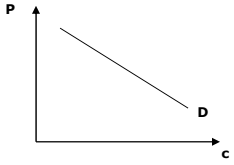
CONSUMPTION-SAVINGS FRAMEWORK (CONTINUED)

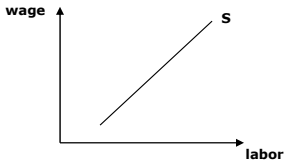
FEBRUARY 13, 2012

THE THREE MACRO (AGGREGATE) MARKETS

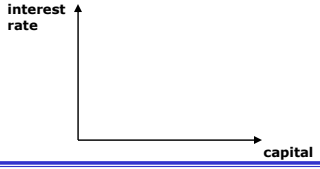
- **Goods Markets**
  - Demand derived from C-L framework



- **Labor Markets**
  - Supply derived from C-L framework

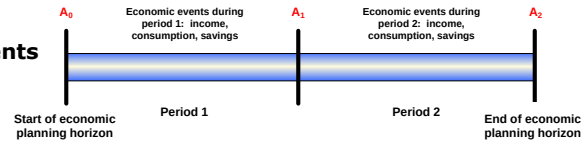


- **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**



BASICS

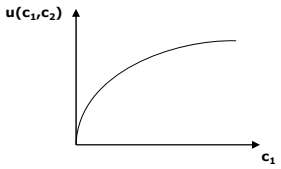
- **Timeline of events**

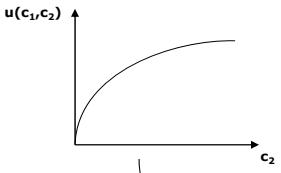


- **Notation**
  - c_1: consumption in period 1
  - c_2: consumption in period 2
  - P_1: nominal price of consumption in period 1
  - P_2: nominal price of consumption in period 2
  - Y_1: nominal income in period 1 ("falls from the sky")
  - Y_2: nominal income in period 2 ("falls from the sky")
  - A_0: nominal wealth at the beginning of period 1/end of period 0
  - A_1: nominal wealth at the beginning of period 2/end of period 1
  - A_2: nominal wealth at the beginning of period 3/end of period 2
  - i: nominal interest rate between periods
  - r: real interest rate between periods
  - n_2: net inflation rate between period 1 and period 2 $\pi_2 = \frac{P_2 - P_1}{P_1}\left(= \frac{P_2}{P_1} - 1\right)$
  - y_1: real income in period 1 (= Y_1/P_1)
  - y_2: real income in period 2 (= Y_2/P_2)

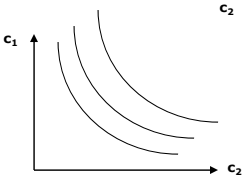
UTILITY

- **Preferences $u(c_1, c_2)$ with all the "usual properties"**
  - **Lifetime utility function**
  - **Strictly increasing in c_1**
  - **Strictly increasing in c_2**
  - **Diminishing marginal utility in c_1**
  - **Diminishing marginal utility in c_2**





  - **Plotted as indifference curves**



  - **Utility side of consumption-savings framework identical to Chapter 1 framework**

BUDGET CONSTRAINT(S)

- Suppose again *Y* "falls from the sky"
  - Y_1 in period 1, Y_2 in period 2
- Need **two** budget constraints to describe economic opportunities and possibilities
  - One for each period
  - Period-1 budget constraint

$$P_1c_1 + A_1 = Y_1 + (1+i)A_0 \quad \text{can rewrite as} \quad P_1c_1 + A_1 - A_0 = Y_1 + iA_0$$

Savings during period 1 (a flow); Asset income during period 1 (a flow)

Total expenditure in period 1: period-1 consumption + wealth to *carry into period* 2

Total income in period 1: period-1 *Y* + income from wealth *carried into period* 1 (inclusive of interest)

  - Period-2 budget constraint

$$P_2c_2 + A_2 = Y_2 + (1+i)A_1 \quad \text{can rewrite as} \quad P_2c_2 + A_2 - A_1 = Y_2 + iA_1$$

Savings during period 2 (a flow); Asset income during period 2 (a flow)

Total expenditure in period 2: period-2 consumption + wealth to *carry into period* 3

Total income in period 2: period-2 *Y* + income from wealth *carried into period* 2 (inclusive of interest)

DEFINITION: A consumer's savings during a given period is the *change* in his wealth during that period

BUDGET CONSTRAINT(S)

- Adopt a **lifetime** view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 budget constraint $P_1c_1 + A_1 = Y_1 + (1+i)A_0$
  - Period-2 budget constraint $P_2c_2 + A_2 = Y_2 + (1+i)A_1$

Asset position at end of period 1/beginning of period 2 the key link - will think further about this soon...

