THE FINANCIAL ACCELERATOR: FINANCIAL MARKETS AND THE MACROECONOMY

NOVEMBER 28, 2011

FINANCIAL ACCELERATOR

- "Financial accelerator" framework
  - The most widely-used and applied framework in macroeconomic theory and policy for thinking about financial markets
  - Developed in series of studies by Bernanke and Gertler in late 1980's and early 1990's
- Popular-press language
  - "Financial accelerator"
  - "Financial feedback loops"
  - "Loan spirals"
- Describes well many of the financial-macroeconomic linkages underpinning the dynamics of
  - The Great Depression
  - Current macroeconomic conditions
- Will develop idea in context of firm theory (Chapter 6)
- Can also develop idea in context of consumer theory (Chapter 3, Chapter 4, Chapter 8)
  - Recall "credit constraint" analysis of consumption/savings decisions (Chapter 3 and 4)

OUTLINE OF FRAMEWORK

Major ideas underlying Financial Accelerator Framework

1. **Firms' financial assets (i.e., stocks and bonds) matter for their ability to purchase physical assets (i.e., machines and equipment)**

2. **Market prices of financial assets matter for firm financing constraints**

3. **Government regulation affects the linkage between financial markets and real (i.e., goods and physical capital) markets *through* financing constraints**

OUTLINE OF FRAMEWORK

Four Building Blocks of the Financial Accelerator Framework

1. **Two-Period Model of Firm Profit Maximization**
   - **Based on Chapter 6**
   - **Enriched to allow for both physical assets (machines and equipment) and financial assets (stocks and bonds)**

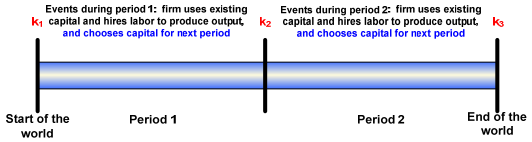
2. **Financing Constraint – conceptually the key building block**
   - **Quantity of physical capital firms can purchase depends on the market value of their financial assets**
   - **Reflects market and regulatory structures designed to mitigate informational asymmetries**
   - **(Recall basic Chapter 6 theory of firms featured no constraints on firm profit maximization)**

3. **Government Regulation/Oversight of Financial Relationships**

4. **Relationship between Firm Profits and Dividends**

BASIC FIRM THEORY

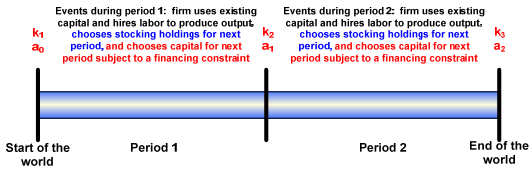
- **Timeline of events**



- **Notation**
  - k_1: capital used for production in period 1 (decided upon in "period 0")
  - n_1: labor used for production in period 1
  - w_1: real wage rate for labor in period 1 ($w_1 = W_1/P_1$)
  - i: nominal interest rate (between period 1 and period 2)
  - P_1: nominal price of output produced and sold by firm in period 1
    AND nominal price of one unit of capital bought by the firm in period 1 for use in period 2

ENRICHING THE BASIC FIRM THEORY

- **Timeline of events**



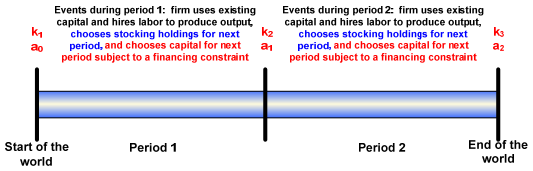
- **Notation**
  - k_1: capital used for production in period 1 (decided upon in "period 0")
  - n_1: labor used for production in period 1
  - w_1: real wage rate for labor in period 1 ($w_1 = W_1/P_1$)
  - i: nominal interest rate (between period 1 and period 2)
  - P_1: nominal price of output produced and sold by firm in period 1
    AND nominal price of one unit of capital bought by the firm in period 1 for use in period 2
  - a_0: real wealth (stock) holdings at beginning of period 1/end of period 0

The "defining features" of stock

  - S_1: nominal price of a unit of stock in period 1
  - D_1: nominal dividend paid in period 1 by each unit of stock held at the start of period 1

ENRICHING THE BASIC FIRM THEORY

- **Timeline of events**



- **Notation**
  - k_2: capital used for production in period 2 (decided upon in period 1)
  - n_2: labor used for production in period 2
  - w_2: real wage rate for labor in period 2 ($w_2 = W_2/P_2$)
  - i: nominal interest rate (between period 1 and period 2)
  - P_2: nominal price of output produced and sold by firm in period 2
    AND nominal price of one unit of capital bought by the firm in period 2 for use in period 3
  - a_1: real wealth (stock) holdings at beginning of period 2/end of period 1

The "defining features" of stock:
  - S_2: nominal price of a unit of stock in period 2
  - D_2: nominal dividend paid in period 2 by each unit of stock held at the start of period 2

  - π_2: net inflation rate between period 1 and period 2 (recall: $\pi_2 = P_2/P_1 - 1$)

RATES OF RETURN

- "Interest rates" can be defined for any type of asset
  - There is no ***single*** interest rate in the economy

- Interpret/understand the ***two*** types of "interest rates" that co-exist in this richer framework of firm profit maximization
  - i: nominal interest rate on bonds
    - Recall from Chapter 14

$$1+i=\frac{1}{P_1^b}$$

    - Thus can think of bonds (one ***type*** of financial asset) as being in the background of the analysis

  - i^{STOCK}: nominal return on stock – i.e., "interest rate on stock" (though bad terminology)
    - Define according to

$$1+i^{STOCK}=\frac{S_2+D_2}{S_1}$$

    - Measures the net dollar return (in period 2) on one share of stock (whose purchase price was S_1 in period 1)

