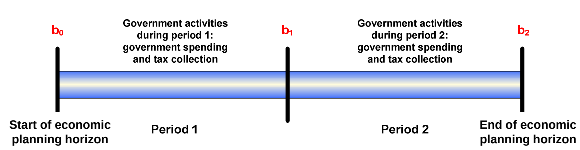
GOVERNMENT AND FISCAL POLICY IN THE TWO-PERIOD FRAMEWORK (CONTINUED)

FEBRUARY 20, 2012

A DYNAMIC MODEL OF THE GOVERNMENT

- So far only consumers in our two-period framework
- Introduce government in very simple form
  - Exists for both periods
  - Has spending in each period it needs to finance – can be financed via
    - Taxes
    - Issuing government debt / assets



- Notation
  - g_1: real government spending in period 1
  - g_2: real government spending in period 2
  - b_0: government asset position at beginning of period 1 / end of period 0
  - b_1: government asset position at beginning of period 2 / end of period 1
  - b_2: government asset position at beginning of period 3 / end of period 2
  - r: real interest rate between periods

Government Budget Constraint(s)

- Adopt a lifetime view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 government budget constraint $g_1 + b_1 = t_1 + (1+r)b_0$
  - Period-2 government budget constraint $g_2 + b_2 = t_2 + (1+r)b_1$

Asset position at end of period 1 / beginning of period 2 the key link

Assume = 0 (no defaults + strictly increasing "utility")

  - Combine into lifetime budget constraint (LBC)
    - Solve period-2 budget constraint for b_1…
    - …and substitute into period-1 budget constraint

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} + (1+r)b_0$$

Present discounted value (PDV) of all lifetime government expenditure

Present discounted value (PDV) of all lifetime government income

IMPORTANT: Government must balance its budget over its *lifetime*, not necessarily in each period

For graphical simplicity, will often assume $b_0 = 0$ (i.e., government begins life with zero net wealth). Note this is a *different* assumption than $b_2 = 0$.

Consumer Budget Constraint(s)

- Introduce tax payments into consumer side of framework
  - All in real terms for simplicity – can cast in nominal terms by multiplying by ***P***
  - Period-1 budget constraint $c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$
  - Period-2 budget constraint $c_2 + t_2 + a_2 - a_1 = y_2 + ra_1$
  - Combine into lifetime budget constraint (LBC)
    - Solve period-2 budget constraint for a_1…
    - …and substitute into period-1 budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)a_0$$

Present discounted value (PDV) of all lifetime expenditure

Present discounted value (PDV) of all lifetime disposable income (i.e., after-tax income)

ECONOMY-WIDE RESOURCE FRONTIER

- Consumer lifetime budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)a_0$$

- Government lifetime budget constraint

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} + (1+r)b_0$$

- Summing the two yields economy-wide resource frontier

$$c_1 + \frac{c_2}{1+r} = y_1 - g_1 + \frac{y_2 - g_2}{1+r} + (1+r)(a_0 + b_0)$$

  - aka "production possibilities frontier" (PPF)
  - The GDP accounting equation in two-period form

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- Focus here on changes in taxes *t*
  - Not on (PDV of) government spending *g*
  - Framework here focuses just on effects of ***changes in t*** on consumer decisions over time, not effects of changes in *g*
    - How does the government make its spending decisions?....

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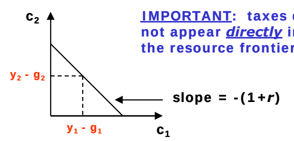
Suppose = 0 for graphical simplicity

  - **aka "production possibilities frontier" (PPF)**
  - **The GDP accounting equation in two-period form**



Resource Frontier

IMPORTANT: taxes do not appear *directly* in the resource frontier…

THEOREM (intermediate micro): If taxes are *lump-sum*, then consumer optimal choices can be obtained by analyzing *either* the consumer LBC *or* the economy-wide resource frontier (superimpose indifference map), and either approach will yield the same predictions.

