifference-curve/budget constraint diagram, with period-1 consumption plotted on the horizontal axis, period-2 consumption plotted on the vertical axis, and the downward-sloping line representing, as always, the consumer's LBC. Throughout all of the analysis here, assume that ***r* = 0 always.** Furthermore, there is no government, hence never any taxes.

Suppose that the representative consumer has lifetime utility function $u(c_1, c_2) = \ln c_1 + \ln c_2$, and that the **real** income of the consumer in period 1 and period 2 is $y_1$ = 12 and $y_2$ = 8. Finally, suppose that the initial amount of net assets the consumer has is $a_0$ = 0. **EVERY** consumer in the economy is described by this utility function and these values of $y_1$, $y_2$, and $a_0$.

a. **(5 points)** If there are no problems in credit markets whatsoever (so that consumers can borrow or save as much or as little as they want), compute the numerical value of the optimal quantity of period-1 consumption. (**Note:** if you can solve this problem without setting up a Lagrangian, you are free to do so as long as you explain your logic.)

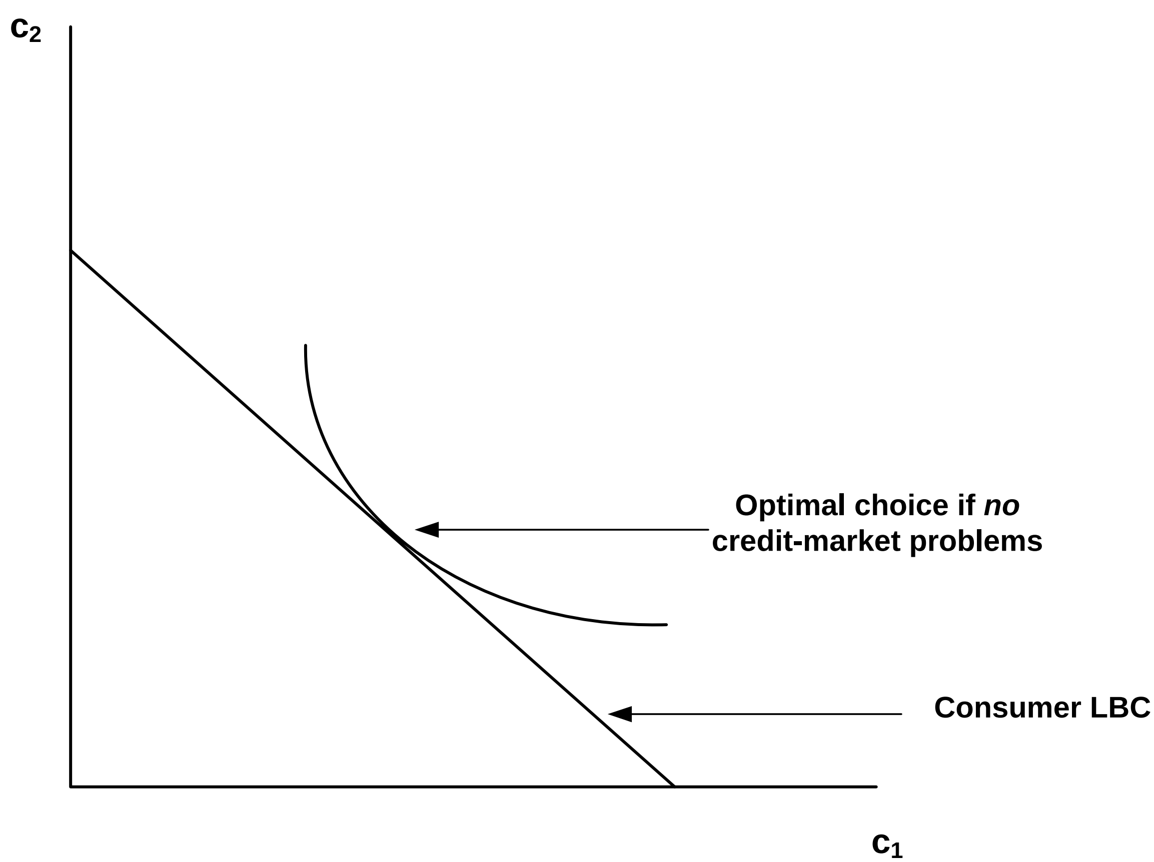
**Solution:** The consumption-savings optimality condition (given the natural-log utility function) is given by $c_2/c_1 = 1+r = 1$ (the second equality follows because $r = 0$ here). Thus, at the optimal choice, it is the case that $c_1 = c_2$. Using this relationship (and again using the fact that $r = 0$ here), we can express the consumer's LBC as $c_1 + c_1 = y_1 + y_2 = 20$, which obviously implies the optimal choice of period-1 consumption is $c_1 = 10$.