Assume = 0 (no bankruptcies + strictly increasing utility)

  - Combine into **lifetime budget constraint (LBC)**
    - Solve period-2 budget constraint for A_1...
    - ...and substitute into period-1 budget constraint

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i} + (1+i)A_0$$

Present discounted value (PDV) of all lifetime expenditure

Present discounted value (PDV) of all lifetime income

LIFETIME BUDGET CONSTRAINT

Graphically

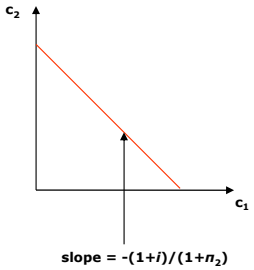
$$P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

Solve for c_2

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right) c_1 + \left(\frac{1+i}{P_2}\right) Y_1 + \frac{Y_2}{P_2}$$

Rearrange further using definition of inflation: $1+\pi_2 = \frac{P_2}{P_1} \Rightarrow \frac{1}{1+\pi_2} = \frac{P_1}{P_2}$

$$c_2 = -\left(\frac{1+i}{1+\pi_2}\right) c_1 + \left(\frac{1+i}{P_2}\right) Y_1 + \frac{Y_2}{P_2}$$

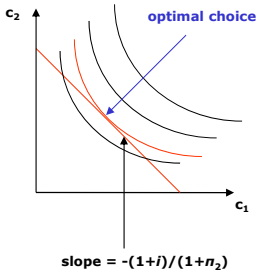


The larger is $(1+i)/(1+\pi_2)$, the steeper is the budget line

IMPORTANT: Changes in nominal interest rates (Fed) and/or inflation affect the slope of the LBC

CONSUMER OPTIMIZATION

- **Consumer's decision problem:** maximize lifetime utility subject to lifetime budget constraint – bring together both **cost** side and **benefit** side
  - Choose c_1 and c_2 subject to $P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$
  - Plot budget line
  - Superimpose indifference map
- At the optimal choice



CONSUMPTION-SAVINGS OPTIMALITY CONDITION

- A key building block of modern macro models

$$\underbrace{\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)}}_{\text{MRS (between consumption in consecutive time periods)}} = \underbrace{\frac{1+i}{1+\pi_2}}_{\text{price ratio (across consecutive time periods)}}$$

LAGRANGE ANALYSIS: LIFETIME APPROACH

- Apply Lagrange tools to consumption-savings optimization
- Objective function: $u(c_1,c_2)$
- Constraint (assuming $A_0 = 0$): $g(c_1,c_2) = Y_1 + \frac{Y_2}{1+i} - P_1c_1 - \frac{P_2c_2}{1+i} = 0$
- **Step 1:** Construct Lagrange function

$$L(c_1,c_2,\lambda) = u(c_1,c_2) + \lambda\left[Y_1 + \frac{Y_2}{1+i} - P_1c_1 - \frac{P_2c_2}{1+i}\right]$$

- **Step 2:** Compute first-order conditions with respect to c_1, c_2, λ
- **Step 3:** Combine (with focus on eliminating multiplier)

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

$$\underbrace{\frac{u_1(c_1^*,c_2^*)}{u_2(c_1^*,c_2^*)}}_{\text{MRS (between consumption in consecutive time periods)}} = \underbrace{\frac{1+i}{1+\pi_2}}_{\text{price ratio (across consecutive time periods)}} = 1+r$$

using Fisher equation

TWO-PERIOD FRAMEWORK

- Depending on application, may be useful to work with framework (independent of lifetime vs. sequential approach) in nominal terms or in real terms