RATES OF RETURN

- "Interest rates" can be defined for any type of asset
  - There is no ***single*** interest rate in the economy
- Interpret/understand the ***two*** types of "interest rates" that co-exist in this richer framework of firm profit maximization
  - ***i***: nominal interest rate on bonds
    - Recall from Chapter 14

REAL INTEREST RATE ON GOVERNMENT BONDS: A "RISKLESS" ASSET

$$1+i=\frac{1}{P_1^b} \quad \text{can rewrite as} \quad i=\frac{1}{P_1^b}-1 \quad \text{express as } \textit{real} \text{ interest rate} \quad 1+r=\frac{1+i}{1+\pi}$$

    - Thus can think of bonds (one ***type*** of financial asset) as being in the background of the analysis
  - i^{STOCK}: nominal return on stock – i.e., "interest rate on stock" (though bad terminology)
    - Define according to

REAL INTEREST RATE ON STOCKS: A "RISKY" ASSET

$$1+i^{STOCK}=\frac{S_2+D_2}{S_1} \quad \text{can rewrite as} \quad i^{STOCK}=\frac{S_2+D_2}{S_1}-1 \quad \text{express as } \textit{real} \text{ interest rate} \quad 1+r^{STOCK}=\frac{1+i^{STOCK}}{1+\pi}$$

    - Measures the net dollar return (in period 2) on one share of stock (whose purchase price was S_1 in period 1)
- Can distinguish two measures of real interest rates in this framework

FIRM PROFIT FUNCTION

- A ***dynamic*** profit maximization problem
  - Because firm exists for both periods
  - All analysis conducted from the perspective of the very beginning of period 1
  - → Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits
- Dynamic profit function
  - (specified in nominal terms – could specify in real terms…)

Period-1 profits

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1$$

- $P_1 f(k_1, n_1)$: Total revenue in period 1 (price x output)
- $P_1 k_1$: Value of pre-existing physical capital (an asset for firms)
- $(S_1 + D_1) a_0$: Value (inclusive of dividends) of pre-existing financial assets (i.e., stock-holdings in *other* firms)
- $P_1 w_1 n_1$: Total labor cost in period 1
- $P_1 k_2$: Total cost of buying physical capital for period 2 (time to build → must purchase period-2 capital in period 1)
- $S_1 a_1$: Total cost of buying financial assets (i.e., stock-holdings in *other* firms) for period 2

FIRM PROFIT FUNCTION

- A ***dynamic*** profit maximization problem
  - Because firm exists for both periods
  - All analysis conducted from the perspective of the very beginning of period 1
  - → Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits

- Dynamic profit function
  - (specified in nominal terms – could specify in real terms…)

As usual: no physical or financial assets needed for "period 3"

= 0 = 0

Period-1 profits

(PDV of) period-2 profits

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

Total revenue in period 1 (price x output)

Value of pre-existing physical capital (an asset for firms)

Total labor cost in period 1

Total cost of buying physical capital for period 2 (time to build → must purchase period-2 capital in period 1)

Total revenue in period 2 (price x output)

Value of pre-existing physical capital (an asset for firms)

Total labor cost in period 2

Total cost of buying physical capital for period 3 (time to build → must purchase period-3 capital in period 2)

Value (inclusive of dividends) of pre-existing financial assets (i.e., stock-holdings in *other* firms)

Total cost of buying financial assets (i.e., stock-holdings in *other* firms) for period 2

Value (inclusive of dividends) of pre-existing financial assets (i.e., stock-holdings in *other* firms)

Total cost of buying financial assets (i.e., stock-holdings in *other* firms) for period 3

INFORMATIONAL ASYMMETRIES

- "Informational asymmetries" pervasive in borrowing/lending relationships

- Borrower (whether consumer, firm, or financial institution) ***much*** more likely to know his own ability/willingness to repay a loan
  - Lenders only know little about the "quality" or "trustworthiness" of a borrower
  - Asymmetry of information – cannot be eliminated

- To mitigate consequences of informational asymmetries, lenders often require borrower to have a stake in "succeeding" in the project/purpose for which funds are being borrowed
  - Consumers
    - e.g., down payment on house purchase
    - e.g., down payment on car purchase
    - If stop making payments on house or car
      - Borrower loses down payment (in addition to the car or house…)…
      - Affects individual's incentives ***before*** borrowing

INFORMATIONAL ASYMMETRIES

- "Informational asymmetries" pervasive in borrowing/lending relationships
- Borrower (whether consumer, firm, or financial institution) ***much*** more likely to know his own ability/willingness to repay a loan
  - Lenders only know little about the "quality" or "trustworthiness" of a borrower
  - Asymmetry of information – cannot be eliminated
- To mitigate consequences of informational asymmetries, lenders often require borrower to have a stake in "succeeding" in the project/purpose for which funds are being borrowed
  - Consumers
    - e.g., down payment on house purchase
    - e.g., down payment on car purchase
    - Total amount of loan (typically) depends on individual's collateral
  - Firms
    - Capital investment (factories, technology upgrades, etc) outlays
    - Payroll outlays
    - Financing inventories
    - Total amount of loan (typically) depends on firm's collateral

"Working capital"

  - Financial institutions: borrow in order to make (big) loans
    - By raising "small" quantities of funds from many different sources

FINANCING CONSTRAINT

- Capture this idea through a financing constraint on firm's ability to purchase capital between period 1 and period 2
- Financing constraint
  - Total expenditures on period-1 physical investment must be equal to market value of firm's financial (stock) holdings
  - (Technically, smaller than or equal to, so an inequality constraint…but will formally analyze constraint with equality)

$$P_1 \cdot inv_1 = S_1 \cdot a_1$$

$inv_1 = k_2 - k_1$ (investment is ***change in*** quantity of physical capital)

