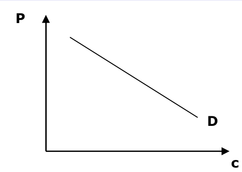
CONSUMPTION-SAVINGS FRAMEWORK

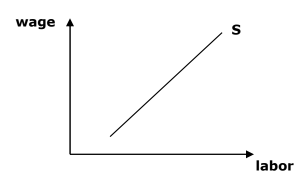
FEBRUARY 6, 2012

THE THREE MACRO (AGGREGATE) MARKETS

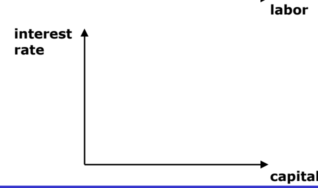
- **Goods Markets**
  - **Demand derived from C-L framework**



- **Labor Markets**
  - **Supply derived from C-L framework**



- **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**



IN THE NEWS

- Understanding debts and assets a prerequisite for understanding financial-market outcomes
- Assets are positive wealth
- Debts are negative wealth
- Net wealth = assets – debt
- Impossible to understand the economics of asset and debt accumulation without reference to decision-making over time

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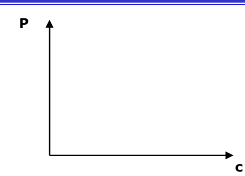

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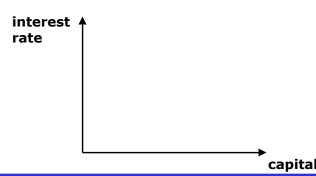

BASICS

- Consumption-Savings Framework – provides foundation for
  - Goods-market demand function (again…but w/different interpretation)
  - Financial-market supply function
  - An application of basic consumer analysis…
  - …we will put a macro interpretation on it
  - Two time periods
    - Important: all analysis conducted from the perspective of the very beginning of period 1…
    - …so a "future" (period 2) for which to save

Introduction

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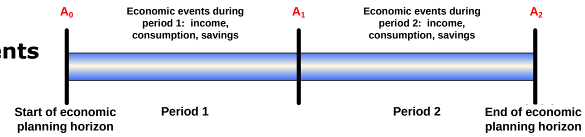
- Dynamic models are central to modern macroeconomic analysis

- An explicit accounting of time

- Two periods are sufficient to illustrate the basic principles
  - Soon will extend beyond two periods (Chapter 8)

BASICS

- **Timeline of events**



- **Notation**
  - c_1: consumption in period 1
  - c_2: consumption in period 2
  - P_1: nominal price of consumption in period 1
  - P_2: nominal price of consumption in period 2
  - Y_1: nominal income in period 1 ("falls from the sky")
  - Y_2: nominal income in period 2 ("falls from the sky")
  - A_0: nominal wealth at the beginning of period 1/end of period 0
  - A_1: nominal wealth at the beginning of period 2/end of period 1
  - A_2: nominal wealth at the beginning of period 3/end of period 2

BASICS

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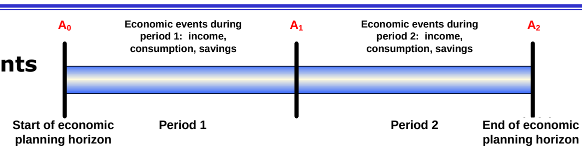

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  - A_2: nominal wealth at the beginning of period 3/end of period 2
  - i: nominal interest rate between periods
  - r: real interest rate between periods
  - π_2: net inflation rate between period 1 and period 2 $\pi_2 = \frac{P_2 - P_1}{P_1}\left(= \frac{P_2}{P_1} - 1\right)$
  - y_1: real income in period 1 ($= Y_1/P_1$)
  - y_2: real income in period 2 ($= Y_2/P_2$)

STOCKS VS. FLOWS

- **Stock variables (aka accumulation variables)**
  - **Quantity variables whose natural measurement occurs at a particular moment in time**

- **Flow variables**
  - **Quantity variables whose natural measurement occurs over the course of a given interval of time**