	IN NOMINAL TERMS	IN REAL TERMS
Period-1 budget constraint	$P_1c_1 + A_1 = Y_1 + (1+i)A_0$	$c_1 + a_1 = y_1 + (1+r)a_0$
Period-2 budget constraint	$P_2c_2 + A_2 = Y_2 + (1+i)A_1$	$c_2 + a_2 = y_2 + (1+r)a_1$
LBC	$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i} + (1+i)A_0$	$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + (1+r)a_0$

TWO-PERIOD FRAMEWORK IN REAL TERMS

- Depending on application, may be useful to work with model (independent of lifetime vs. sequential approach) in nominal terms or in real terms

$$P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$ LBC in nominal terms (assuming $A_0 = 0$)

↓ Divide by P_1

$$c_1 + \left(\frac{P_2}{P_1(1+i)}\right) c_2 = \frac{Y_1}{P_1} + \frac{Y_2}{P_1(1+i)}$$

↓ Multiply *and* divide last term on right-hand-side by P_2

$$c_1 + \left(\frac{P_2}{P_1(1+i)}\right) c_2 = \frac{Y_1}{P_1} + \left(\frac{P_2}{P_1(1+i)}\right)\frac{Y_2}{P_2}$$

↓ Use definitions: $y_1 = Y_1/P_1$, $y_2 = Y_2/P_2$, and $1+r_2 = P_2/P_1$

$$c_1 + \left(\frac{1+\pi_2}{1+i}\right) c_2 = y_1 + \left(\frac{1+\pi_2}{1+i}\right) y_2$$

↓ Use Fisher equation: $(1+r_2)/(1+i) = 1/(1+r)$

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$ LBC in real terms (assuming $A_0 = 0$)

Maximize $u(c_1,c_2)$ subject to the real LBC → identical consumption-savings optimality condition

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Sequential formulation highlights the role of net wealth (A_1) between period 1 and period 2**
  - Accords better with the explicit timing of economic events than the lifetime approach…
  - …but yields the same result
  - Advantage: allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory in Chapter 8)
- Apply Lagrange tools to consumption savings optimization
- Objective function: $u(c_1,c_2)$
- Constraints:
  - Period 1 budget constraint: $Y_1 + (1+i)A_0 - P_1 c_1 - A_1 = 0$
  - Period 2 budget constraint: $Y_2 + (1+i)A_1 - P_2 c_2 - A_2 = 0$

  } *TWO* constraints
- Sequential Lagrange formulation requires two multipliers
  - See Math Refresher, Chapter -1
- Can apply sequential analysis in either nominal or real terms

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1:** Construct Lagrange function

$$L(c_1, c_2, A_1, \lambda_1, \lambda_2) = u(c_1, c_2) + \lambda_1 \left[Y_1 + (1+i)A_0 - P_1 c_1 - A_1 \right] + \lambda_2 \left[Y_2 + (1+i)A_1 - P_2 c_2 - A_2 \right]$$

- **Step 2:** Compute FOCs with respect to c_1, c_2, A_1, λ_1, λ_2

(FOC on A_1 will be the key to asset pricing in Chapter 8)

- **Step 3:** Combine (with focus on eliminating multipliers)

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using Fisher equation

Recall: real interest rate measures the *price* of period-1 consumption in terms of period-2 consumption

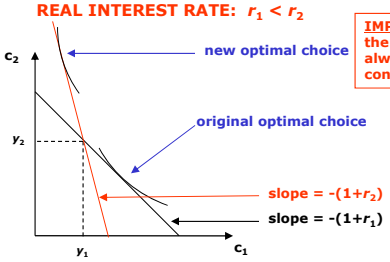
r the most important price for macroeconomic analysis

- **Identical to result of lifetime formulation**

MICRO-LEVEL SAVINGS

How do micro-level consumption/savings choices change as the real interest rate changes (continue assuming $A_0 = 0$ for simplicity)?