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

FINANCING CONSTRAINT

- Capture this idea through a financing constraint on firm's ability to purchase capital between period 1 and period 2
- Financing constraint
  - Total expenditures on period-1 physical investment must be equal to market value of firm's financial (stock) holdings
  - (Technically, smaller than or equal to, so an inequality constraint…but will formally analyze constraint with equality)

$$P_1 \cdot inv_1 = S_1 \cdot a_1$$

$inv_1 = k_2 - k_1$ (investment is *change in* quantity of physical capital)

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

  - Important: a_1 appears in the financing constraint, *not* a_0
    - Idea this assumption captures: firm might try to strategically manipulate the value of *financial* assets it holds in order to affect the quantity of *physical* investment it can engage in
    - (From the perspective of beginning of period 1, a_1 has not yet been chosen, whereas a_0 is pre-determined)

GOVERNMENT OVERSIGHT OF FINANCIAL MARKETS

- Government oversight of informational asymmetries in borrower/lender relationships
  - Filing of proper documentation
  - Full disclosure ("truth-in-lending") laws
  - Direct lending in some markets
  - …
- Capture government Regulation of financial dealings in our framework in very simple way
  - Firm can only borrow up to a multiple *R* of the market value of its financial assets for physical investment purposes
  - e.g., if government regulates that expenditures on investment cannot be larger than 5 times market value of financial assets, *R* = 5
- Terminology: *R* is leverage ratio
  - Will think of it as government regulation…
  - …but can and does also reflect market and institutional arrangements

GOVERNMENT OVERSIGHT OF FINANCIAL MARKETS

- Capture this idea through a **financing constraint** on firm's ability to purchase capital between period 1 and period 2

- Financing constraint
  - Total expenditures on period-1 physical investment must be equal to market value of firm's financial (stock) holdings
  - (Technically, smaller than or equal to, so an inequality constraint…but will formally analyze constraint with equality)

$$P_1 \cdot inv_1 = S_1 \cdot a_1$$

$inv_1 = k_2 - k_1$ (investment is *change in* quantity of physical capital)

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

Government regulation of leverage R

$$\boxed{P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1}$$

Impose this financing constraint on firm profit maximization problem

FINANCIAL ACCELERATOR FRAMEWORK

- Four Building Blocks of the Financial Accelerator Framework

1. Firm Profit Function

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

= 0 = 0

2. Financing Constraint

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

3. Government Regulation of Financial Relationships (imposition of R on financing constraint)

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

4. Relationship between firm profits and dividends

LATER

FIRM PROFIT MAXIMIZATION

Maximize two-period profits

= 0 = 0

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

Subject to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

Construct Lagrangian

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i}$$
$$+ \lambda \left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1) \right]$$

Lagrange multiplier on financing constraint

CRUCIAL OBSERVATION: in basic firm theory (i.e., Chapter 6), value of this multiplier was....

λ = 0 i.e., there was no financing constraint!

LATER: will study which regulatory and/or market features make the financing constraint effectively "disappear" (i.e., cause λ = 0)

FIRM PROFIT MAXIMIZATION

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i}$$
$$+ \lambda \left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1) \right]$$

- **FOCs with respect to n_1, n_2**

Identical except for time subscripts

- with respect to n_1: $P_1 f_n(k_1, n_1) - P_1 w_1 = 0$ — Equation 1
- with respect to n_2: $\frac{P_2 f_n(k_2, n_2)}{1+i} - \frac{P_2 w_2}{1+i} = 0$ — Equation 2

  - Financing constraint does not affect profit-maximizing choices of labor hiring...
  - ...thus same analysis from Chapter 6 of labor demand curve, etc, applies

- **FOCs with respect to k_2, a_1**
  - The interesting aspects of the framework
  - The heart of the financial accelerator framework

FIRM PROFIT MAXIMIZATION

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} + \lambda\left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)\right]$$

- **FOCs with respect to k_2, a_1**

  with respect to k_2: $-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \lambda P_1 = 0$ **Equation 3**

  with respect to a_1: $-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0$ **Equation 4**

FIRM PROFIT MAXIMIZATION

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} + \lambda\left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)\right]$$

- **FOCs with respect to k_2, a_1**

  with respect to k_2: $-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \lambda P_1 = 0$ **Equation 3**

  with respect to a_1: $-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0$ **Equation 4**

- **Analysis of Equation 4 in isolation**
  - **Answers the central question: under what conditions does $\lambda = 0$?**
  - **Reveals how stock market returns affect financing constraints**
  - **Reveals how government regulation affects financing constraints**
- **Analysis of Equation 3 and Equation 4 jointly**
  - **Demonstrates how/why financial market prices (i.e., stock prices/returns) matter for macroeconomic activity**
  - **The financial accelerator effect**

WHY IS FINANCING A *CONSTRAINT*?

$$-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0$$ Equation 4

↓ Solve for λ

$$\lambda = \left[S_1 - \frac{S_2 + D_2}{1+i} \right] \cdot \frac{1}{R \cdot S_1}$$

↓ Pull $1/S_1$ term inside

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R}$$

↓ Multiply *and* divide second term in parentheses by P_1 *and* P_2

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{P_2}{P_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R}$$

↓ Use definition of inflation, $1 + \pi_2 = P_2 / P_1$, and regroup terms

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{1+\pi_2}{1+i} \right] \cdot \frac{1}{R}$$

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{1+\pi_2}{1+i} \right] \cdot \frac{1}{R}$$ (from previous page)

↓ Use definition of "nominal interest rate on stock", $1 + i^{STOCK} = (S_2 + D_2)/ S_1$
Use definition of inflation, $1 + \pi_2 = P_2 / P_1$

$$\lambda = \left[1 - \frac{1+i^{STOCK}}{1+\pi_2} \cdot \frac{1+\pi_2}{1+i} \right] \cdot \frac{1}{R}$$

↓ Fisher equation for stock: $1 + r^{STOCK} = (1 + i^{STOCK})/ (1 + \pi_2)$
Fisher equation for bonds: $1 + r = (1 + i) / (1 + \pi_2)$

$$\lambda = \left[1 - \frac{1+r^{STOCK}}{1+r} \right] \cdot \frac{1}{R}$$

↓ Final rewrite!