STOCKS VS. FLOWS

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  - **Quantity variables whose natural measurement occurs at a particular moment in time**
  - Economic examples:
    - **Checking account balance**
    - **Credit card indebtedness**
    - **Mortgage loan payoff**
    - Interpret *A* in our model as net wealth (= total assets – total debts)

- **Flow variables**
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  - Economic examples:
    - **Income**
    - **Consumption**
    - **Savings**

STOCKS VS. FLOWS

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Economic examples

Interpret *A* in our model as net wealth
(= total assets – total debts)

- Flow variables
  - Quantity variables whose natural measurement occurs over the course of a given interval of time
  - Income
  - Consumption
  - Savings

Economic examples

- The two broad categories of income
  - Labor income
  - Asset income (generated by interest rate(s) on (components of) wealth)

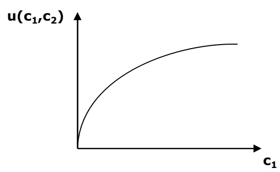
All income is a FLOW regardless of source

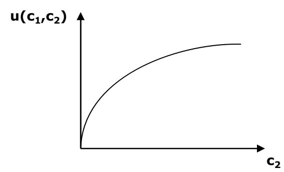
BASICS

- Building blocks of consumption-savings framework
- Utility
  - Describes the benefits of engaging in financial market (and other) activities
- Budget constraint
  - Describes the costs of engaging in financial market (and other) activities
- Utility and budgets two ***DISTINCT*** concepts
  - As in basic consumer analysis (Chapter 1)
- Only after describing utility and budgets separately do we bring the two together to obtain predictions from the framework

UTILITY

- Preferences $u(c_1, c_2)$ with all the "usual properties"
  - Lifetime utility function
  - Strictly increasing in c_1
  - Strictly increasing in c_2
  - Diminishing marginal utility in c_1
  - Diminishing marginal utility in c_2





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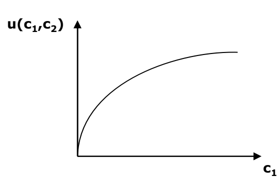
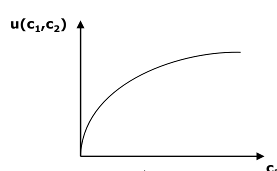
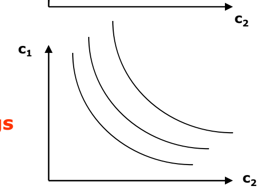




  - Plotted as indifference curves



  - Utility side of consumption-savings framework identical to Chapter 1 framework

BUDGET CONSTRAINT(S)

- **Suppose again *Y* "falls from the sky"**
  - **Y_1 in period 1, Y_2 in period 2**
- **Need two budget constraints to describe economic opportunities and possibilities**
  - **One for each period**
  - **Period-1 budget constraint**

$$\underbrace{P_1c_1 + A_1}_{} = \underbrace{Y_1 + (1+i)A_0}_{}$$

Total expenditure in period 1: period-1 consumption + wealth to *carry into period* 2

Total income in period 1: period-1 *Y* + income from wealth *carried into period* 1 (inclusive of interest)

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  - **Period-2 budget constraint**

$$\underbrace{P_2c_2 + A_2}_{} = \underbrace{Y_2 + (1+i)A_1}_{}$$

Total expenditure in period 2: period-2 consumption + wealth to *carry into period* 3

Total income in period 2: period-2 *Y* + income from wealth *carried into period* 2 (inclusive of interest)

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Total expenditure in period 1: period-1 consumption + wealth to *carry into period* 2

Total income in period 1: period-1 Y + income from wealth *carried into period* 1 (inclusive of interest)

Savings during period 1 (a flow) ($A_1 - A_0$)

Asset income during period 1 (a flow) (iA_0)

  - **Period-2 budget constraint**

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Total expenditure in period 2: period-2 consumption + wealth to *carry into period* 3

Total income in period 2: period-2 Y + income from wealth *carried into period* 2 (inclusive of interest)