REAL INTEREST RATE: $r_1 < r_2$



IMPORTANT: LBC *pivots* around the point (y_1, y_2) because (y_1, y_2) is always a *possible* choice of consumption

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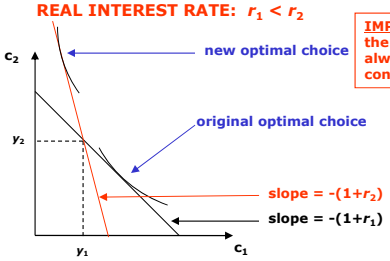

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RESULT: optimal choice of c_1 *decreases* as r rises → optimal choice of *savings* ($= y_1 - c_1$) *increases* as r rises

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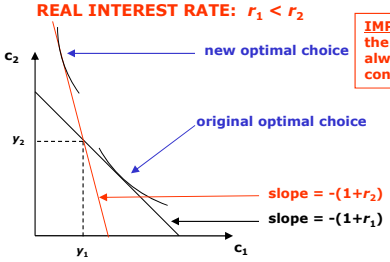

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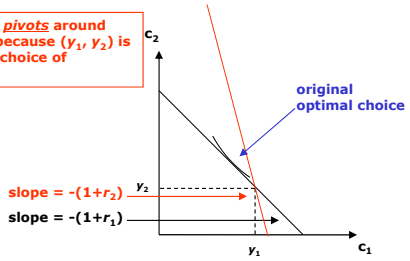

RESULT: optimal choice of c_1 *decreases* as r rises → optimal choice of *savings* (= $y_1 - c_1$) *increases* as r rises

Optimal choice of c_1 increases or decreases?

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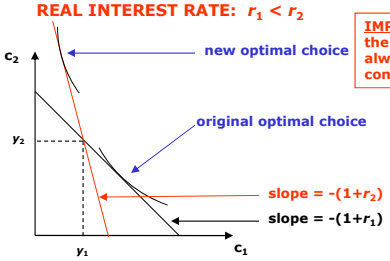

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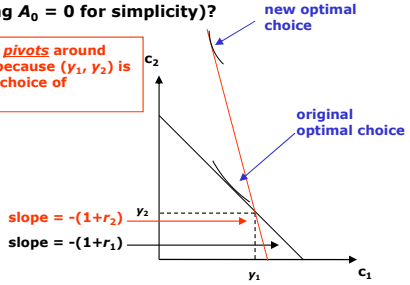

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Empirical evidence shows that when r rises, period-1 (i.e., "short-run") *consumption* of all types of consumers decreases

→ **implying that when r rises, period-1 (i.e., "short-run") *savings* of all types of consumers increases…**

MICRO-LEVEL SAVINGS

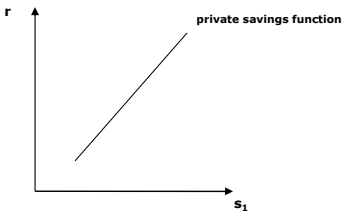
- **Define private savings function (in period 1) for an individual**

(Recall alternative (equivalent) definition: savings is the change in wealth during a period)

$$s_1^{priv}(r, y_1, y_2) = y_1 - c_1(r, y_1, y_2)$$

(+ for y_1, - for c_1)

Emphasizing functional relationships



SAVINGS

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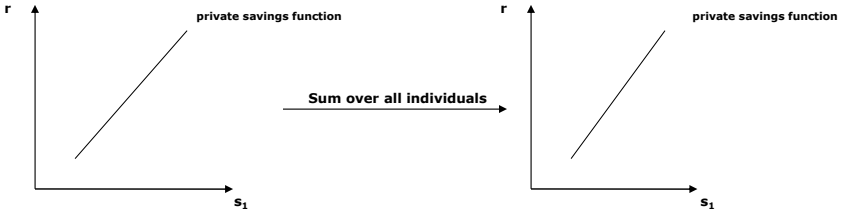

Individual-level savings function

Aggregate-level savings function

(No tension between the micro and macro as there is for labor supply)

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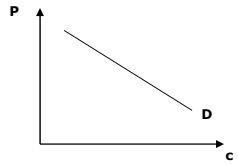

- **Labor Markets**
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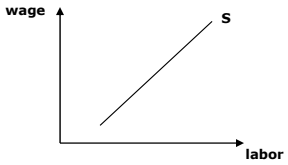