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r} \right] \cdot \frac{1}{R}$$ The Lagrange multiplier on firm's financing constraint

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- **Basic firm theory (Chapter 6)**
  - **No financing constraint**
  - **Can interpret basic firm theory analysis as featuring $\lambda = 0$**

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i}$$
$$+ \lambda\left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)\right]$$

$\lambda = 0$

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- **Basic firm theory (Chapter 6)**
  - **No financing constraint**
  - **Can interpret basic firm theory analysis as featuring $\lambda = 0$**
    - **Interpretation: under "normal market conditions," financing constraints don't matter (much…)**
    - **Identify "normal market conditions" as steady state**

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i}$$
$$+ \lambda\left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)\right]$$

$\lambda = 0$

  - **If $\lambda = 0$ (i.e., "normal market conditions," aka steady state)**
    - **Labor demand decisions unaffected by financial market conditions**
    - **Capital demand decisions unaffected by financial market conditions**

- **Key question: what causes $\lambda = 0$?**

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- **Two conditions for $\lambda = 0$**
  - **Market returns on risky assets equal returns on riskless assets**
    - **Risky assets: stocks**
    - **Riskless assets**
      - **Bonds (financial)**
      - **Machines and equipment (physical) – most directly relevant for firms' production and sales activity**
        - **Basic firm theory prediction: *r = mpk***

Can think of both government bonds (financial assets) and machines & equipment (physical assets) as "riskless": you (pretty much…) know what you're going to get from them.

$$r = r^{STOCK} \longrightarrow \lambda = 0$$

Interpretation: if returns on financial assets are aligned with returns on physical assets, financing constraints "don't matter"

Finance Fundamentals

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- **Two conditions for $\lambda = 0$**
  - **Market returns on risky assets equal returns on riskless assets**
    - **Risky assets: stocks**
    - **Riskless assets**
      - **Bonds (financial)**
      - **Machines and equipment (physical) – most directly relevant for firms' production and sales activity**
        - **Basic firm theory prediction: *r = mpk***

Can think of both government bonds (financial assets) and machines & equipment (physical assets) as "riskless": you (pretty much…) know what you're going to get from them.

$$r = r^{STOCK} \longrightarrow \lambda = 0$$

Interpretation: if returns on financial assets are aligned with returns on physical assets, financing constraints "don't matter"

  - **Government oversight of borrowing/lending relationships very lax**
    - **The larger is *R*, the lower is *λ***
    - **Financing constraint:** $P_1 \cdot (k_2 - k_1) = R(S_1 \cdot a_1)$

Market value of financial assets

      - **Holding constant market value of financial assets, higher *R* allows higher k_2**

In practice, not literally infinity…

$$R = \infty \longrightarrow \lambda = 0$$

Interpretation: if government regulations allow high borrowing with little assets, financing constraints "don't matter"

FINANCING CONSTRAINT AND CAPITAL DEMAND

- **Suppose $R = 1$ in "steady state" (but keep R in rest of analysis)**
  - $R > 1$ is "lax regulation" (because it lowers λ, all else constant)
  - $R < 1$ is "tight regulation" (because it increases λ, all else constant)
  - → Whether or not financing constraint matters (i.e., whether or not $\lambda = 0$) all depends on whether or not $r^{STOCK} = r$ or not

KEY IDEA: if returns on riskless assets = returns on risky assets → financing constraints "don't matter" for firm production decisions

FINANCING CONSTRAINT AND CAPITAL DEMAND

- **Suppose $R = 1$ in "steady state" (but keep R in rest of analysis)**
  - $R > 1$ is "lax regulation" (because it lowers λ, all else constant)
  - $R < 1$ is "tight regulation" (because it increases λ, all else constant)
  - → Whether or not financing constraint matters (i.e., whether or not $\lambda = 0$) all depends on whether or not $r^{STOCK} = r$ or not

KEY IDEA: if returns on riskless assets = returns on risky assets → financing constraints "don't matter" for firm production decisions

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \lambda P_1 = 0 \qquad \text{Equation 3 (FOC on } k_2\text{)}$$

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R} \qquad \text{Equation 4 (FOC on } a_1\text{)}$$

- **Basic firm theory (Chapter 6)**
  - Capital demand function derived from Equation 3
  - Idea same as in Chapter 6…but now complicated by the financing constraint

Substitute λ from Equation 4 into Equation 3

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} P_1 = 0$$

Rearrange

FINANCING CONSTRAINT AND CAPITAL DEMAND

$$-P_1+\frac{P_2 f_k(k_2,n_2)}{1+i}+\frac{P_2}{1+i}-\left[\frac{r-r^{STOCK}}{1+r}\right]\frac{1}{R}P_1=0 \quad \text{(from previous page)}$$

Divide by P_1

$$\frac{P_2 f_k(k_2,n_2)}{P_1(1+i)}+\frac{P_2}{P_1(1+i)}-\left[\frac{r-r^{STOCK}}{1+r}\right]\frac{1}{R}=1$$

Use definition of inflation, $1+\pi_2=P_2/P_1$

$$\left(\frac{1+\pi_2}{1+i}\right)f_k(k_2,n_2)+\frac{1+\pi_2}{1+i}-\left[\frac{r-r^{STOCK}}{1+r}\right]\frac{1}{R}=1$$

Apply Fisher relation for "riskless" assets

$$\frac{f_k(k_2,n_2)}{1+r}+\frac{1}{1+r}-\left[\frac{r-r^{STOCK}}{1+r}\right]\frac{1}{R}=1$$

FINANCING CONSTRAINT AND CAPITAL DEMAND

$$-P_1+\frac{P_2 f_k(k_2,n_2)}{1+i}+\frac{P_2}{1+i}-\left[\frac{r-r^{STOCK}}{1+r}\right]\frac{1}{R}P_1=0 \quad \text{(from previous page)}$$