An important theoretical result for the analysis of tax policy.

NATIONAL SAVINGS

- **National savings = savings by consumers + savings by government + savings by firms**
  - **No firms in our model (yet..), so** $s_1^{firm} = 0$
  - $s_1^{priv} = y_1 - t_1 - c_1 + ra_0$
  - $s_1^{govt} = t_1 - g_1 + rb_0$
  - $s_1^{nat} = s_1^{priv} + s_1^{govt} = y_1 - t_1 - c_1 + t_1 - g_1 + r(a_0 + b_0)$

Effects of Tax Policy

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- Policy Experiment: Is national savings affected by a decrease in t_1?
  - Suppose g_1 and g_2 do not change
  - Question 1: Effect on t_2?
    - t_2 must rise (examine government lifetime budget constraint)
  - Question 2: Effect of tax changes on consumers' optimal choice of period-1 consumption?
    - Crucial logic: Using micro theorem, NO EFFECT ON optimal c_1
      - Taxes are lump sum (will define / discuss soon…)
      - Economy-wide resource constraint does not depend ***directly*** on taxes → optimal choice of c_1 unaffected by the change in tax policy

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Analyzing effects of changes in tax policy on optimal consumption choices is the key

- Policy Experiment: Is national savings affected by a decrease in t_1?
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  - Question 3: Effect of tax changes on period-1 national savings?
    - NONE – because neither g_1 nor c_1 changed

RICARDIAN EQUIVALENCE

- **Ricardian Equivalence Theorem: For a given present discounted value of government spending, neither consumption nor national savings is affected by the precise timing of lump-sum taxes**
- **A benchmark result / concept in the theory of macroeconomic policy**
- **Economic Interpretation: Rational consumers understand that a tax cut in short run means a tax increase in the future (because PDV of government spending is unchanged)**
  - **Thus entire tax cut is saved by consumers in order to pay higher taxes in the future**
  - **Private savings and government savings move in exactly offsetting ways**

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- **Analytically: key is that fiscal policy does not affect *real* i.r.**
- **Ricardian Equivalence is to tax theory as perfect competition is to basic economic theory**
  - **Prediction relies crucially on lump-sum taxes**

NATURE OF TAXATION

- **Lump-Sum Tax**
  - **A tax whose total incidence (i.e., total amount paid) does not depend in any way on any economic decisions/choices an individual makes**
  - **Real-world examples: ?...**

- **Taxes in our two-period framework so far**
  - **Lump-sum!** **Total amounts t_1 and t_2 paid by consumer are independent of any of their decisions/choices**

Period-1 budget constraint

$$c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$$

Period-2 budget constraint

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- **Proportional (aka distortionary) Tax**
  - **A tax whose total incidence depends on economic decisions/choices an individual makes**
  - **In simple two-period framework: consumers only make consumption choices c_1 and c_2**

τ is consumption tax *rate* (aka sales tax rate)

Period-1 budget constraint

$$(1+\tau_1)c_1 + a_1 - a_0 = y_1 + ra_0$$

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- **Combine into consumer LBC**

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Fiscal Policy and National Savings

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- **Slope is** $-\left(\frac{1+\tau_1}{1+\tau_2}\right)(1+r)$
- **Non-lump-sum taxes: optimal consumption choices must be determined using consumer LBC, not economy's resource frontier (i.e., intermediate micro theorem does not apply)**
- **Changes in tax *rates* do affect optimal consumption choices because they change slope ("effective *real* i.r!") of consumer LBC**

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- **Ricardian Equivalence Theorem does not apply**
  - **Changes in tax rates *do* affect national savings**

RICARDIAN EQUIVALENCE?