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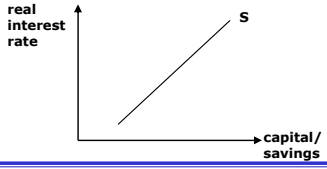


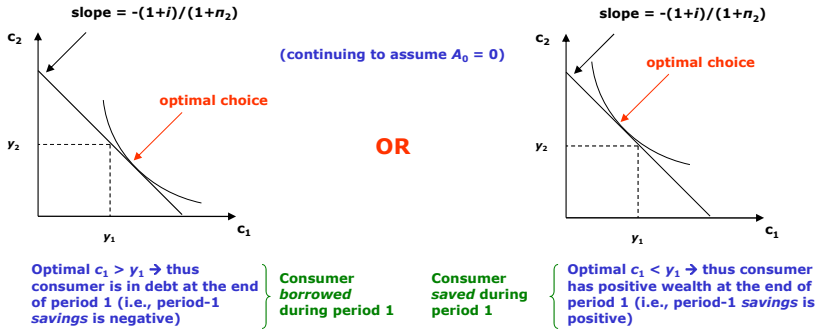
ASSESSING SOME EFFECTS OF THE "CREDIT CRUNCH"

FEBRUARY 13, 2012

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH

- "Credit crunch" – financial sector has restricted the quantity of loans it is willing to extend to consumers in the "short run"
  - Financial market data and bank surveys show quantity of loans made to consumers (car loans, home loans, personal loans, etc.) shrunk dramatically starting in 2008
- Can analyze macroeconomic consequences of shrinkage of credit availability using two-period model
  - Interpret "short run" to be period 1 (i.e., 2008-2011)

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



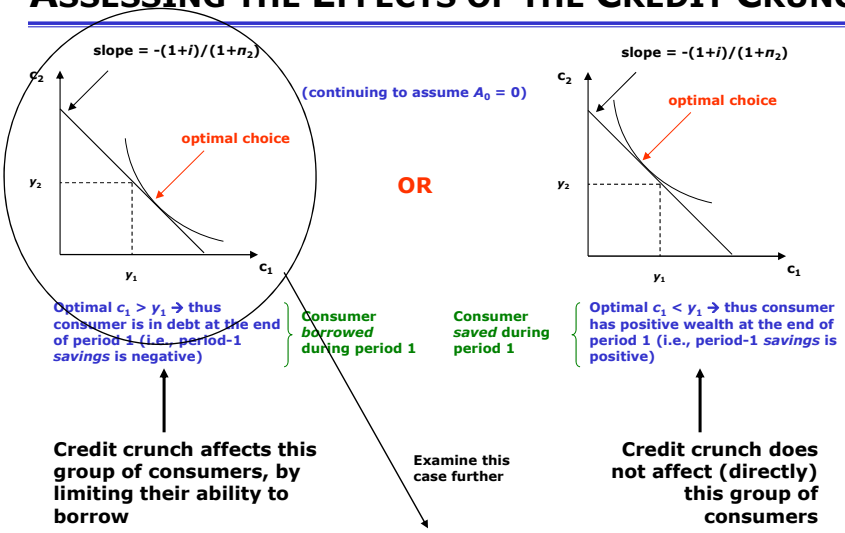
Optimal $c_1 > y_1$ → thus consumer is in debt at the end of period 1 (i.e., period-1 *savings* is negative)

Consumer *borrowed* during period 1

Consumer *saved* during period 1

Optimal $c_1 < y_1$ → thus consumer has positive wealth at the end of period 1 (i.e., period-1 *savings* is positive)

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



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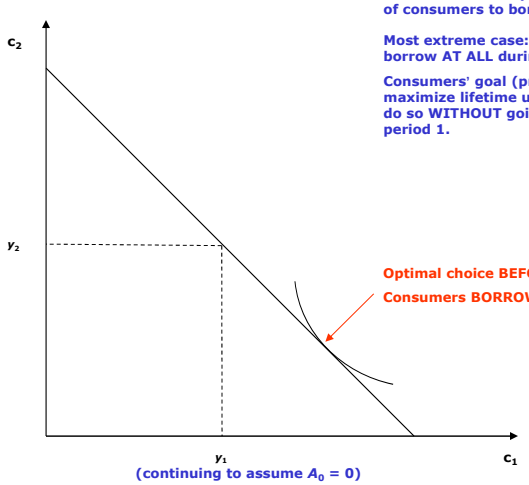