Divide by P_1

$$\frac{P_2 f_k(k_2,n_2)}{P_1(1+i)}+\frac{P_2}{P_1(1+i)}-\left[\frac{r-r^{STOCK}}{1+r}\right]\frac{1}{R}=1$$

Use definition of inflation, $1+\pi_2=P_2/P_1$

$$\left(\frac{1+\pi_2}{1+i}\right)f_k(k_2,n_2)+\frac{1+\pi_2}{1+i}-\left[\frac{r-r^{STOCK}}{1+r}\right]\frac{1}{R}=1$$

Apply Fisher relation for "riskless" assets

$$\frac{f_k(k_2,n_2)}{1+r}+\frac{1}{1+r}-\left[\frac{r-r^{STOCK}}{1+r}\right]\frac{1}{R}=1$$

Multiply by (1+r)

Marginal product of capital, *mpk*

$$f_k(k_2,n_2)+1-\frac{r-r^{STOCK}}{R}=1+r$$

Suppose $R = 1$ in "steady state" but keep R in the analysis

COBB-DOUGLAS PRODUCTION FUNCTION

- **Commonly-used functional form in quantitative macroeconomic analysis**

$$f(k,n) = k^{\alpha} n^{1-\alpha}$$

- **Describes the empirical relationship between aggregate GDP, aggregate capital, and aggregate labor quite well**
- $\alpha \in (0,1)$ **measures capital's share of output**
  - **Hence** $(1-\alpha) \in (0,1)$ **measures labor's share of output**
  - **Interpretation**
    - **The relative importance of (either) capital (or labor) in the production process**
  - **Estimates for U.S. economy:** $\alpha \approx 0.3$
  - **Estimates for Chinese economy:** $\alpha \approx 0.15$ **(not (yet) a very capital-rich economy)**
- **Cobb-Douglas form useful for illustrating factor demands**
  - $mpn = f_n(k,n) = (1-\alpha)k^{\alpha} n^{-\alpha}$
  - $mpk = f_k(k,n) = \alpha k^{\alpha-1} n^{1-\alpha}$

FINANCING CONSTRAINT AND CAPITAL DEMAND

- **Firm-level demand for capital defined by the relation**

$$r = \alpha k^{\alpha-1} n^{1-\alpha} - \left[\frac{r - r^{STOCK}}{R}\right] \left(= mpk - \left[\frac{r - r^{STOCK}}{R}\right]\right)$$

$$\downarrow$$

$$r = \alpha k^{\alpha-1} n^{1-\alpha} - \frac{r}{R} + \frac{r^{STOCK}}{R}$$

$$\downarrow$$

$$\left[1 + \frac{1}{R}\right] r = \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R}$$

$$\downarrow$$

$$\left[\frac{R+1}{R}\right] r = \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R}$$

$$\downarrow$$

$$r = \left(\frac{R}{R+1}\right) \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Solve for *r* (return on "riskless" physical assets)

FINANCING CONSTRAINT AND CAPITAL DEMAND

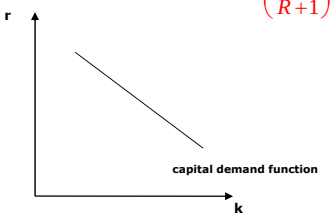
- Firm-level demand for capital **defined** by the relation

$$r = \left(\frac{R}{R+1}\right)\alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Because exponent ($\alpha - 1$) is a negative number, can move to denominator

$$r = \left(\frac{R}{R+1}\right)\alpha \left(\frac{n_t}{k_t}\right)^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

CHAPTER 6: NEGATIVE RELATIONSHIP BETWEEN *r* AND *k*



FINANCING CONSTRAINT AND CAPITAL DEMAND

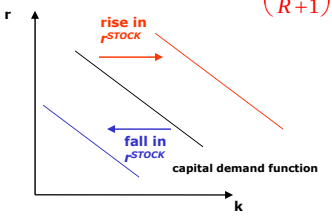
- Firm-level demand for capital **defined** by the relation

$$r = \left(\frac{R}{R+1}\right)\alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Because exponent ($\alpha - 1$) is a negative number, can move to denominator

$$r = \left(\frac{R}{R+1}\right)\alpha \left(\frac{n_t}{k_t}\right)^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Rise (fall) in return on stock leads to shift out (in) of capital demand function



FINANCING CONSTRAINT AND CAPITAL DEMAND

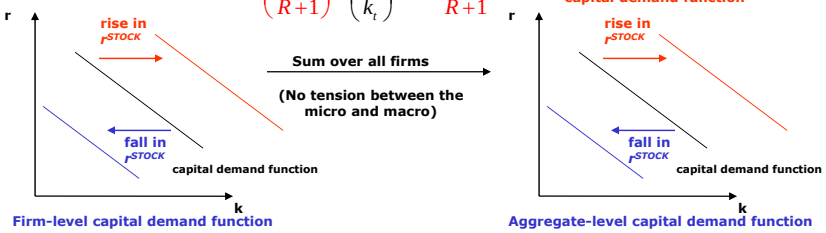
- Firm-level demand for capital **defined** by the relation

$$r = \left(\frac{R}{R+1}\right)\alpha k^{\alpha-1}n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Because exponent ($\alpha - 1$) is a negative number, can move to denominator

$$r = \left(\frac{R}{R+1}\right)\alpha\left(\frac{n_t}{k_t}\right)^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Rise (fall) in return on stock leads to shift out (in) of capital demand function



Firm-level capital demand function — Aggregate-level capital demand function

- **IMPORTANT**: changes in financial market returns shift capital demand (and hence investment demand – recall $inv_t = k_{t+1} - k_t$)