- **So why the fascination with Ricardian Equivalence?**
- **A benchmark result/concept in the theory of macroeconomic policy**
  - **Effects of actual policy proposals can be compared to the Ricardian Equivalence benchmark**
  - **In practice, *does* seem like tax rebates are sometimes saved**

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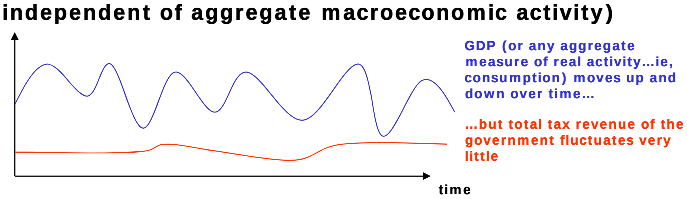

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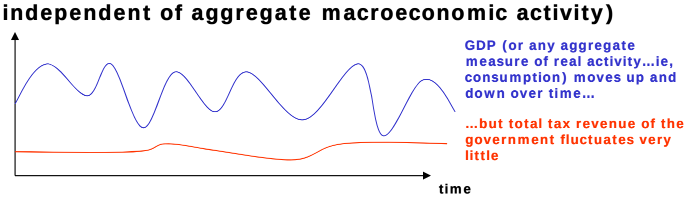


- **Ricardian Equivalence**
  - **Is a theoretical benchmark**
  - **Is an empirical benchmark**

Ricardian Equivalence is about the (lack of) effects of *<u>changes in tax policy</u>*, holding total government liabilities fixed. If g_1 and / or g_2 change, Ric. Equiv. does *<u>not</u>* apply.

INFINITE-PERIOD CONSUMER ANALYSIS

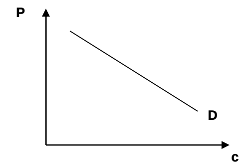
FEBRUARY 20, 2012

BASICS

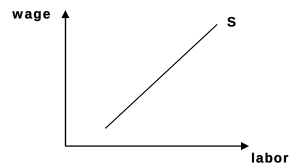
- Modern workhorse macroeconomic models feature an infinite number of periods
  - A more realistic (?) view of time
- Especially useful for thinking about asset accumulation and asset pricing
  - The intersection of modern macro theory and modern finance theory
- Here, assume just one real asset
  - Call it a "stock" – i.e., a share in the S&P 500
  - (In Chapter 14, two nominal assets: bonds and money)
- Index time periods by arbitrary indexes t, $t+1$, $t+2$, etc.
  - Important: all analysis conducted from the perspective of the very beginning of period t...
  - ...so an "infinite future" (period $t+1$, period, $t+2$, period $t+3$, ...) for which to save

THE THREE MACRO (AGGREGATE) MARKETS

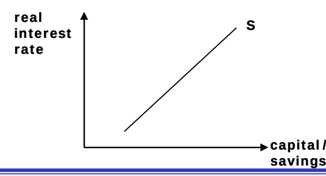
- **Goods Markets**
  - **Demand derived from C-L framework**



- **Labor Markets**
  - **Supply derived from C-L framework**



- **Capital / Savings / Funds / Asset Markets (aka Financial Markets)**
  - **Supply derived from C-S framework**



THE THREE MACRO (AGGREGATE) MARKETS

Many **different** ***types*** **of assets exist…hence many different** ***types of financial markets***

1. **Stock markets (Chapter 8)**
2. **Real-estate markets (Problem Set 5)**
3. **Bond markets (Chapter 14)**
4. **Money markets (Chapter 14)**

- **Capital / Savings / Funds / Asset Markets (aka Financial Markets)**
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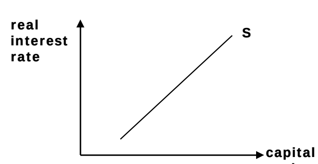
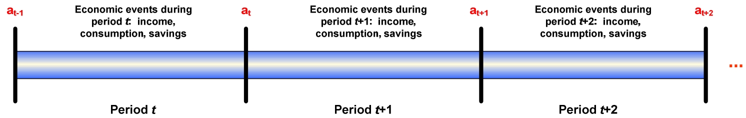


