CONSUMPTION-LABOR FRAMEWORK
(aka CONSUMPTION-LEISURE FRAMEWORK)

JANUARY 30, 2012

BASICS

- Consumption-Leisure framework – provides foundation for
  - Labor-market supply function
  - Goods-market demand function
  - An application of the basic consumer theory model…
  - …we will put a macro interpretation on it
  - Only one time period – no "future" for which to save

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- Notation
  - *c*: consumption ("all stuff")
  - *n*: number of hours spent working per week
  - *l*: number of hours leisure per week (time spent not working)
  - *P*: dollar price of one unit of consumption (a nominal variable)
  - *W*: hourly wage rate in terms of dollars (a nominal variable)
  - *t*: tax rate on labor income

  $n + l = 168$

- "Weekly" model a detail
  - Could have called it a daily model, a monthly model, a yearly model, …
  - Just need to take SOME stand on the length of a "period"

BASICS

- Building blocks of consumption-leisure framework

- Utility
  - Describes the benefits of engaging in labor market (and other) activities

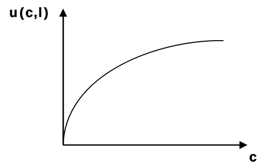
- Budget constraint
  - Describes the costs of engaging in labor market (and other) activities

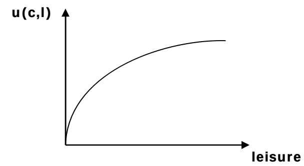
- Utility and budgets two ***DISTINCT*** concepts
  - As in basic consumer analysis (Chapter 1)

- Only after describing utility and budgets separately do we bring the two together to obtain predictions from the framework

UTILITY

- Preferences *u*(*c*, *l*) with all the "usual properties"
  - Strictly increasing in *c*
  - Strictly increasing in *l*
  - Diminishing marginal utility in *c*
  - Diminishing marginal utility in *l*
  - Plotted in good-by-good spaces:





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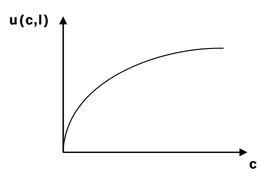
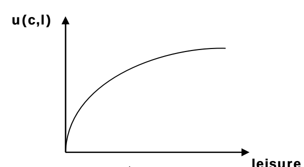
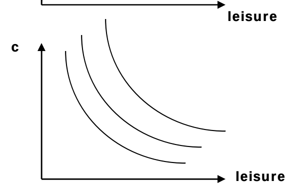




  - Plotted as indifference curves



  - Utility side of consumption-leisure framework identical to Chapter 1 framework

BUDGET CONSTRAINT

- **Consumer must work for his income**
  - ***Y*** **no longer "falls from the sky"**

$$Pc = Y$$

↓ $Y = (1-t)Wn$ (all income is after-tax labor income)

$$Pc = (1-t)Wn$$

↓ $n = 168 - l$

$$Pc = (1-t)W(168-l)$$

↓ Rearrange

$$Pc + (1-t)Wl = 168(1-t)W$$

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Pc: Spending on consumption

$168(1-t)W$: A constant from the point of view of the individual (price-taker)

$$P_1c_1 + P_2c_2 = Y$$

Chapter 1 budget constraint

P_1c_1: Spending on c_1

P_2c_2: Spending on c_2

Y: A constant from the point of view of the individual

BUDGET CONSTRAINT

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↓ *n* = 168 - *l*

$$Pc = (1-t)W(168-l)$$

↓ Rearrange

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Spending on consumption (Pc) | "Spending" on leisure ($(1-t)Wl$) | A constant from the point of view of the individual (price-taker) ($168(1-t)W$)

(After-tax) wage is *opportunity cost* of leisure, hence the "price" of leisure

- opportunity costs *are real economic costs/prices*

Simply an application/re-interpretation of our basic consumer theory framework

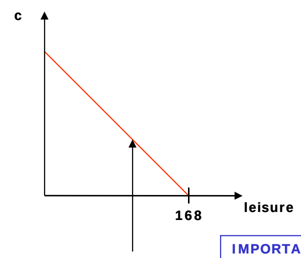
Chapter 1 budget constraint

$$P_1c_1 + P_2c_2 = Y$$

Spending on c_1 | Spending on c_2 | A constant from the point of view of the individual

CONSUMER OPTIMIZATION

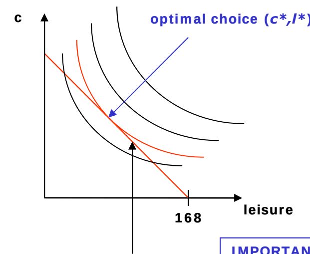
- **Consumer's decision problem: maximize utility subject to budget constraint – bring together both cost side and benefit side**
  - **Choose *c* and *l* subject to** $Pc + (1-t)Wl = 168(1-t)W$
  - **Plot budget line**