FINANCING CONSTRAINT AND CAPITAL DEMAND

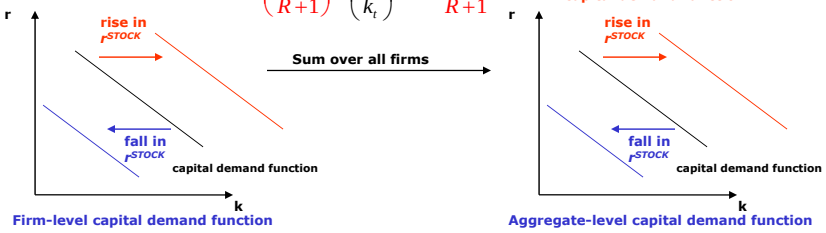
- Firm-level demand for capital **defined** by the relation

$$r = \left(\frac{R}{R+1}\right)\alpha k^{\alpha-1}n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Because exponent ($\alpha - 1$) is a negative number, can move to denominator

$$r = \left(\frac{R}{R+1}\right)\alpha\left(\frac{n_t}{k_t}\right)^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Rise (fall) in return on stock leads to shift out (in) of capital demand function



Firm-level capital demand function — Aggregate-level capital demand function

- **Basis for the financial accelerator effect**
- **Basis for understanding the role of financial oversight**

FINANCIAL ACCELERATOR FRAMEWORK

- **Four Building Blocks of the Financial Accelerator Framework**

1. **Firm Profit Function**

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

(= 0 = 0)

2. **Financing Constraint**

$$P_1 \cdot (k_2 - k_1) = \quad S_1 \cdot a_1$$

3. **Government Regulation of Financial Relationships (imposition of *R* on financing constraint)**

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

4. **Relationship between firm profits and dividends**

NOW

DIVIDENDS AND PROFITS

- **Dividend: payment made by a corporation to its shareholders; the portion of corporate profits paid out to stockholders (Wikipedia definition)**
- **Corporate dividend policies differ widely across industries and companies**
  - **Some companies retain most of their profits (to re-invest in ongoing projects)**
  - **Some industries' dividend policies subject to government regulation**
    - **Recently: financial companies receiving government support had dividend payments limited to $0.01 per share**
- **Recent average: ≈35 percent of profits disbursed as dividends**
  - **Based on recent data collected by U.S. Bureau of Economic Analysis for corporations listed on S&P 500**

DIVIDENDS AND PROFITS

- **Dividend: payment made by a corporation to its shareholders; the portion of corporate profits paid out to stockholders (Wikipedia definition)**
- **Corporate dividend policies differ widely across industries and companies**
  - **Some companies retain most of their profits (to re-invest in ongoing projects)**
  - **Some industries' dividend policies subject to government regulation**
    - **Recently: financial companies receiving government support had dividend payments limited to $0.01 per share**
- **Recent average: ≈35 percent of profits disbursed as dividends**
  - **Based on recent data collected by U.S. Bureau of Economic Analysis for corporations listed on S&P 500**
- **Simplifying assumption for our analytical framework**
  - **All (100 percent) firm profits distributed as dividends**
  - **In period *t*, D_t = nominal profits$_t$**
- **Building Block 4: Relationship between firm profits and dividends**

$$D_t = P_t \cdot profit_t$$

REAL profits of firm in period *t*

FINANCIAL ACCELERATOR FRAMEWORK

- **Four Building Blocks of the Financial Accelerator Framework**

1. **Firm Profit Function**

= 0 = 0

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

2. **Financing Constraint**

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

3. **Government Regulation of Financial Relationships (imposition of *R* on financing constraint)**

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

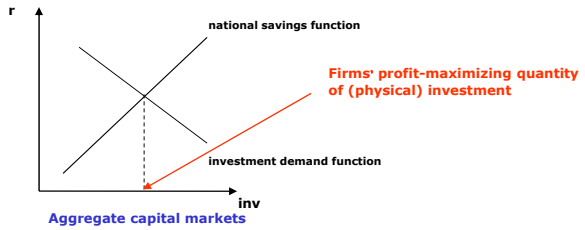
4. **Relationship between firm profits and dividends**

$$D_t = P_t \cdot profit_t$$

REAL profits of firm in period *t*

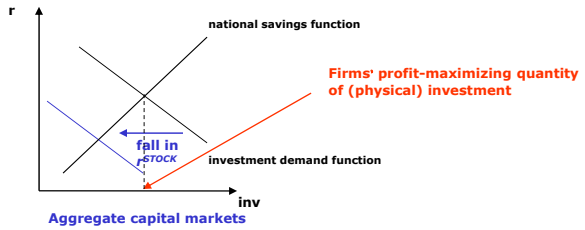
FINANCIAL ACCELERATOR IN ACTION

- Suppose economy is in a "steady-state" in which $r = r^{STOCK}$...



FINANCIAL ACCELERATOR IN ACTION

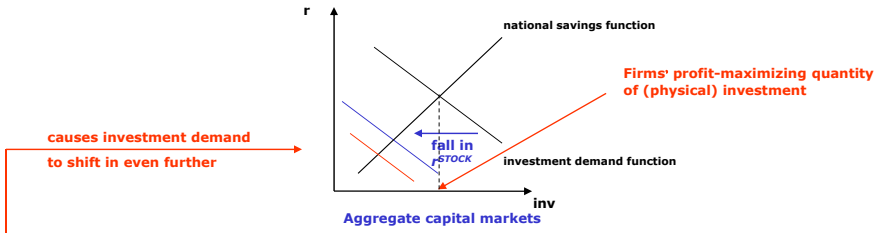
- Suppose economy is in a "steady-state" in which $r = r^{STOCK}$...
- ...then a shock causes r^{STOCK} to decline
  - i.e., broad range of financial asset returns suddenly fall...
  - ...perhaps because of problems stemming from one or a few classes of financial assets (i.e., mortgage-backed bonds)



- Equilibrium quantity of (physical) investment *falls*
  - Investment ≈ 15% of GDP