Savings during period 2 (a flow) ($A_2 - A_1$)

Asset income during period 2 (a flow) (iA_1)

> **DEFINITION:** A consumer's savings during a given period is the *change* in his wealth during that period

BUDGET CONSTRAINT(S)

- **Adopt a lifetime view of the budget constraint(s)**
  - **All analysis conducted from perspective of beginning of period 1**
  - **Period-1 budget constraint** $P_1c_1 + A_1 = Y_1 + (1+i)A_0$
  - **Period-2 budget constraint** $P_2c_2 + A_2 = Y_2 + (1+i)A_1$

Asset position at end of period 1/beginning of period 2 the key link - will think further about this soon…

Assume A_2 = 0 (no bankruptcies + strictly increasing utility)

Budget Constraint(s)

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  - Combine into **lifetime budget constraint (LBC)**
    - Solve period-2 budget constraint for A_1…
    - …and substitute into period-1 budget constraint

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i} + (1+i)A_0$$

Present discounted value (PDV) of all lifetime expenditure (left side)

Present discounted value (PDV) of all lifetime income (right side)

For graphical simplicity, will often assume $A_0 = 0$ (i.e., consumer begins planning horizon with zero net wealth). Note this is a *different* assumption than $A_2 = 0$.

Lifetime Budget Constraint

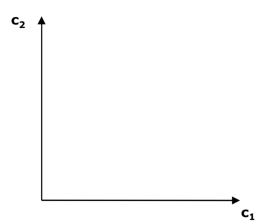
Graphically

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

Solve for c_2

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right)c_1 + \left(\frac{1+i}{P_2}\right)Y_1 + \frac{Y_2}{P_2}$$

Rearrange further using definition of inflation: $1+\pi_2 = \frac{P_2}{P_1} \Rightarrow \frac{1}{1+\pi_2} = \frac{P_1}{P_2}$



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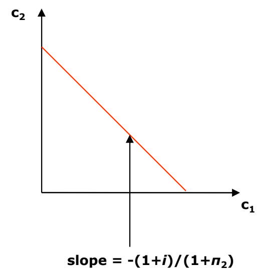
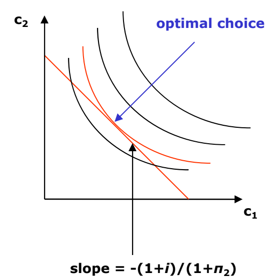


The larger is (1+*i*)/(1+*π*$_2$), the steeper is the budget line

IMPORTANT: Changes in nominal interest rates (Fed) and/or inflation affect the slope of the LBC

CONSUMER OPTIMIZATION

- **Consumer's decision problem: maximize lifetime utility subject to lifetime budget constraint – bring together both cost side and benefit side**
  - **Choose c_1 and c_2 subject to** $P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$
  - **Plot budget line**
  - **Superimpose indifference map**



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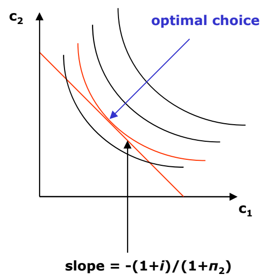

- **At the optimal choice**

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

- key result in modern macro analysis

$$\underbrace{\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)}}_{\text{MRS (between consumption in consecutive time periods)}} = \underbrace{\frac{1+i}{1+\pi_2}}_{\text{price ratio (across consecutive time periods)}}$$

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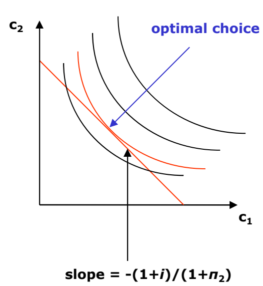

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Derive consumption-savings optimality condition using Lagrange analysis

LAGRANGE ANALYSIS

- **Apply Lagrange tools to consumption-savings optimization**
- **Objective function:** $u(c_1,c_2)$
- **Constraint:** $g(c_1,c_2) = Y_1 + \frac{Y_2}{1+i} - P_1 c_1 - \frac{P_2 c_2}{1+i} = 0$
- **Step 1: Construct Lagrange function**

$$L(c_1,c_2,\lambda) = u(c_1,c_2) + \lambda\left[Y_1 + \frac{Y_2}{1+i} - P_1 c_1 - \frac{P_2 c_2}{1+i}\right]$$