IMPORTANT: the larger is (1-*t*)*W*/*P*, the steeper is the budget line

Consumer Optimization

- **Consumer's decision problem:** maximize utility subject to budget constraint – bring together both **cost** side and **benefit** side
  - Choose *c* and *l* subject to $Pc + (1-t)Wl = 168(1-t)W$
  - Plot budget line
  - Superimpose indifference map



IMPORTANT: the larger is (1-*t*)*W* / *P*, the steeper is the budget line

- At the optimal choice

CONSUMPTION-LEISURE OPTIMALITY CONDITION

- A key building block of modern macro models

$$\frac{u_l(c^*, l^*)}{u_c(c^*, l^*)} = \frac{(1-t)W}{P}$$

MRS (between consumption and leisure) — price ratio (inclusive of taxes)

Real Wage

- ***W* / *P*** a key variable for macroeconomic analysis
- Unit Analysis (i.e., analyze algebraic units of variables)
  - Units(***W***) = $ / hour of work
  - Units(***P***) = $ / unit of consumption

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$$= \frac{\text{unit of consumption}}{\text{hour of work}}$$

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Will sometimes denote using *w* (lower-case…)

- **Economic decisions depend on *real* wages (*W / P*), not nominal wages (*W*)**
  - **Measures the purchasing power of (nominal) wage earnings…**
  - **…which is presumably what people most care about**

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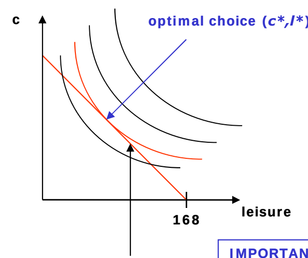

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CONSUMPTION-LEISURE OPTIMALITY CONDITION

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$$\underbrace{\frac{u_l(c^*, l^*)}{u_c(c^*, l^*)}}_{\text{MRS (between consumption and leisure)}} = \underbrace{\frac{(1-t)W}{P}}_{\text{After-tax real wage}}$$

IMPORTANT: the larger is (1-t)W/P, the steeper is the budget line

Derive consumption-leisure optimality condition using Lagrange analysis

LAGRANGE ANALYSIS

- Apply Lagrange tools to consumption-leisure optimization
- Objective function: $u(c,l)$
- Constraint: $g(c,l) = 168(1-t)W - Pc - (1-t)Wl = 0$

- **Step 1:** Construct Lagrange function

$$L(c,l,\lambda) = u(c,l) + \lambda\left[168(1-t)W - Pc - (1-t)Wl\right]$$

- **Step 2:** Compute first-order conditions with respect to c, l, λ

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- **Step 2: Compute first-order conditions with respect to c, l, λ**

- **Step 3: Solve (with focus on eliminating multiplier)**

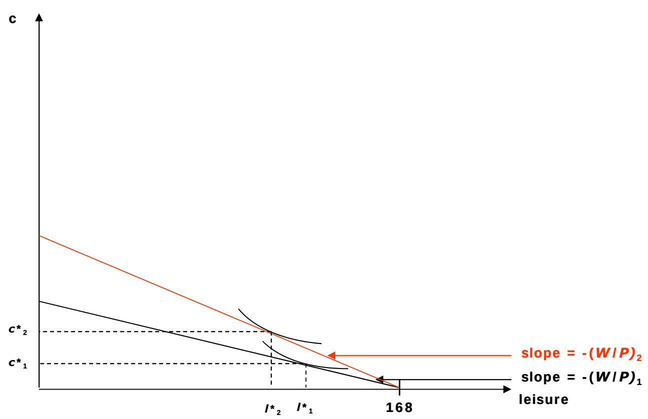
$$\underbrace{\frac{u_l(c^*,l^*)}{u_c(c^*,l^*)}}_{\text{MRS (between consumption and leisure)}} = \underbrace{\frac{(1-t)W}{P}}_{\text{After-tax real wage}}$$

CONSUMPTION-LEISURE OPTIMALITY CONDITION

MICRO-LEVEL LABOR SUPPLY

<u>An experiment:</u> how do micro-level consumption/leisure choices change as the real wage changes (assume t = 0 here for simplicity)