BASICS

- Timeline of events



- Notation
  - c_t: consumption in period t
  - P_t: nominal price of consumption in period t
  - Y_t: nominal income in period t ("falls from the sky")
  - a_{t-1}: real wealth (stock) holdings at beginning of period t / end of period t-1

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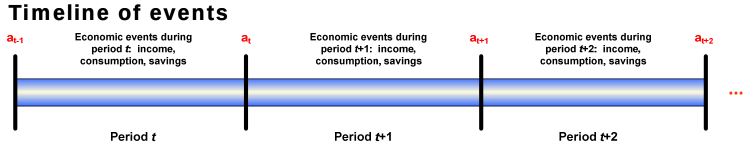

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  - S_t: nominal price of a unit of stock in period t
  - D_t: nominal dividend paid in period t by each unit of stock held at the start of t

The "defining features" of stock

BASICS

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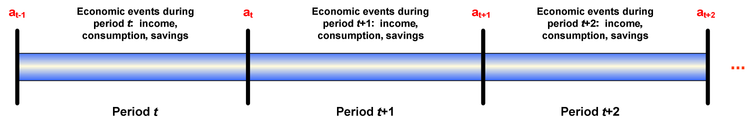

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  The "defining features" of stock

  - π_{t+1}: net inflation rate between period t and period t+1

$$\pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} \left(= \frac{P_{t+1}}{P_t} - 1 \right)$$

  - y_t: real income in period t (= Y_t / P_t)

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BASICS

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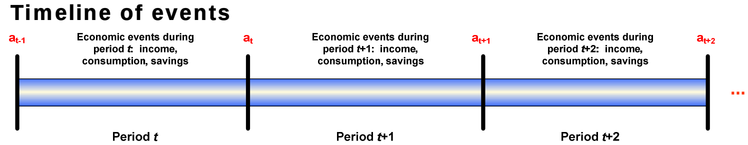

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  The "defining features" of stock

  - π_{t+2}: net inflation rate between period t+1 and period t+2

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Basics

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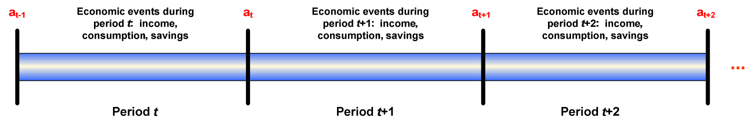

- Notation
  - And so on for period $t+2$, $t+3$, etc…

Subjective Discount Factor

- Infinite number of periods a more serious view of time
- Impatience potentially an issue when taking a serious view of time
- Individuals (i.e., consumers) are impatient
  - All else equal, would rather have experience ***X*** utils today than identical ***X*** utils at some future date
  - An introspective statement about the world
  - An empirical statement about the world

SUBJECTIVE DISCOUNT FACTOR

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- **Subjective discount factor**
  - **A simple model of consumer impatience**
  - **β (a number between zero and one) measures impatience**
    - **The lower is β, the less does individual value future utility**

Macro Fundamentals

SUBJECTIVE DISCOUNT FACTOR

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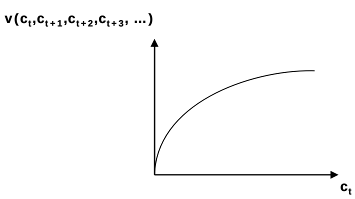
- **Subjective discount factor**
  - **A simple model of consumer impatience**
  - **β (a number between zero and one) measures impatience**
    - **The lower is β, the less does individual value future utility**
  - **Simple assumption about how "impatience" builds up over time**
    - **Multiplicatively: i.e., discount one period ahead by β, discount two periods ahead by β^2, discount three periods ahead by β^3, etc.**
    - **Do individuals' impatience really build up over time in this way?...limited empirical evidence so really don't know…**