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- **Step 3: Combine (with focus on eliminating multiplier)**

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CONSUMPTION-SAVINGS OPTIMALITY CONDITION

SAVINGS AND ASSET POSITIONS

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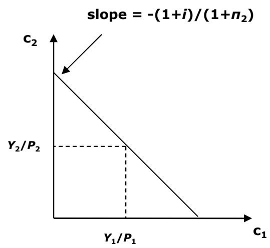
- **Assets/wealth (whether positive or negative) are a means for "transferring income over time"**



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Macro Fundamentals

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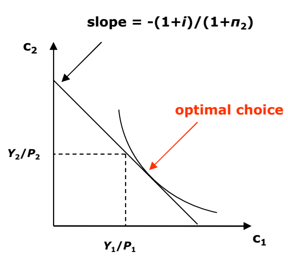

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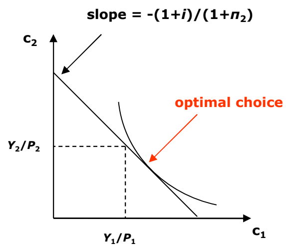

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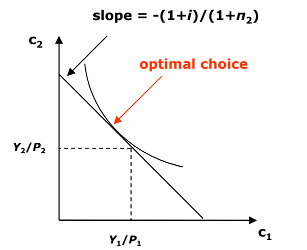

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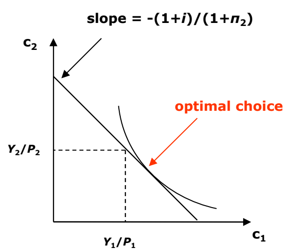
Consumer *saved* during period 1 { **Optimal $c_1 < Y_1/P_1$ → thus consumer has positive wealth at the end of period 1 (i.e., period-1 *savings* is positive)**

ASSESSING THE CREDIT CRUNCH



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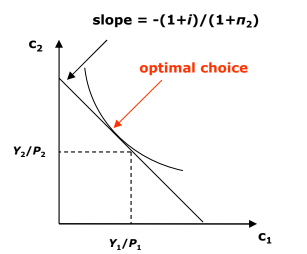

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Use this framework to analyze the channel by which financial market downturn dampened consumer spending

Fisher Equation

- Nominal interest rate – measured in dollars
- Real interest rate – measured in goods

- Fisher equation: a link between nominal interest rate, inflation rate, and real interest rate
  - "Strips out the effect of inflation"
  - Exact Fisher equation (will see foundations in Chapter 14)

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  - Approximate Fisher equation (intro macro)

$$(1+r)(1+\pi)=1+i$$

$$1+r+\pi+\underbrace{r\pi}_{\approx 0}=1+i$$

In advanced economies, *r* and *π* are both generally small → *rπ* ≈ 0

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More useful for our analytical framework

  - Approximate Fisher equation (intro macro)

$$(1+r)(1+\pi)=1+i$$

$$\cancel{1}+r+\pi+\underbrace{\cancel{r\pi}}_{\approx 0}=\cancel{1}+i$$

In advanced economies, *r* and *π* are both generally small → *rπ* ≈ 0

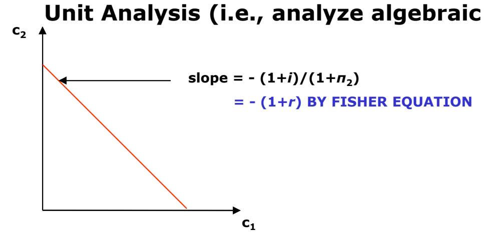
$$r=i-\pi$$

A useful rule of thumb

REAL INTEREST RATE

- *r* a key variable for macroeconomic analysis

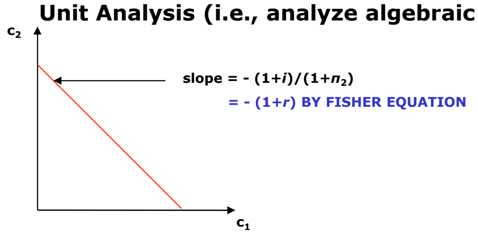
- Unit Analysis (i.e., analyze algebraic units of variables)