FINANCIAL ACCELERATOR IN ACTION

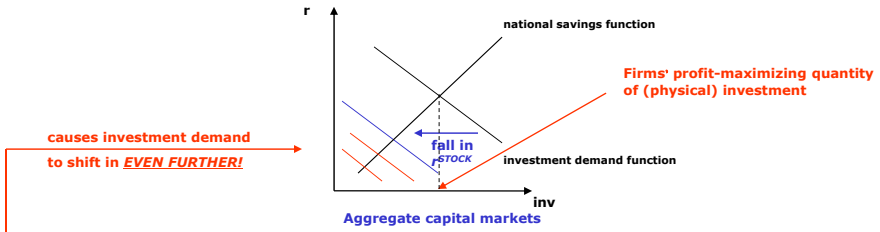
- Suppose economy is in a "steady-state" in which $r = r^{STOCK}$...
- ...then a shock causes r^{STOCK} to decline
  - i.e., broad range of financial asset returns suddenly fall...
  - ...perhaps because of problems stemming from one or a few classes of financial assets (i.e., mortgage-backed bonds)



- Equilibrium quantity of (physical) investment ***falls***
  - Investment ≈ 15% of GDP
- Firm profits ***fall*** (i.e., investment no longer at profit-maximizing choice)
  - → Dividends ***fall*** (Building Block 4: dividends = profits)
  - → r^{STOCK} falls even further! (because *D* a component of r^{STOCK})

FINANCIAL ACCELERATOR IN ACTION

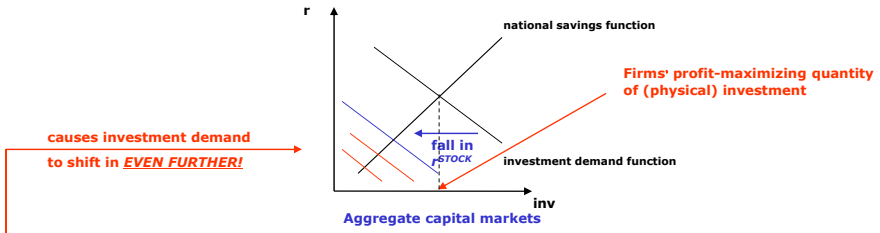
- Suppose economy is in a "steady-state" in which $r = r^{STOCK}$...
- ...then a shock causes r^{STOCK} to decline
  - i.e., broad range of financial asset returns suddenly fall...
  - ...perhaps because of problems stemming from one or a few classes of financial assets (i.e., mortgage-backed bonds)



- Equilibrium quantity of (physical) investment ***falls further***
  - Investment ≈ 15% of GDP
- Firm profits ***fall further***
  - → Dividends ***fall further*** (Building Block 4: dividends = profits)
  - → r^{STOCK} falls ***EVEN FURTHER***! (because *D* a component of r^{STOCK})

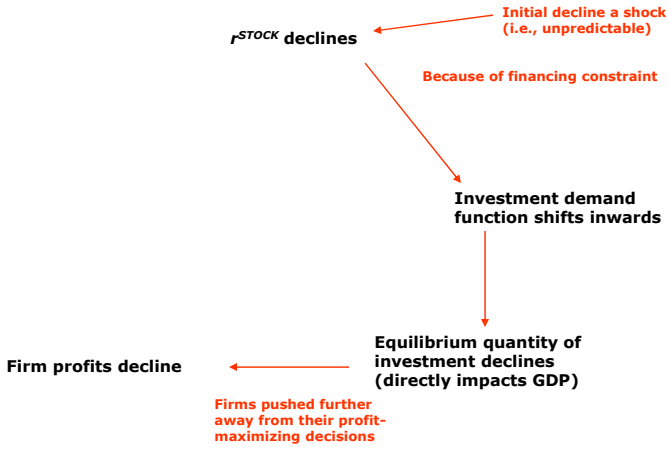
FINANCIAL ACCELERATOR IN ACTION

- Suppose economy is in a "steady-state" in which $r = r^{STOCK}$...
- ...then a shock causes r^{STOCK} to decline
  - i.e., broad range of financial asset returns suddenly fall...
  - ...perhaps because of problems stemming from one or a few classes of financial assets (i.e., mortgage-backed bonds)

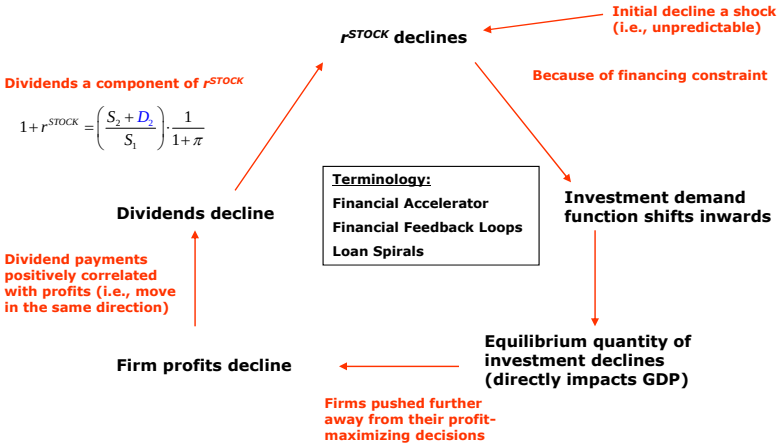


AND ON AND ON...