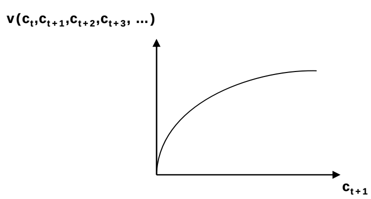
BASICS

- **Building blocks of infinite-horizon consumption-savings framework**
- **Utility**
  - **Describes the benefits of engaging in financial market (and other) activities**
- **Budget constraint**
  - **Describes the costs of engaging in financial market (and other) activities**
- **Utility and budgets two *DISTINCT* concepts**
  - **As in basic consumer analysis (Chapter 1)**
- **Only after describing utility and budgets separately do we bring the two together to obtain predictions from the framework**

UTILITY

- **Preferences $v(c_t, c_{t+1}, c_{t+2}, \ldots)$ with all the "usual properties"**
  - **Lifetime utility function**
  - **Strictly increasing in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots$**
  - **Diminishing marginal utility in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots$**





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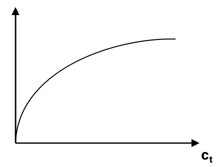
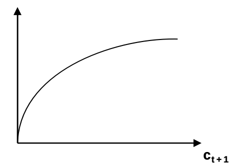
  - Lifetime utility function
  - Strictly increasing in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots$
  - Diminishing marginal utility in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots$





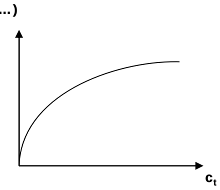
etc.

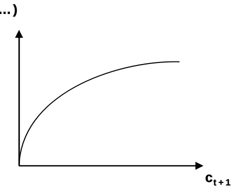
- Lifetime utility function additively-separable across time (a simplifying assumption), starting at time *t*

$$v(c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots) = u(c_t) + \quad u(c_{t+1}) + \quad u(c_{t+2}) + \quad u(c_{t+3}) + \ldots$$

UTILITY

- Preferences $v(c_t, c_{t+1}, c_{t+2}, \ldots)$ with all the "usual properties"
  - Lifetime utility function
  - Strictly increasing in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots$
  - Diminishing marginal utility in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots$





etc.

- Lifetime utility function additively-separable across time (a simplifying assumption), starting at time *t*

$$v(c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots) = u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \ldots$$

- **Utility side of infinite-period model no different than Chapter 1 model – except no longer possible to represent graphically**

BUDGET CONSTRAINT(S)

- Suppose again ***Y*** "falls from the sky"
  - Y_t in period t, Y_{t+1} in period $t+1$, Y_{t+2} in period $t+2$, etc.
- Need infinite budget constraints to describe economic opportunities and possibilities
  - One for each period
  - Period-*t* budget constraint

$$\underbrace{P_t c_t + S_t a_t}_{} = \underbrace{Y_t + S_t a_{t-1} + D_t a_{t-1}}_{}$$

Total expenditure in period *t*: period-*t* consumption + wealth to *carry into period t*+1

Total income in period *t*: period-*t* *Y* + income from stock-holdings *carried into period t* (has value S_t and pays dividend D_t)

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Total income in period *t*: period-*t* *Y* + income from stock-holdings *carried into period t* (has value S_t and pays dividend D_t)

  - Period *t*+1 budget constraint

$$\underbrace{P_{t+1} c_{t+1} + S_{t+1} a_{t+1}}_{} = \underbrace{Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t}_{}$$

Total expenditure in period *t*+1: period-*t*+1 consumption + wealth to *carry into period t*+2

Total income in period *t*+1: period-*t*+1 *Y* + income from stock-holdings *carried into period t*+1 (has value S_{t+1} and pays dividend D_{t+1})

BUDGET CONSTRAINT(S)

- **Suppose again *Y* "falls from the sky"**
  - **Y_t in period *t*, Y_{t+1} in period *t*+1, Y_{t+2} in period *t*+2, etc.**

- **Need infinite budget constraints to describe economic opportunities and possibilities**
  - **One for each period**
  - **Period-*t* budget constraint**

$$P_t c_t + S_t a_t = Y_t + S_t a_{t-1} + D_t a_{t-1}$$

can rewrite as

$$P_t c_t + S_t(a_t - a_{t-1}) = Y_t + D_t a_{t-1}$$

Total expenditure in period *t*: period-*t* consumption + wealth to *carry into period t*+1