REAL WAGES: $(W/P)_1 < (W/P)_2$



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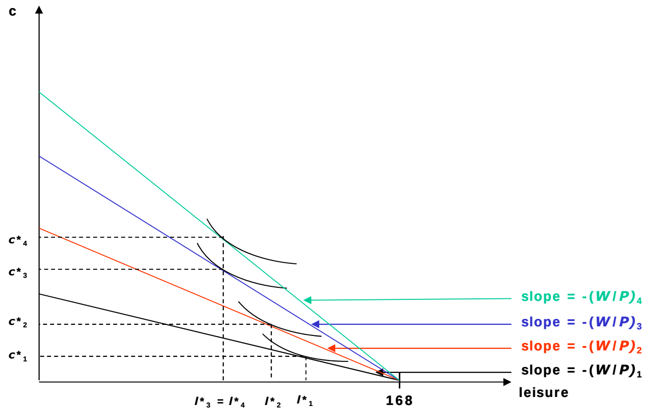

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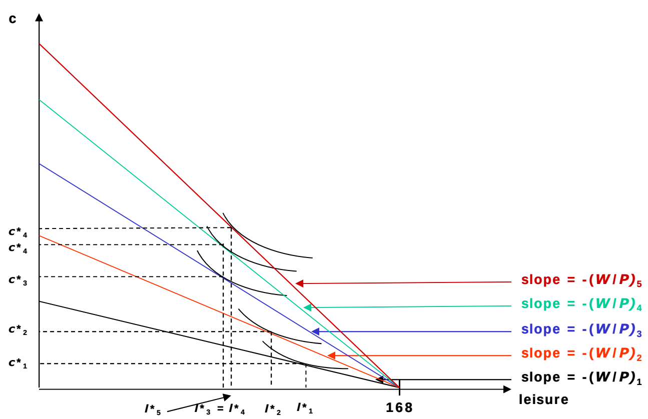

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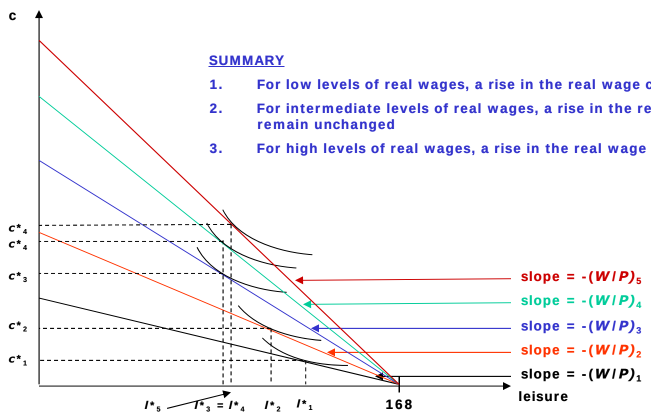
REAL WAGES: $(W/P)_1 < (W/P)_2 < (W/P)_3 < (W/P)_4 < (W/P)_5$

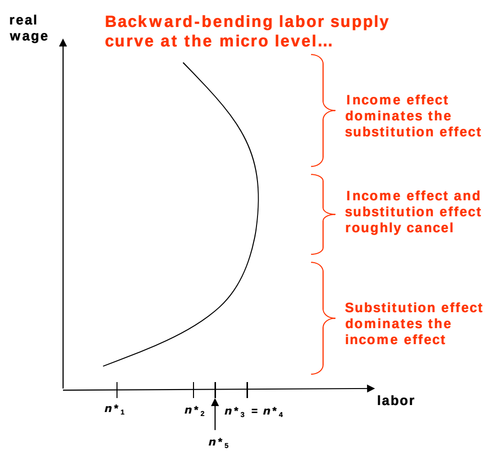
SUMMARY

1. For low levels of real wages, a rise in the real wage causes optimal leisure to decrease
2. For intermediate levels of real wages, a rise in the real wage causes optimal leisure to remain unchanged
3. For high levels of real wages, a rise in the real wage causes optimal leisure to increase



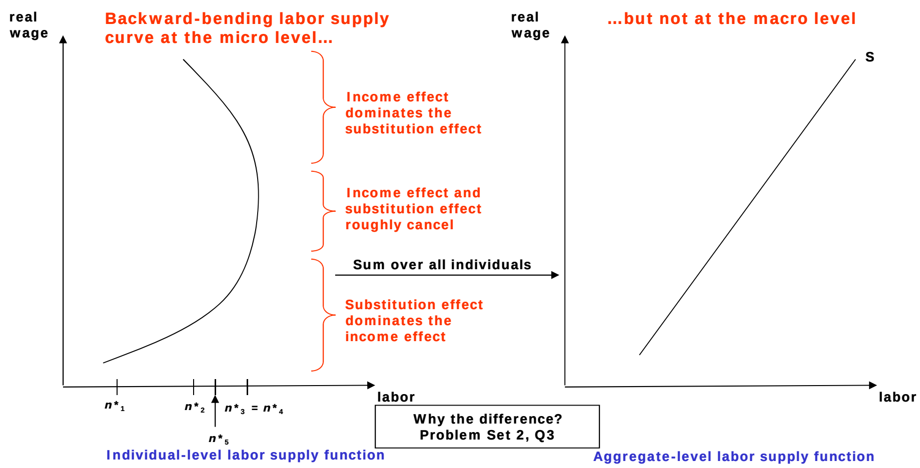
MICRO-LEVEL LABOR SUPPLY

Using the relation $n = 168 - l$



Labor Supply

Using the relation $n = 168 - l$



Macro vs. Micro Labor Quantities

$$\text{Average hours per worker} = \frac{\text{Aggregate Hours Worked}}{\text{Number of individuals working}}$$

$$\text{Aggregate Hours Worked} = \text{Average hours per worker} \times \text{Number of individuals working}$$