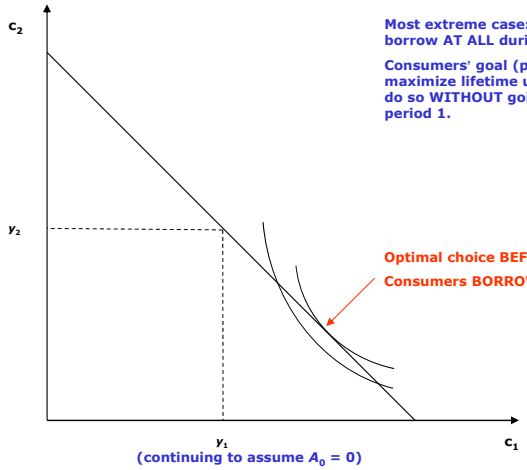
"Credit crunch" in period 1 limits the ability of consumers to borrow during period 1.

Most extreme case: consumers cannot borrow AT ALL during period 1.

Consumers' goal (presumably…) is STILL to maximize lifetime utility…but now they must do so WITHOUT going into debt at the end of period 1.

Optimal choice BEFORE credit crunch:
Consumers BORROWING during period 1

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



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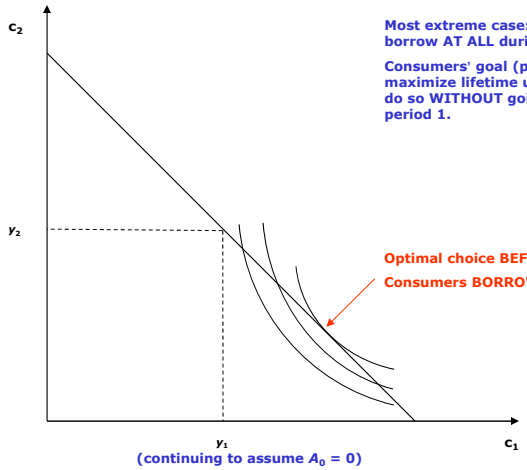
(continuing to assume $A_0 = 0$)

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



"Credit crunch" in period 1 limits the ability of consumers to borrow during period 1.

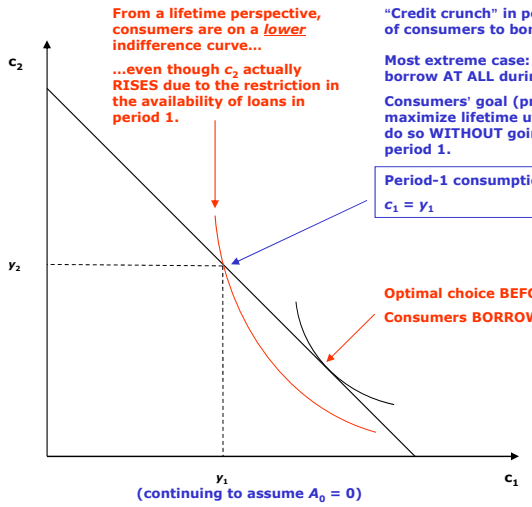
Most extreme case: consumers cannot borrow AT ALL during period 1.

Consumers' goal (presumably…) is STILL to maximize lifetime utility…but now they must do so WITHOUT going into debt at the end of period 1.