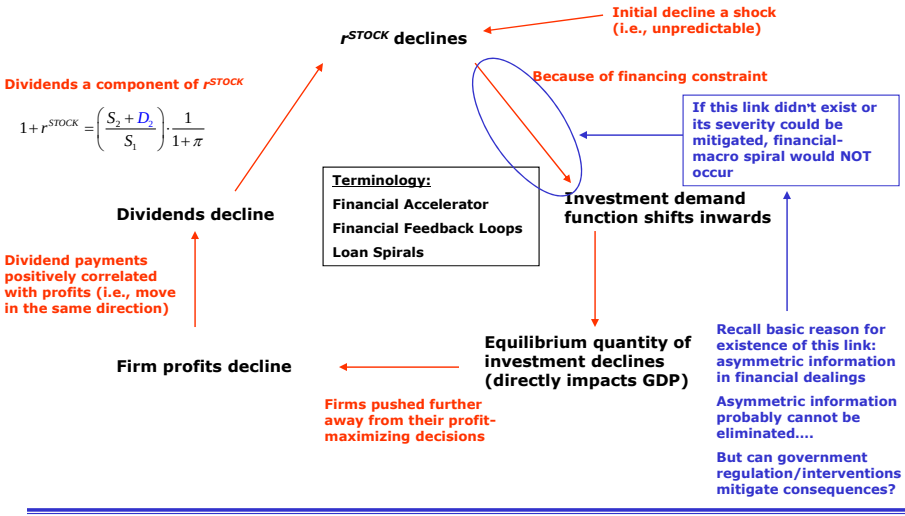
FINANCIAL ACCELERATOR



FINANCIAL ACCELERATOR



FINANCIAL ACCELERATOR



POLICY AND REGULATORY RESPONSES

- Entire accelerator mechanism due to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

- Lagrange multiplier related to asset returns and government regulation by

$$\lambda = \left[\frac{r - r^{STOCK}}{1 + r} \right] \cdot \frac{1}{R}$$

- If r^{STOCK} falls below r (which causes accelerator mechanism to begin)
  - λ increases
  - Optimal regulatory response:

POLICY AND REGULATORY RESPONSES

- Entire accelerator mechanism due to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

- Lagrange multiplier related to asset returns and government regulation by

$$\lambda = \left[\frac{r - r^{STOCK}}{1 + r} \right] \cdot \frac{1}{R}$$

- If r^{STOCK} falls below r (which causes accelerator mechanism to begin)
  - λ increases
  - Optimal regulatory response: raise R, which would cause λ to decline!
  - If designed properly, a rise in R can perfectly offset the fall in r^{STOCK}, thus choking off the damaging effects of the accelerator
- Interpretation of rise in R
  - For a given market value of financial assets, $S_1 a_1$, a higher R allows firms to borrow more from private lenders, in turn allowing them to purchase more (physical) capital
  - One interpretation: government "guarantees" private loans
  - Allows firms to produce more for the same level of financial resources

POLICY AND REGULATORY RESPONSES

- Entire accelerator mechanism due to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

- Interpretation of rise in *R*
  - For a given market value of financial assets, $S_1 a_1$, a higher *R* allows firms to borrow more in order to purchase more (physical) capital
  - Allows firms to produce more for the same exact financial resources
- Changes in *R* can be time-consuming to implement
  - Simultaneously controlled by Federal Reserve, Treasury, Securities and Exchange Commission (SEC), Comptroller of the Currency, and several other regulatory agencies – huge coordination delays!

POLICY AND REGULATORY RESPONSES

- Entire accelerator mechanism due to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

- Interpretation of rise in *R*
  - For a given market value of financial assets, $S_1 a_1$, a higher *R* allows firms to borrow more in order to purchase more (physical) capital
  - Allows firms to produce more for the same exact financial resources
- Changes in *R* can be time-consuming to implement
  - Simultaneously controlled by Federal Reserve, Treasury, Securities and Exchange Commission (SEC), Comptroller of the Currency, and several other regulatory agencies – huge coordination delays!
- Another "policy action" that has the same effect as raising *R*
  - Design policies to raise financial asset prices (i.e., S_1) directly!
  - Exactly the intention of U.S. Troubled Asset Relief Program (TARP)
    - Direct purchases by Treasury of a wide variety of financial assets
    - The increased demand for these assets would lift their price
  - Exactly the intention of Federal Reserve's programs to buy a wide variety of financial assets – increased demand would lift prices

REAL INTEREST RATE

- ***r*** **a key variable for macroeconomic analysis**
- **Chapter 4: *r* measures the price of period-1 consumption in terms of period-2 consumption**
- **Chapter 8: *r* reflects degree of impatience**
- ***r*** **often reflects rate of consumption growth between periods**
- **Chapter 6: *r* measures the price/return of physical assets (i.e., machines and equipment) of firms**
  - **"Riskless" assets**

REAL INTEREST RATE

- ***r*** **a key variable for macroeconomic analysis**
- **Chapter 4: *r* measures the price of period-1 consumption in terms of period-2 consumption**
- **Chapter 8: *r* reflects degree of impatience**
- ***r*** **often reflects rate of consumption growth between periods**
- **Chapter 6: *r* measures the price/return of physical assets (i.e., machines and equipment) of firms**
  - **"Riskless" assets**
- **Now: *r* *also* measures price/return of risky assets (i.e., stock) in "steady state"**
  - **If $r = r^{STOCK}$, financing issues don't affect (very much) macroeconomic outcomes**
  - **If *r* and r^{STOCK} deviate significantly**
    - **Financial conditions of firms matter for investment/output**
    - **And can matter very importantly!**

REAL INTEREST RATE

- ***r*** **a key variable for macroeconomic analysis**
- **Chapter 4: *r* measures the price of period-1 consumption in terms of period-2 consumption**
- **Chapter 8: *r* reflects degree of impatience**
- ***r*** **often reflects rate of consumption growth between periods**
- **Chapter 6: *r* measures the price/return of physical assets (i.e., machines and equipment) of firms**
  - **"Riskless" assets**
- **Now: *r* *also* measures price/return of risky assets (i.e., stock) in "steady state"**
- **Can also think of *λ* itself as a type of real interest rate – an interest *SPREAD***
  - **The price of bringing funds from "outside sources" (i.e., lenders) "inside" the firm (i.e., the borrower) to finance operations**
    - **If $r = r^{STOCK}$, this price equals zero**
    - **Cost of "external funding sources" vs. "internal funding sources" due to info. asymmetry**