Aggregate Hours Worked: Macro / representative-agent framework has typically been most concerned with this

Average hours per worker: Micro studies measure this — "Intensive margin"

Number of individuals working: "Extensive margin"

Macro vs. Micro Labor Quantities

Average hours per worker = Aggregate Hours Worked / Number of individuals working

Aggregate Hours Worked = Average hours per worker x Number of individuals working

Aggregate Hours Worked: Macro / representative-agent framework has typically been most concerned with this

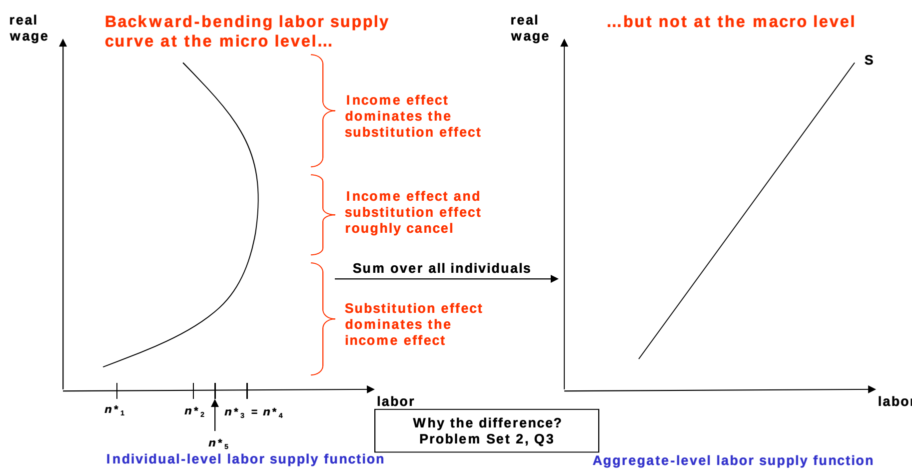
Average hours per worker: Micro studies measure this — "Intensive margin"

Number of individuals working: "Extensive margin"

- Standard rep-agent framework offers an aggregate theory of employment
  - But not necessarily of unemployment
- Search and matching theory is a theory of unemployment

Labor Supply

Using the relation $n = 168 - l$



CONSUMPTION DEMAND

- **Optimal choice of consumption was always rising as real wage was rising**

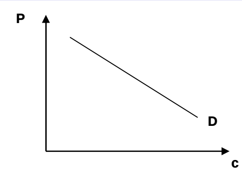
- **Could have conducted the entire analysis assuming nominal *W* was held fixed and nominal *P* was falling**
  - **Which means real wage *W* / *P* is rising**

- **Result: Fall in *P* → rise in optimal *c* always**
  - **Implies downward-sloping consumption demand function at the micro level…**
  - **…and at the aggregate level**

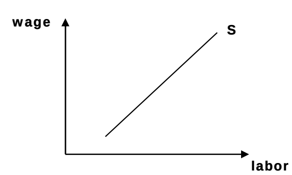
- **Consumption demand over two-thirds of aggregate demand in developed countries**

THE THREE MACRO (AGGREGATE) MARKETS

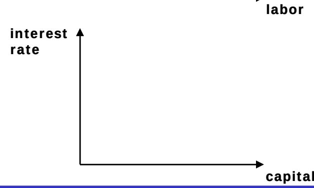
- **Goods Markets**
  - **Demand derived from C-L framework**



- **Labor Markets**
  - **Supply derived from C-L framework**



- **Financial / Capital / Savings / Asset Markets**



THE MACROECONOMICS OF TIME

- Consumption-leisure model a static (i.e., one time period) model
- Dynamic models the core of modern macroeconomic theory
- Explicit consideration of how economic decisions/behaviors/outcomes unfold over multiple time periods
- Two-period framework (Chapters 3 and 4) the simplest possible multi-period framework
  - Will allow us to begin analyzing issues regarding interest rates and inflation (phenomena that occur across time)
  - Will allow us to think about credit restrictions and the "credit crunch"
- Infinite-period framework (Chapter 8)
  - Allows a richer quantitative description of the macroeconomy
  - Highlights the role of assets and the intersection between finance and macroeconomics