Optimal choice BEFORE credit crunch:
Consumers BORROWING during period 1

(continuing to assume $A_0 = 0$)

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



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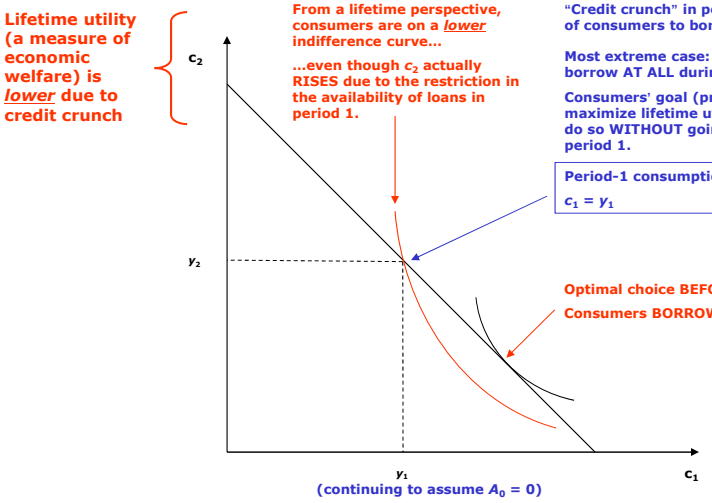

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH

- "Credit crunch" – financial sector has restricted the quantity of loans it is willing to extend to consumers in the "short run"
  - Financial market data and bank surveys show quantity of loans made to consumers (car loans, home loans, personal loans, etc.) shrunk dramatically starting in 2008
- Can analyze macroeconomic consequences of shrinkage of credit availability using two-period model
  - Interpret "short run" to be period 1 (i.e., 2008-2011)
- Consequences
  - A large fraction of consumers (though not all) unable to borrow to pay for their desired period-1 consumption → their period-1 (i.e., "short run") consumption falls
  - Consumption ≈ 2/3 of GDP → period-1 (i.e., "short run") GDP falls

GOVERNMENT AND FISCAL POLICY IN THE TWO-PERIOD FRAMEWORK

FEBRUARY 13, 2012

A Dynamic Model of the Government

- So far only consumers in the two-period framework
- Introduce government in very simple form
  - Exists for both periods
  - Has spending in each period it needs to finance – can be financed via
    - Taxes
    - Issuing government debt/assets

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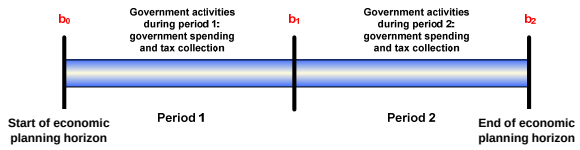

- Notation
  - g_1: real government spending in period 1
  - g_2: real government spending in period 2
  - b_0: government asset position at beginning of period 1/end of period 0
  - b_1: government asset position at beginning of period 2/end of period 1
  - b_2: government asset position at beginning of period 3/end of period 2
  - r: real interest rate between periods

A Dynamic Model of the Government

- Utility function of the government
- ?
- Mainstream macroeconomics generally leaves this to political economy and political science
- One view to adopt: government's utility function is identical to that of the average consumer
- But not necessarily the only or even correct view to adopt

A Dynamic Model of the Government

- Economic activities/actions described by period budget constraints
  - Period-1 government budget constraint

$$\underbrace{g_1 + b_1}_{\text{Total expenditure in period 1:}} = \underbrace{(1+r)b_0 + t_1}_{\text{Total income in period 1:}}$$

Total expenditure in period 1: period-1 spending + wealth to *carry into period* 2

Total income in period 1: period-1 tax collections + income from wealth *carried into period* 1 (inclusive of interest)

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Total income in period 1: period-1 tax collections + income from wealth *carried into period* 1 (inclusive of interest)

  - Period-2 government budget constraint

$$\underbrace{g_2 + b_2}_{} = \underbrace{(1+r)b_1 + t_2}_{}$$

Total expenditure in period 2: period-2 spending + wealth to *carry into period* 3

Total income in period 2: period-2 tax collections + income from wealth *carried into period* 2 (inclusive of interest)

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$$\underbrace{g_1 + b_1}_{} = \underbrace{(1+r)b_0 + t_1}_{} \quad \text{can rewrite as} \quad g_1 + \underbrace{b_1 - b_0}_{} = t_1 + rb_0$$

Savings **during period 1 (a flow)** — **Asset income during period 1 (a flow)**

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Savings during period 2 (a flow)

Asset income during period 2 (a flow)