Total income in period *t*: period-*t* *Y* + income from stock-holdings *carried into period t* (has value S_t and pays dividend D_t)

Savings during period *t* (a flow)

Dividend income during period *t* (a flow)

  - **Period *t*+1 budget constraint**

$$P_{t+1} c_{t+1} + S_{t+1} a_{t+1} = Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t$$

can rewrite as

$$P_{t+1} c_{t+1} + S_{t+1}(a_{t+1} - a_t) = Y_{t+1} + D_{t+1} a_t$$

Total expenditure in period *t*+1: period-*t*+1 consumption + wealth to *carry into period t*+2

Total income in period *t*+1: period-*t*+1 *Y* + income from stock-holdings *carried into period t*+1 (has value S_{t+1} and pays dividend D_{t+1})

Savings during period *t*+1 (a flow)

Dividend income during period *t*+1 (a flow)

And identical-looking budget constraints for *t*+2, *t*+3, *t*+4, etc…

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Sequential formulation highlights the role of stock holdings (a_t) between period *t* and period *t*+1**
  - **Accords better with the explicit timing of economic events than the lifetime approach…**
  - **…but yields the same result**
  - **Advantage: allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory)**

- **Apply Lagrange tools to consumption-savings optimization**
- **Objective function: $v(c_t, c_{t+1}, c_{t+2}, \ldots)$**
- **Constraints:**
  - **Period-*t* budget constraint:** $Y_t + S_t a_{t-1} + D_t a_{t-1} - P_t c_t - S_t a_t = 0$
  - **Period-*t*+1 budget constraint:** $Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} = 0$
  - **Period-*t*+2 budget constraint:** $Y_{t+2} + S_{t+2} a_{t+1} + D_{t+2} a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2} = 0$
  - **etc…**

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Sequential formulation highlights the role of stock holdings (a_t) between period t and period $t+1$**
  - **Accords better with the explicit timing of economic events than the lifetime approach…**
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- **Apply Lagrange tools to consumption-savings optimization**
- **Objective function: $v(c_t, c_{t+1}, c_{t+2}, \ldots)$**
- **Constraints:**
  - **Period-t budget constraint:** $Y_t + S_t a_{t-1} + D_t a_{t-1} - P_t c_t - S_t a_t = 0$
  - **Period-$t+1$ budget constraint:** $Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} = 0$
  - **Period-$t+2$ budget constraint:** $Y_{t+2} + S_{t+2} a_{t+1} + D_{t+2} a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2} = 0$
  - **etc…**

INFINITE constraints

- **Sequential Lagrange formulation requires infinite multipliers**

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1: Construct Lagrange function (starting from t)**

$$u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \ldots$$

First the lifetime utility function….

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1: Construct Lagrange function (starting from *t*)**

$$u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \ldots$$
$$+\lambda_t \left[Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t \right]$$

First the lifetime utility function….

…then the period *t* constraint…

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

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$$u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \ldots$$
$$+\lambda_t \left[Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t \right]$$
$$+\beta\lambda_{t+1} \left[Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} \right]$$

First the lifetime utility function….

…then the period *t* constraint…

…then the period *t*+1 constraint…

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1: Construct Lagrange function (starting from *t*)**

$$u(c_t)+\beta u(c_{t+1})+\beta^2 u(c_{t+2})+\beta^3 u(c_{t+3})+\ldots$$ First the lifetime utility function....
$$+\lambda_t\left[Y_t+(S_t+D_t)a_{t-1}-P_t c_t-S_t a_t\right]$$ ...then the period *t* constraint...
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$$+\beta^3\lambda_{t+3}\left[Y_{t+3}+(S_{t+3}+D_{t+3})a_{t+2}-P_{t+3}c_{t+3}-S_{t+3}a_{t+3}\right]$$ ...then the period *t*+3 constraint...
$$+\ldots$$ Infinite number of terms