**Note**: although you were not asked to compute it, you could have computed the implied value of the consumer's asset position at the end of period one. Because $a_0 = 0$, $y_1 = 12$, and we just computed $c_1 = 10$, the asset position at the end of period one is $a_1 = y_1 - c_1 = 2$ (i.e., **positive** 2).

b. **(8 points)** Now suppose that because of problems in the financial sector, no consumers are allowed to be in debt at the end of period 1. With this credit restriction in place, compute the numerical value of the optimal quantity of period-1 consumption. **ALSO,** on the diagram on the next page, qualitatively and **clearly** sketch the optimal choice with this credit restriction in place (qualitatively sketched already for you is the optimal choice if there are no problems in credit markets). Your sketch should indicate **both** the new optimal choice **and** an appropriately-drawn and labeled indifference curve that contains the new optimal choice. (**Note:** if you can solve this problem without setting up a Lagrangian, you are free to do so as long as you explain your logic.)

**Solution:** Because in part a (ie, without any credit restrictions), the representative consumer was choosing to NOT be in debt at the end of period 1 (ie, $a_1 > 0$ under the optimal choice in part a), the imposition of the credit restriction, **nothing changes compared to part a.** That is, the optimal choice of period-1 consumption is still 10. Hence, in the diagram below, the optimal choice in the presence of credit constraints is **exactly the same as the optimal choice without credit constraints.** The general lesson to draw from this example and our analysis in class is that it is not *necessarily* the case that financial market problems *must* and *always* spill over into real economic activity (i.e., consumption in this case)

**Problem 4b continued**



**Problem 5: Government Debt Ceilings (12 points).** Just like we extended our two-period analysis of consumer behavior to an infinite number of periods, we can extend our two-period analysis of fiscal policy to an infinite number of periods.

The government's budget constraints (expressed in real terms) for the years 2011 and 2012 are

$$g_{2011} + b_{2011} = t_{2011} + (1+r)b_{2010}$$
$$g_{2012} + b_{2012} = t_{2012} + (1+r)b_{2011}$$

and analogous conditions describe the government's budget constraints in the years 2013, 2014, 2015, etc. The notation is as in Chapter 7: $g$ denotes real government spending during a given time period, $t$ denotes real tax revenue during a given time period (all taxes are assumed to be lump-sum here), $r$ denotes the real interest rate, and $b$ denotes the government's asset position ($b_{2010}$ is the government's asset position at the end of the year 2010, $b_{2011}$ is the government's asset position at the end of 2011, and so on).

Describing numerics qualitatively, at the end of 2010, the government's asset position was roughly a **debt** of \$14 trillion (that is, $b_{2010}$ = -\$14 trillion).

The current fiscal policy plans/projections call for: $g_{2011}$ = \$4 trillion, $t_{2011}$ = \$2 trillion, $g_{2012}$ = \$3 trillion, and $t_{2012}$ = \$2 trillion.

Finally, given how low interest rates are right now and how low they are projected to remain for at least the next couple of years, suppose that the real interest rate is always zero (i.e., $r = 0$ always).

a. **(3 points)** Assuming the projections above prove correct, what will be the numerical value of the federal government's asset position at the end of 2011? Briefly explain/justify **in no more than two sentences.**

**Solution:** Using the given numerical values and using the 2011 government budget constraint given above, it is straightforward to calculate $b_{2011}$ = -\$16 trillion.

b. **(3 points)** Assuming the projections above prove correct, what will be the numerical value of the federal government's asset position at the end of 2012? Briefly explain/justify **in no more than two sentences.**

**Solution:** Using the given numerical values, the value for $b_{2011}$ found in part a, and using the 2011 government budget constraint given above, it is straightforward to calculate $b_{2011}$ = -\$17 trillion.

**Problem 5 continued**

Under current federal law, the U.S. government's debt cannot be larger than $16 trillion at any point in time. This limit is known as the "debt ceiling."

c. **(3 points) Based on your answer in part a above,** does the debt ceiling pose a problem for the government's fiscal policy plans during the course of the year 2011? If it poses a problem, briefly describe the problem; if it poses no problem, briefly describe why it poses no problem.

**Solution:** No, the debt ceiling poses no problem for the fiscal policy plans for the year 2011. This is because the *t* and *g* plans call for a debt at the end of 2011 of $16 trillion, which does not exceed the ceiling.

d. **(3 points) Based on your answer in part b above,** does the debt ceiling pose a problem for the government's fiscal policy plans during the course of the year 2012? If it poses a problem, briefly describe the problem; if it poses no problem, briefly describe why it poses no problem.

**Solution:** Yes, the debt ceiling poses a problem for the fiscal policy plans for the year 2012. This is because the *t* and *g* plans call for a debt at the end of 2012 of $17 trillion, which violates the ceiling.

**END OF EXAM**