- Definition: A government's **savings** during a given period is the ***change*** **in its wealth** during that period
  - "Fiscal surplus" if government savings is positive
  - "Fiscal deficit" if government savings is negative

Surplus/deficit is a *flow* measure

Government Budget Constraint(s)

- Adopt a **lifetime** view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 government budget constraint $g_1 + b_1 = t_1 + (1+r)b_0$
  - Period-2 government budget constraint $g_2 + b_2 = t_2 + (1+r)b_1$

Asset position at end of period 1/beginning of period 2 the key link

Assume = 0 (no defaults + strictly increasing "utility")

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  - Combine into lifetime budget constraint (LBC)
    - Solve period-2 budget constraint for b_1...
    - ...and substitute into period-1 budget constraint

$$\underbrace{g_1 + \frac{g_2}{1+r}}_{\text{Present discounted value (PDV) of all lifetime government expenditure}} = \underbrace{t_1 + \frac{t_2}{1+r}}_{\text{Present discounted value (PDV) of all lifetime government income}} + (1+r)b_0$$

IMPORTANT: Government must balance its budget over its *lifetime*, not necessarily in each period

Model Structure

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IMPORTANT: Government must balance its budget over its *lifetime*, not necessarily in each period

For graphical simplicity, will often assume $b_0 = 0$ (i.e., government begins life with zero net wealth).

Note this is a *different* assumption than $b_2 = 0$.

CONSUMER BUDGET CONSTRAINT(S)

- **Introduce tax payments into consumer side of framework**
  - **All in real terms for simplicity – can cast in nominal terms by multiplying by *P***
  - **Period-1 budget constraint** $c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$
  - **Period-2 budget constraint** $c_2 + t_2 + a_2 - a_1 = y_2 + ra_1$

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ECONOMY-WIDE RESOURCE FRONTIER

- **Consumer lifetime budget constraint**

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)a_0$$

- **Government lifetime budget constraint**

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  - **aka "production possibilities frontier" (PPF)**
  - **The GDP accounting equation in two-period form**

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- **Focus here on changes in taxes *t***
  - **Not on (PDV of) government spending *g***
  - **Framework here focuses just on effects of *changes in t* on consumer decisions over time, not effects of changes in *g***
    - **How does the government make its spending decisions?....**

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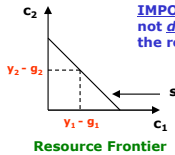

Resource Frontier

IMPORTANT: taxes do not *directly* appear in the resource frontier...

THEOREM (micro theory): If taxes are *lump-sum*, then consumer optimal choices can be analyzed using *either* the consumer LBC *or* the economy-wide resource frontier (superimpose indifference map), and either approach will yield the same predictions.

An important theoretical result for the analysis of tax policy.

National Savings

- **National savings = savings by consumers + savings by government + savings by firms**
  - **No firms in our model (yet..), so** $s_1^{firm} = 0$
  - $s_1^{priv} = y_1 - t_1 - c_1 + ra_0$
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EFFECTS OF TAX POLICY

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    - **t_2 must rise (examine government lifetime budget constraint)**

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Crucial logic

- **Using micro theorem, NO EFFECT ON optimal c_1**
  - **Taxes are lump sum (will define/discuss next time…)**
  - **Economy-wide resource constraint does not depend *directly* on taxes → optimal choice of c_1 unaffected by the change in tax policy**

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Analyzing effects of changes in tax policy on optimal consumption choices is the key

- Policy Experiment: Is national savings affected by a decrease in t_1?
  - Suppose g_1 and g_2 do not change
  - Question 1: Effect on t_2?
    - t_2 must rise (examine government lifetime budget constraint)
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Crucial logic

  - Question 3: Effect of tax changes on period-1 national savings?
    - NONE – because neither g_1 nor c_1 changed

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$s_1^{priv} = y_1 - t_1 - c_1 + ra_0$ ← Rises when t_1 decreases, ***GIVEN*** that we have ***CONCLUDED*** that c_1 does not change

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- **Ricardian Equivalence is to tax theory what perfect competition is to standard economic theory**
  - **Idea relies crucially on lump-sum taxes**