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1: Construct Lagrange function (starting from *t*)**

IMPORTANT: Discount factor β multiplies ***both*** future utility ***and*** future budget constraints

Everything (utility ***and*** income) about the future is discounted

$$u(c_t)+\beta u(c_{t+1})+\beta^2 u(c_{t+2})+\beta^3 u(c_{t+3})+\ldots$$ First the lifetime utility function....
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$$+\beta\lambda_{t+1}\left[Y_{t+1}+(S_{t+1}+D_{t+1})a_t-P_{t+1}c_{t+1}-S_{t+1}a_{t+1}\right]$$ ...then the period *t*+1 constraint...

$$+\beta^2\lambda_{t+2}\left[Y_{t+2}+(S_{t+2}+D_{t+2})a_{t+1}-P_{t+2}c_{t+2}-S_{t+2}a_{t+2}\right]$$ ...then the period *t*+2 constraint...

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$$+\ldots$$ Infinite number of terms

- **Step 2: Compute FOCs with respect to c_t, a_t, c_{t+1}, a_{t+1}, c_{t+2}, …**

Identical except for time subscripts

with respect to c_t:	$u'(c_t)-\lambda_tP_t=0$	Equation 1
with respect to a_t:	$-\lambda_tS_t+\beta\lambda_{t+1}(S_{t+1}+D_{t+1})=0$	Equation 2
with respect to c_{t+1}:	$\cancel{\beta}u'(c_{t+1})-\cancel{\beta}\lambda_{t+1}P_{t+1}=0$	Equation 3

THE BASICS OF ASSET PRICING

$$u'(c_t)-\lambda_tP_t=0$$ Equation 1

$$-\lambda_tS_t+\beta\lambda_{t+1}(S_{t+1}+D_{t+1})=0$$ Equation 2

$$u'(c_{t+1})-\lambda_{t+1}P_{t+1}=0$$ Equation 3

- **Equation 2 →** $$S_t=\left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1}+D_{t+1})$$ BASIC ASSET-PRICING EQUATION

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- **Equation 2 →** $$S_t = \left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1} + D_{t+1})$$ BASIC ASSET-PRICING EQUATION

Period-t stock price = Pricing kernel x Future return

- **Much of finance theory concerned with pricing kernel**
  - **Theoretical properties**
  - **Empirical models of kernels**
- **Pricing kernel where macro theory and finance theory intersect**
  - **Allows studying common "macro factors" that affect "all" asset markets/asset prices**
  - **Chapter 14: link between pricing kernel and bond prices**

THE BASICS OF ASSET PRICING

$$u'(c_t) - \lambda_t P_t = 0$$ Equation 1

$$-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0$$ Equation 2

$$u'(c_{t+1}) - \lambda_{t+1} P_{t+1} = 0$$ Equation 3

- **Equation 2 →** $$S_t = \left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1} + D_{t+1})$$ BASIC ASSET-PRICING EQUATION

Period-t stock price = Pricing kernel x Future return

Two components:
1. Future price of stock
2. Future dividend payment

- **Much of finance theory concerned with pricing kernel**
  - **Theoretical properties**
  - **Empirical models of kernels**
- **Pricing kernel where macro theory and finance theory intersect**
- **Why buy an asset?**
  - **May pay a dividend in the future**
  - **Market value may rise in the future: $S_{t+1}/S_t > 1$ is "capital gain"**

The Basics of Asset Pricing

$$u'(c_t) - \lambda_t P_t = 0 \quad \text{Equation 1}$$
$$-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0 \quad \text{Equation 2}$$
$$u'(c_{t+1}) - \lambda_{t+1} P_{t+1} = 0 \quad \text{Equation 3}$$

- **Equation 2 →** $S_t = \left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1} + D_{t+1})$ BASIC ASSET-PRICING EQUATION