Slope measures how much c_2 must be given up in order to obtain one more unit of c_1 ("rise over run") when borrowing or lending at market prices

REAL INTEREST RATE

- ***r*** **a key variable for macroeconomic analysis**
- **Unit Analysis (i.e., analyze algebraic units of variables)**



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More generally: r measures the price of current goods in terms of future goods

Macro Fundamentals

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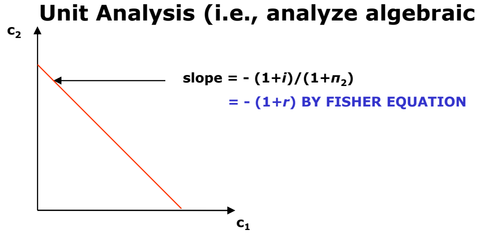

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- **Economic decisions depend on *real* interest rates (*r*), not nominal interest rates (*i*)**
  - **Measures the cost of borrowing/lending in terms of goods…**
  - **…which is presumably what people most care about**

Two-Period Model in Real Terms

- Depending on application, may be useful to work with model (independent of lifetime vs. sequential approach) in nominal terms or in real terms

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

LBC in nominal terms (assuming $A_0 = 0$)

↓ Divide by P_1

$$c_1 + \left(\frac{P_2}{P_1(1+i)}\right)c_2 = \frac{Y_1}{P_1} + \frac{Y_2}{P_1(1+i)}$$

↓ Multiply *and* divide last term on right-hand-side by P_2

$$c_1 + \left(\frac{P_2}{P_1(1+i)}\right)c_2 = \frac{Y_1}{P_1} + \left(\frac{P_2}{P_1(1+i)}\right)\frac{Y_2}{P_2}$$

↓ Use definitions: $y_1 = Y_1/P_1$, $y_2 = Y_2/P_2$, and $1+\pi_2 = P_2/P_1$

$$c_1 + \left(\frac{1+\pi_2}{1+i}\right)c_2 = y_1 + \left(\frac{1+\pi_2}{1+i}\right)y_2$$

↓ Use Fisher equation: $(1+\pi_2)/(1+i) = 1/(1+r)$

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

LBC in real terms (assuming $A_0 = 0$)

Maximize $u(c_1,c_2)$ subject to the real LBC → identical consumption-savings optimality condition (work out details yourself)

Consumption-Savings Optimality Condition

- Emphasizing *i* and *π* $\qquad \dfrac{u_1(c_1^*,c_2^*)}{u_2(c_1^*,c_2^*)} = \dfrac{1+i}{1+\pi}$

↓ Fisher equation

- Emphasizing *r* $\qquad \dfrac{u_1(c_1^*,c_2^*)}{u_2(c_1^*,c_2^*)} = 1+r$

- Can also analyze two-period model sequentially, rather than from a lifetime perspective

Lagrange Analysis: Sequential Approach

- **Sequential formulation highlights the role of net wealth (A_1) between period 1 and period 2**
  - **Accords better with the explicit timing of economic events than the lifetime approach…**
  - **…but yields the same result**
  - **Advantage: allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory in Chapter 8)**

- **Apply Lagrange tools to consumption savings optimization**
- **Objective function: $u(c_1,c_2)$**
- **Constraints:**
  - **Period 1 budget constraint:** $Y_1 + (1+i)A_0 - P_1c_1 - A_1 = 0$
  - **Period 2 budget constraint:** $Y_2 + (1+i)A_1 - P_2c_2 - A_2 = 0$

  TWO constraints

- **Sequential Lagrange formulation requires two multipliers**
  - **See Math Refresher, Chapter -1**