Period-*t* stock price = Pricing kernel x Future return

Two components:
1. Future price of stock
2. Future dividend payment

- **Much of finance theory concerned with pricing kernel**
  - **Theoretical properties**
  - **Empirical models of kernels**
- **Pricing kernel where macro theory and finance theory intersect**
- **To take more macro-centric view**
  - **Solve equations 1 and 3 for λ_t and λ_{t+1}**
  - **Insert in asset-pricing equation**

Macroeconomic Events Affect Asset Prices

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1} + D_{t+1})\left(\frac{P_t}{P_{t+1}}\right)$$

↓ Using definition of inflation: $1+\pi_{t+1} = P_{t+1} / P_t$

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1} + D_{t+1})\left(\frac{1}{1+\pi_{t+1}}\right)$$

- **Consumption growth rate affects stock prices**
  - **Fluctuations in aggregate growth rate impact S_t**
- **Inflation affects stock prices**
  - **Fluctuations over time in inflation impact S_t**

MACROECONOMIC EVENTS AFFECT ASSET PRICES

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1}+D_{t+1})\left(\frac{P_t}{P_{t+1}}\right)$$

Using definition of inflation: $1+\pi_{t+1} = P_{t+1} / P_t$

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1}+D_{t+1})\left(\frac{1}{1+\pi_{t+1}}\right)$$

VIEW AS A CONSUMPTION-SAVINGS OPTIMALITY CONDITION →

- **Consumption growth rate affects stock prices**
  - **Fluctuations in aggregate growth rate impact S_t**
- **Inflation affects stock prices**
  - **Fluctuations over time in inflation impact S_t**
- **ANY factor (monetary policy, fiscal policy, globalization, etc.) that affects inflation and GDP in principle impacts stock/asset markets**
- **Direction of causality?...**

CONSUMER OPTIMIZATION

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1}+D_{t+1})\left(\frac{P_t}{P_{t+1}}\right)$$

Move $u'(c_t)$ and $\beta u'(c_{t+1})$ terms to left-hand-side, and S_t to right-hand-side

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \left(\frac{S_{t+1}+D_{t+1}}{S_t}\right)\left(\frac{1}{1+\pi_{t+1}}\right)$$

i.e., ratio of marginal utilities → MRS between period t consumption and period $t+1$ consumption

Some sort of price ratio...

Consumer Optimization

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1} + D_{t+1})\left(\frac{P_t}{P_{t+1}}\right)$$

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CONSUMPTION-SAVINGS OPTIMALITY CONDITION

i.e., ratio of marginal utilities → MRS between period t consumption and period $t+1$ consumption

Analogy with Chapters 3 & 4: must be $(1+r_t)$ ← Recall real interest rate is a *price*

- Recover Chapter 3 & 4 framework by setting t =1 and β = 1

Consumption-Savings View

Consumer Optimization

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1} + D_{t+1})\left(\frac{P_t}{P_{t+1}}\right)$$

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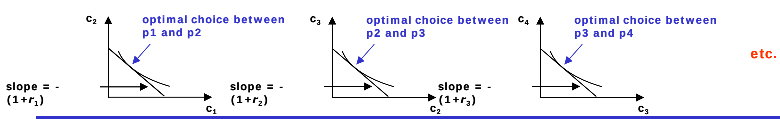
Analogy with Chapters 3 & 4: must be $(1+r_t)$ ← Recall real interest rate is a *price*

- Recover Chapter 3 & 4 framework by setting t =1 and β = 1
- Infinite-period framework is sequence of overlapping two-period frameworks



A LONG-RUN THEORY OF MACRO

- Consumption-savings optimality condition at the heart of modern macro models
  - Emphasize the dynamic nature of aggregate economic events
  - Foundation for understanding the periodic ups and downs ("business cycles") of the economy
  - (Chapter 13: business cycle theories)

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = 1 + r_t$$

NEXT: Impose "steady state" and examine long-run relationship between interest rates and consumer impatience

$$\frac{1}{\beta} = 1 + r$$